CHAOTIC BEHAVIOR OF CHARGE VARYING DUST GRAINS IN PLASMAS

Yoshifumi Saitou and Tadao Honzawa

Department of Electrical and Electronic Engineering,
Utsunomiya University, Utsunomiya, Tochigi 321-8585, Japan

1. Introduction

Researches on plasma-dust grain systems are developing new research fields in plasma physics [1] - [7]. The systems also attract our attention because of their wide range of applicable fields. In particular, they are investigated in relations to the process and the space plasmas. Furthermore, in basic plasma physics, investigations on new wave modes [8] and on self-organizations of the dust grains [9] are very interesting subjects. It is noticeable that the dust grain has a heavier mass than that of the usual gaseous plasma particle and that its charge is determined by balance of the capture and emission of charged particles. The above fact suggests that the dust grain can have a charge which is temporally varying.

In this paper, we report derivation of an equation which describes a density fluctuation of dust grains in a simplified plasma-dust grain system. The derived equation is numerically analyzed using the fourth-order Runge-Kutta method. We show by estimating the Lyapnov exponent that a solution of the equation possibly becomes chaotic. In addition, we discuss by means of algebraic analysis why the system becomes chaotic.

2. Derivation of Basic Equation

Consider a plasma-dust grain system which is spatially one-dimensional and has no external electric and magnetic fields. Dust grains, whose masses are much heavier than the ion and the electron ones, are at rest on average. Plasma ions and electrons surrounding the dust grains have sufficiently large densities in comparison with those of the dust grains, and as a result, the ion and the electron densities do not fluctuate during interactions between plasma particles or between the plasma particles and the dust grains. The charge neutrality is always satisfied. The charge of each dust grain, q, is a time dependent variable and is much larger than that of the ion or the electron, so that q continuously changes with time.

Equations of motion and of continuity with a source term for the dust grains and Poisson's equation are given as follows:

$$m_d n_0 \frac{\partial v}{\partial t} = q n_0 E, \tag{1}$$

$$\frac{\partial n}{\partial t} + n_0 \frac{\partial v}{\partial x} = \alpha n - \frac{1}{3}\beta n^3, \tag{2}$$

$$\varepsilon_0 \frac{\partial E}{\partial x} = qn, \tag{3}$$

where m_d, n_0, n, and v denote the mass, the unperturbed and perturbed densities and the average velocity of the dust grains, respectively. The coefficient α corresponds to a production rate of the dust grains, and β to their loss rate. Usually, the term n^3 represents a loss by the three body recombination such as $X^+ + e^- + Z \rightarrow X^* + Z$, where X and Z are atoms [11]. Assuming that the density fluctuation depends only on time and the dust charge varies temporally as expressed in a relation $q = q_0\,(\delta - \varepsilon \cos \omega t)^{1/2}$ and replacing n by x, we obtain from Eqs. (1) $\sim$ (3) a basic equation as follows:

$$\frac{d^2x}{dt^2} - (\alpha - \beta x^2)\frac{dx}{dt} + x\omega_0^2(\delta - \varepsilon \cos \omega t) = 0, \tag{4}$$

where q_0 is a constant and ω_0 is the plasma frequency corresponding to the dust grains. The second term of the left-hand side of this equation is similar to that of the van der Pol equation, and the third term to the Mathieu one. Henceforth, we call this equation “van der Pol-Mathieu equation": That is, this is considered to be an equation including the van der Pol equation describing the self-excited oscillations, excited by a negative resistance, and the Mathieu equation describing the parametric type oscillations.

3. Results of Numerical Calculations

We solve the van der Pol-Mathieu equation, Eq. (4), using the fourth-order Runge-Kutta method [10]. To do this, we rewrite the equation as a set of two first-order ordinary differential equations:

$$\begin{cases} \dfrac{dx}{dt} &= y, \\ \dfrac{dy}{dt} &= (\alpha - \beta x^2)y - \omega_0^2 x(\delta - \varepsilon \cos \omega t). \end{cases} \tag{5}$$

Here, we use fixed parameters as follows: $\delta = 1.0$, $\varepsilon = 1.0$, $\omega_0 = 1.0$, and $\omega = 1.0$. The parameters α and β are changeable.

