NONLINEAR PROPAGATION OF LOW FREQUENCY WAVES IN STRONGLY COUPLED DUSTY PLASMAS

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1. Introduction

The collective properties of a dusty plasma in the strongly coupled regime (with $\Gamma = (Z_d e)^2/aT_d$ of the order of 1 or larger, where $Z_d e$ is the charge on the dust grain, a is the intergrain distance and T_d is the dust temperature) is of considerable current interest both theoretically and experimentally. A number of recent laboratory experiments[1] have succesfully demonstrated the formation of 'cystalline' states in dusty plasmas and have also attempted to study the wave propagation characteristics in such plasmas. Past theoretical studies[2] based on linear wave analysis indicate that strong correlations can introduce a number of significant modifications in the propagation characteristics of low frequency waves such as new dispersive corrections, an overall reduction of the frequency and phase velocity and the existence of parameter regions where $\partial\omega/\partial k < 0$. In this paper, we extend this study into the weakly nonlinear regime and investigate the propagation characteristics of finite amplitude wave packets in a strongly coupled dusty plasma which is modelled by a generalised hydrodynamic description[3]. For longitudinal wave propagation we obtain a generalised nonlinear propagation equation which in the limit of plasmas without correlation effects reduces to the well known KdV equation and has solitonic solutions. The additional nonlinearity arising from the correlation effects introduces interesting modifications to the soliton wave solution. When this term is small, we carry out a perturbative analysis and find that the soliton solution continues to exist and propagates with its original amplitude intact but with an enhanced phase velocity. This is confirmed by a numerical solution of the equation.

2. Dusty Plasma Model Equations

To study the dynamics of a strongly correlated dusty plasma we use the so called generalized hydrodynamic model(GH) for the dust component. This phenomenological model provides a simple physical picture of the effects of strong correlations through the introduction of viscoelastic coefficients and is generally valid over a large range of the coupling parameter Γ, all the way from the weakly coupled gaseous phase ($\Gamma \ll 1$) to the strongly coupled liquid state ($1 \ll \Gamma < \Gamma_c$) and may even be used in the supercooled regime (beyond the critical Γ_c for crystallisation) as long as the plasma retains its fluid characteristics. Typically, the viscoelastic coefficients are functions of the coupling parameter Γ. When Γ is small and we are in the weakly coupled plasma regime, these coefficients simply lead to viscous damping of the collective modes. As Γ increases, these very coefficients give restoring force contributions related to elasticity effects. The electrons and ions on the other hand constitute a light fluid in comparison to the dust and can be modelled by Boltzmann distributions. This is justified because when the Γ for dust is large the corresponding coupling parameter for the electrons and ions can still be small due to their higher temperatures and smaller electric charges. Thus our model equations are,

$$n_e = n_{e0} e^{e\phi/T_e} \qquad n_i = n_{i0} e^{-e\phi/T_i} \tag{1}$$

$$\frac{\partial n}{\partial t} + \frac{\partial (nv)}{\partial x} = 0 \tag{2}$$

$$\left[1 + \tau_m \left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)\right] \left[\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right) v - \frac{eZ_d}{m_d}\frac{\partial \phi}{\partial x}\right] = \frac{\frac{4\eta}{3} + \zeta}{n_{d0} m_d} \frac{\partial^2 v}{\partial x^2} \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e(n_e + Z_d n_d - n_i) \tag{4}$$

where $n_{e,i,d}$ are densities of electrons, ions and dust respectively, v is the dust velocity, $T_{e,i}$ are the electron and ion temperatures, ϕ is the electrostatic potential and $Z_d e$ is the electronic charge of the dust. m_d is the dust mass. τ_m is the viscoelastic relaxation time given by

$$\tau_m = \frac{(\frac{4}{3}\eta + \zeta)}{n_{d0} T_d} \; \frac{1}{\left(1 - \gamma_d \mu_d + \frac{4}{15} u\right)} \tag{5}$$

where η, ζ are the bulk and shear modulii of elasticity respectively, T_d is the dust temperature, γ_d is the adiabatic index, μ_d is the compressibility, $u = E_c/n_{d0}T_d$ is the excess energy which is related to the correlation energy E_c. The finite characteristic time for internal relaxations, τ_m, as well as the other transport coefficients such as η, ζ etc. in the GH model are functions of Γ. The functional dependencies are obtained either from direct MD simulations or from various analytic statistical schemes and expressed in terms of analytically fitted formulae. For example, typically for weakly coupled plasmas ($\Gamma < 1$)[4], $u(\Gamma) \approx -\frac{\sqrt{3}}{2}\Gamma^{3/2}$, whereas in the the range of $1 \leq \Gamma \leq 200$, Slattery $et\ al$[5] have given the relation, $u(\Gamma) = -0.89\Gamma + 0.95\Gamma^{1/4} + 0.19\Gamma^{-1/4} - 0.81$.

