# MIE AND DEBYE SCATTERING IN A DUSTY PLASMA

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A dust immersed in plasma becomes charged, and the number $Z$ of elementary charges attached to its surface can go as high as hundreds or thousands [1]. Therefore, the scattering of electromagnetic waves in a dusty plasma can be strongly enhanced if the electrons in the Debye cloud scatter coherently [2].

Therefore, a clear picture of electromagnetic scattering by charged dusts is important in the context of the study of cosmic dusty plasma [3], particularly in the explanation of some radar measurements of the Earth's mesopause [4]. It can also be important in some laboratory environments, where the process of dust growing is monitored through light scattering [5].

In this work we address the problem of the scattering of electromagnetic waves by the Debye shielding cloud of a charged dust adopting a different approach from previous work [2], which relied upon the Fourier transform of the scattered wave equation and on the Vlasov equation for a dusty plasma. Here we have treated the Debye scattering (DS) problem in parallel to that of Mie scattering (MS), expressing the Debye field in a Mie-like series of spherical vector harmonics [6].

We consider that the Debye shielding cloud around the dust grain is a static pertubation $\tilde{n}$ of the electronic density. It is well known [7] that coupling the Maxwell equations with the fluid equations for the plasma gives the wave equation for the transverse part of the scattered field $\mathbf{E}_T$ as $\nabla \times \nabla \times \mathbf{E}_T - k^2\mathbf{E}_T = i\mu_0\omega\mathbf{J}_T$, where $k = \omega\sqrt{\epsilon(\omega)}/c$, $\epsilon(\omega) = 1 - \omega_{pe}^2/\omega^2$ and $\mathbf{J}_T$ is the transverse part of $\mathbf{J} = (ie^2)/(m\omega)\tilde{n}\mathbf{E}_0$ ($\mathbf{E}_0$ being the incident field), which is the current source for the scattered field, arising from the nonlinear interaction between the incident field and the Debye density perturbation. $\mathbf{E}_T$ is then given by

$$\mathbf{E}_T(\mathbf{r}) = i\omega\mu_0 \int_{V'} \overline{\overline{G}}_0(\mathbf{r},\mathbf{r}) \cdot \mathbf{J}_T(\mathbf{r}')dV', \tag{1}$$

where $\overline{\overline{G}}_0(\mathbf{r},\mathbf{r}')$ is the free Dyadic Green function (DGF) satisfying [8] $\nabla \times \nabla \times \overline{\overline{G}}(\mathbf{r},\mathbf{r}') - k^2\overline{\overline{G}}(\mathbf{r},\mathbf{r}') = \overline{\overline{I}}\delta(\mathbf{r}-\mathbf{r}')$. The total DGF is the sum of the free part and of the scattered DGF. We will neglect the contribution of the latter because it gives a second order (first Debye and then Mie) scattered field. $\overline{\overline{G}}_0(\mathbf{r},\mathbf{r}')$ is given by

$$\overline{\overline{G}}_0(\mathbf{r},\mathbf{r}') = \frac{ik}{4\pi}\sum_{\sigma,m,n} C_{mn} \left\{ \begin{array}{ll} \mathbf{M}^{(1)}(k)\mathbf{M}'(k) + \mathbf{N}^{(1)}(k)\mathbf{N}'(k), & r > r' \\ \mathbf{M}(k)\mathbf{M}'^{(1)}(k) + \mathbf{N}(k)\mathbf{N}'^{(1)}(k), & r < r' \end{array} \right. \tag{2}$$

By $\mathbf{M}(k)$ and $\mathbf{N}(k)$ we denote the spherical vector wave functions (SVWF) $\mathbf{M}_{\sigma mn}(k,\mathbf{r})$ and $\mathbf{N}_{\sigma mn}(k,\mathbf{r})$ (with $\sigma = e, o$ an index for parity) very well known in the literature on MS, together with the longitudinal function $\mathbf{L}_{\sigma mn}(h,\mathbf{r})$ [6]. The prime is assigned to the functions whose argument is $\mathbf{r}'$ and the superscript $(1)$ indicates the spherical Hankel function of the first kind.