Typical orbits in a phase space (x, dx/dt) for some different values of α and β are shown in Figs. 1 and 2. >From these figures, it is found that the shape of the attractor changes with changing α (Fig. 1) and the size also changes with changing β (Fig. 2). In the case of small α, the shape of the attractor is similar to the limit cycle of the stable Mathieu equation. On the other hand, the attractor for large α is similar to the limit cycle of the van der Pol equation. As shown in Figs. 1 and 2, the attractor shows quite complicated behavior if $\alpha = 1.0$. In Fig. 1, we fixed the value of β at 0.1.

4. Discussion

From the above results on the attractors (Fig. 1 and 2) and the power spectra (which are not shown here), it is expected that in the case of $\alpha = 1.0$, the system becomes chaotic. To confirm this, we evaluate the Lyapnov exponent in this case. It is found that the Lyapnov exponent is positive, whose value is $+0.007$, only in the case of $\alpha = 1.0$. This corresponds to the case of chaos. In other cases of α, the Lyapnov exponent is negative.

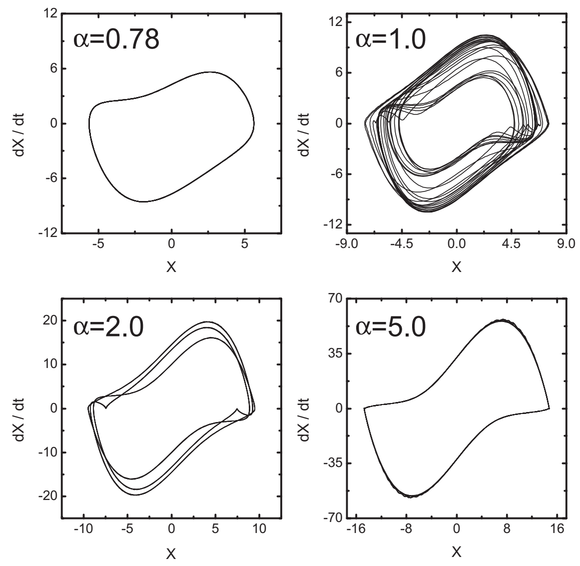

Figure 1. Phase space orbits for $\alpha = 0.78$, 1.0, 2.0, and 5.0, where $\beta = 0.1$.

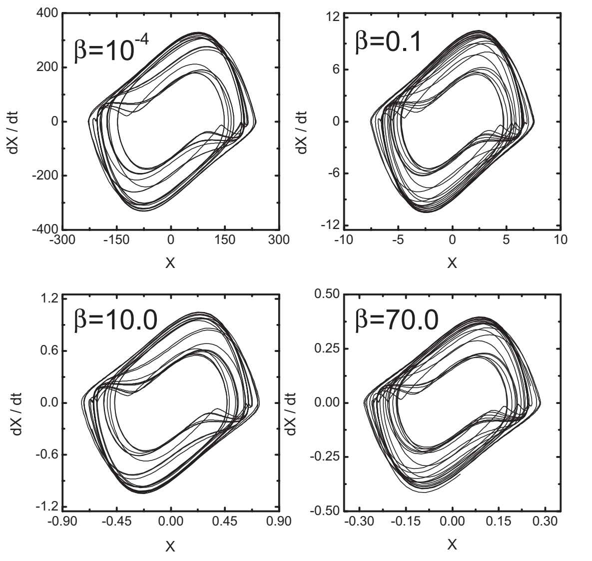

Figure 2. Phase space orbits for $\beta = 10^{-4}$, 0.1, 10.0, and 70.0, where $\alpha = 1.0$.

Now, we consider why the system becomes chaotic. We expand the solution of the van der Pol-Mathieu equation about $t = 0$. If $dx/dt = 0$, then

$$x(t) \simeq x(0)\Big\{1 - \frac{1}{2}\omega_0^2\Big(\delta - \varepsilon\cos\varphi\Big)\Big\}. \tag{6}$$

Notice that we replace the cosine term of Eq. (4) by $\cos(\omega t + \varphi)$. Similarly, the solutions of the van der Pol and the Mathieu equations can be expanded:

$$x(t) \simeq x(0)\Big(1 - \frac{1}{2}\omega_0^2 t^2\Big), \qquad \text{(the van der Pol eq.)} \tag{7}$$

$$x(t) \simeq x(0)\Big\{1 - \frac{1}{2}\omega_0^2\Big(\delta - \varepsilon\cos\varphi\Big)\Big\}. \qquad \text{(the Mathieu eq.)} \tag{8}$$

It is found from Eqs. (6) $\sim$ (8) that the solution of the van der Pol-Mathieu equation behaves similarly to that of the Mathieu one. It is known that the Mathieu equation is unstable when (δ, ε) = (1, 1), [12] which corresponds to the case of Eq. (4). Therefore, the attractor of Eq. (4) tends to expand around $dx/dt = 0$.