3. Nonlinear Propagation Equation

The most interesting physical regime, where strong correlation effects lead to novel elastic restoring forces, is the so called 'kinetic regime' characterised by ($\omega\tau_m \gg 1$). We will work in this limit where the equation (3) becomes

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right) \left[\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right) v - \frac{eZ_d}{m_d}\frac{\partial \phi}{\partial x}\right] = \delta \frac{\partial^2 v}{\partial x^2} \tag{6}$$

where $\delta = (T_d/m_d)(1 - \gamma_d\mu_d + (4/15)u(\Gamma))$. We now carry out a systematic 'reductive perturbative' analysis of the model equations, using the stretched variables, $\xi = \epsilon^{1/2}(x - ut)$, $\tau = \epsilon^{3/2}t$ (where u is the phase velocity) and by expanding the quantities n, v and ϕ as

$$\begin{pmatrix} n_d \\ v \\ \phi \end{pmatrix} = \begin{pmatrix} n_{d0} \\ 0 \\ 0 \end{pmatrix} + \sum_{i=1}^{\infty} \epsilon^i \begin{pmatrix} n_d^{(i)} \\ v^{(i)} \\ \phi^{(i)} \end{pmatrix} \tag{7}$$

with the boundary conditions $n_d^{(i)} \to n_{d0}^{(i)}$ ($n_{d0}^{(0)} \equiv n_{d0}$) and $v^{(i)}$, $\phi^{(i)} \to 0$ as $\xi \to \infty$. Then in the lowest order , ϵ^0, one obtains the charge neutrality condition relating the equilibrium densities. In the next order one obtains an expression for the phase velocity, which in this case is,

$$u^2 = \delta + \frac{C_s^2 Z_d^2 n_{d0}}{n_{e0}\left(1 + \frac{T_e}{T_i}\frac{n_{i0}}{n_{e0}}\right)} \tag{8}$$

where $C_s = \sqrt{T_e/m_d}$. Note that for $\delta = 0$ in (8) one recovers the usual expression for the phase velocity of dust acoustic modes. The δ term is a linear correction to the phase speed brought about by the correlation terms. As a function of Γ, μ_d can change sign and this can bring about interesting modifications in the linear propagation characteristics of the dust acoustic mode as discussed in [2]. Finally, for order ϵ^2 we obtain the following nonlinear equation in the single variable $\phi^{(1)}$:

$$\frac{\partial^2\phi^{(1)}}{\partial\tau\partial\xi} + \frac{\partial}{\partial\xi}\left(\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi}\right) + \frac{\partial^4\phi^{(1)}}{\partial\xi^4} = -\alpha\left(\frac{\partial\phi^{(1)}}{\partial\xi}\right)^2 \tag{9}$$

where all quantities have been appropriately normalised to render them dimensionless and $\alpha \approx \delta/C_d^2$ with C_d the dust acoustic speed.

4. Results

Equation (9) describes the nonlinear propagation of low frequency finite amplitude acoustic waves in a strongly coupled dusty plasma. The equation is derived using the standard weak nonlinear approximation for the wave amplitude and the correlations effects described by the additional nonlinear term on the right hand side are representative of viscous/elastic restoring forces in the so called 'kinetic regime' ($\omega\tau_m \gg 1$). Note that in the absence of correlation effects ($\alpha = 0$), equation (9) reduces to the standard KdV equation with a solitonic solution of the form, $\psi(\xi,\tau) = \psi_0^{(0)}\mathrm{sech}^2[(\sqrt{\psi_0^{(0)}/12})(\xi - \psi_0^{(0)}\tau/3)]$. To study the influence of the α term , we assume it to be small and carry out a perturbative solution of (9). Following Karpman and Maslov [6], we employ the ansatz of introducing a slow time variation in the amplitude and providing for an additional time varying phase term,

$$\psi = \psi_0(\tau)\,\mathrm{sech}^2\left[\left(\frac{\psi_0(\tau)}{12}\right)^{1/2}\left(\xi - \frac{\psi_0}{3}\tau - \Theta(\tau)\right)\right] \tag{10}$$

Substituting this solution in (9) and following standard perturbation analysis we find that,

$$\psi_0(\tau) = \text{constant} \equiv \psi_0 \tag{11}$$

and

$$\Theta(\tau) = \frac{4}{7}\,\alpha\,\psi_0\tau \tag{12}$$

Thus for small α the soliton solution continues to exist and propagates with no change in its original amplitude but with an increase in its phase velocity. This rise in its phase velocity is a nonlinear correction from the correlation effects which is in addition to the linear correction of the correlation effects described by equation (8). We have also confirmed this result by a direct numerical solution of equation (9). As the strength of the correlation effects is increased the soliton solution no longer remains an appropriate nonlinear stationary solution of the system and may rapidly disintegrate due to wake field emissions. We propose to investigate this phenomenon by a direct numerical solution of the nonlinear propagation equation for arbitrary values of the correlation strength, in the near future.

References

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