The use of (1) requires that the current is expressed in the basis of the SVWF. To do that, we begin by choosing the simplest realistic model for the density perturbation, the Debye potential: $\phi(r) = (\pm Ze/4\pi\epsilon_0 r)\exp[-(r-a)/\lambda_D]$, with $+(-)$ for positive (negative) charge. If the incident field is given by $\mathbf{E}_0 = E_0 e^{-i(\omega t - kz)}\mathbf{u}_x$ (in this paper $\mathbf{u}$ means always unit vector), then we obtain for the current ($e^{-i\omega t}$ omitted)

$$\mathbf{J}(\mathbf{r}) = \mp J\frac{e^{-r/\lambda_D}}{r}e^{ikz}\theta(r-a)\mathbf{u}_x, \qquad J = i\frac{e^2E_0}{8\pi m}\,\frac{Ze^{a/\lambda_D}}{\omega\lambda_D^2}, \tag{3}$$

and $\theta(r-a)$ is the step function introduced to take into account the obvious fact that the electron plasma current only exists in the exterior of the sphere.

Now we make an Ohm-Rayleigh [8] decomposition for the current:

$$J\frac{e^{-r/\lambda_D}}{r}e^{ikz}\theta(r-a)\mathbf{u}_x = \tag{4}$$
$$\int_0^\infty dh \sum_{\sigma,m,n}\left[a_{\sigma mn}(h)\mathbf{M}_{\sigma mn}(h,\mathbf{r}) + b_{\sigma mn}(h)\mathbf{N}_{\sigma mn}(h,\mathbf{r}) + c_{\sigma mn}(h)\mathbf{L}_{\sigma mn}(h,\mathbf{r})\right].$$

Using the standard techniques we obtain the coefficients
$a_{\sigma mn}(h) = 2\delta_{\sigma o}\delta_{m1}Ji^{n+2}(2n+1)h^2 I_1^n(h,k)/\pi n(n+1)$,
$b_{\sigma mn}(h) = 2\delta_{\sigma o}\delta_{m1}Ji^{n+1}(2n+1)h^2 I_2^n(h,k)/\pi n(n+1)$, and
$c_{\sigma mn}(h) = 2\delta_{\sigma o}\delta_{m1}Ji^{n+1}h^2 I_3^n(h,k)/\pi$,
where we have made the definitions $I_1^n(h,k) = \int_a^\infty dr\, re^{-r/\lambda_D}j_n(hr)j_n(kr)$, $I_2^n(h,k) = [(n+1)I_1^{n-1}(h,k) + nI_1^{n+1}(h,k)]/(2n+1)$, and $I_3^n(h,k) = I_1^{n-1}(h,k) - I_1^{n+1}(h,k)$.

Performing the integral $\int_{V'}\overline{\overline{G}}_0(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}_T(\mathbf{r}')dV'$ using the orthogonality relations of the SVWF we obtain the transverse Debye scattered field as (already in the asymptotic form, $kr\to\infty$),

$$\mathbf{E}_T(\mathbf{r}) \overset{kr\to\infty}{\longrightarrow} \mp\frac{1}{2}E_0 r_0\frac{Ze^{a/\lambda_D}}{\lambda_D^2}\,\frac{e^{ikr}}{r}\sum_n\frac{2n+1}{\sqrt{n(n+1)}}\left[I_1^n(k)\mathbf{C}_{1n}^o(\theta,\phi) + I_2^n(k)\mathbf{B}_{1n}^e(\theta,\phi)\right] \tag{5}$$

($-/+$ for $+/-$ charge), where $\mathbf{B}_{1n}^e$ and $\mathbf{C}_{1n}^o$ are the spherical vector harmonic functions [6], $r_0 = e^2/(4\pi\epsilon_0 mc^2)$ is the classical electron radius and we have defined $I_i^n(k) \equiv I_i^n(k,k)$, $i = 1,2,3$.