On the other hand, in the case of $x = 0$, each solution can be written as follows:

$$x(t) \simeq \frac{dx}{dt}\Big|_{t=0}\Big(t + \frac{1}{2}\alpha t^2\Big), \qquad \text{(the van der Pol-Mathieu eq.)} \tag{9}$$

$$x(t) \simeq \frac{dx}{dt}\Big|_{t=0}\Big(t + \frac{1}{2}\alpha t^2\Big), \qquad \text{(the van der Pol eq.)} \tag{10}$$

$$x(t) \simeq \frac{dx}{dt}\Big|_{t=0} t. \qquad \text{(the Mathieu eq.)} \tag{11}$$

It is clear that the solution of the van der Pol-Mathieu equation behaves similarly to that of the van der Pol one. In addition, it is found that the α-dependence of the attractor mainly comes from this α-dependence. It is known that the solution of the van der Pol equation without an external force converges to the limit cycle and tracks a stable orbit [13]. Therefore, the attractor of Eq. (4) tends to converge to the limit cycle around $x = 0$.

In view of the above reasons, it is considered that the balance between the instability depending on the Mathieu equation and the stability depending on the van der Pol one determines the shape of the attractor of Eq. (4). Consequently, these instability and stability are competitive when $\alpha = 1.0$, and as a result, the system becomes chaotic.

5. Conclusion

We derived the "van der Pol-Mathieu" equation to describe a simplified plasma-dust grain system with temporally varying dust charges. The solution of this equation shows a variety of behaviors under wide range of parameters' changes. Especially, the solution of the equation shows a chaotic behavior only at $\alpha = 1.0$, where the Lyapnov exponent becomes positive. It is our next work to study this equation at other parameters.

Acknowledgment

This work was partly supported by Grant-in-Aid for Encouragement of Young Scientists from the Ministry of Education, Science, Sports, and Culture, Japan.

References

[1] C.B. Dwivedi and B.P. Pandey: Phys. Plasmas **2**, 4134 (1995).

[2] N.N. Rao, P.K. Shukla and M.Y. Yu: Planet. Space Sci. **38**, 543 (1990).

[3] N. D'Angelo: Planet. Space. Sci. **38**, 1143 (1990).

[4] F. Melandsϕ, T.K. Aslaksen, and O. Havens: Planet. Space Sci. **41**, 321 (1993).

[5] M. Rosenberg: Planet. Space Sci. **41**, 229 (1993).

[6] R.K. Varma, P.K. Shukla, and Krishan: Phys. Rev. E **47**, 3612 (1993).

[7] D. Winske, S. Peter Gray, M.E. Jones, M. Rosenberg, V.W. Chow, and D. A. Mendis: Geophys. Res. Lett. **22**, 2069 (1995).

[8] *for example,* A. Barkan, R.L. Merlino and N. D'Angelo: Phys. Plasmas **2**, 3563 (1995).

[9] *for example,* H.M. Thomas and G.E. Morfill: *in* The Physics of Dusty Plasmas. P.K. Shukla et al. eds., World Scientific, NJ, 1996, pp. 199−222.

[10] W.H. Press, S.A. Teukolsky, W.T. Vettering, and B.P. Flannery: Numerical Recipes in C. Cambridge Unisersity Press, NY, 1994, 2nd ed., pp. 710−714.

[11] J. S. Chang, R. M. Hobson, M. Ichikawa, and T. Kaneda: Atomic and Molecular Processes in Ionised Gases. Tokyo Denki University Press, Tokyo, 1989, Chap. 5 pp. 157−204.

[12] M. Toda: Theory of Oscillations. Baifukan, Tokyo, 1985, Chap. 2, pp. 38−39.

[13] M. Toda: *ibid.*, Chap. 3, pp. 61−64.