Turning now to the Rayleigh limit, the $I_1^n(k)$ coefficient can be written in general as $I_1^n(k) = (1/k^2)Q_n(1+(\lambda^2/2\lambda_D^2)) - \int_0^a dr\, re^{-r/\lambda_D}j_n^2(kr)$, where $Q_n(x)$ is the Legendre polynomial of the second kind. The Rayleigh field corresponds to the approximation for $\lambda \gg \lambda_D$. This allows to approximate $Q_n$ for large values of the argument and $j_n$ for small values. The first term of (5) is dominant and we obtain

$$\mathbf{E}_T^D(\mathbf{r}) \overset{k\lambda_D\ll 1}{\longrightarrow} \mp\frac{1}{2}E_0 r_0 Ze^{a/\lambda_D}\Gamma\left(2,\frac{a}{\lambda_D}\right)\frac{e^{ikr}}{r}(\cos\phi\cos\theta\mathbf{u}_\theta - \sin\phi\mathbf{u}_\phi), \tag{6}$$

where $\Gamma(z,\alpha) = \int_\alpha^\infty e^{-t}t^{z-1}dt$. For most cases $a \ll \lambda_D$ and $\Gamma(2,a/\lambda_D) \simeq 1$. As expected, the amplitude of the scattered wave does not depend on $k$ and only very weakly on $\lambda_D$, which is consistent with the fact that the incident wave does not "see" the inner structure of the Debye cloud; the $Z$ electrons on it scatter coherently.

The total cross section is given by

$$\sigma_{TOT} = \frac{2\pi}{k^2} \sum_{n=1}^{\infty} (2n+1) \left( |A_n|^2 + |B_n|^2 \right), \tag{7}$$

with $A_n$ and $B_n$ being the sum of Mie and Debye coefficients. For example, for a metallic sphere we have $A_n = -j_n(ka)/h_n(ka) \pm ir_0 kZ \exp(a/\lambda_D) I_1^n(k)/2\lambda_D^2$ and $B_n = -(\rho j_n(\rho))'/(\rho h_n(\rho))'|_{\rho=ka} \pm ir_0 kZ \exp(a/\lambda_D) I_2^n(k)/2\lambda_D^2$ ($+/-$ for $+/-$ charge). The total cross section can thus be written as a sum of three terms: $\sigma_{TOT} = \sigma_M + \sigma_D \pm \sigma_{MD}$, where $\sigma_M$ is the usual Mie cross section, $\sigma_D$ is due to the cloud electrons only and $\pm\sigma_{MD}$ is an interference term that is positive (negative) for positive (negative) charge of the dust. This introduces a difference in the total cross section between positive and negative dusts, as shown in Fig. 1. The Rayleigh cross section is $\sigma_0 Z^2/4$, where $\sigma_0$ is the Thomsom cross section, which shows again the coherency of the process.

The radiant intensity (dimensions W sr$^{-1}$) is $i(\theta, \phi) = k/(2\mu_0\omega)|\mathbf{E}_{scatt}(\mathbf{r})|^2 r^2$ and the degree of polarisation (for unpolarised incident light) is given by $\alpha = (i_\perp - i_\parallel)/(i_\perp + i_\parallel)$, where $i_\perp$ ($i_\parallel$) is the radiant intensity for incident wave perpendicular (parallel) to the scattering plane (the scattering plane is defined by the angle $\phi$).

The behaviour of the radiant intensities for Debye scattering is shown in Fig. 2. When $\lambda \gg \lambda_D$ the curves are equal to the Rayleigh-Mie case and for $\lambda \sim \lambda_D$ they are stongly forward peaked. Although Both $i_\parallel$ and $i_\perp$ vary with the incident wavelength $\lambda$, the polarization degree does not (for Debye scattering). Therefore, the curve of the variation of $\alpha$ with the scattering angle $\theta$ is always the same for all incident wavelengths, and has the shape presented by the Rayleigh-Mie case (a symmetric, bell shaped curve).

As a conclusion we point that: *i)* the applicability of our results is very general because even if the dust grain is not spherical the Debye cloud is; *ii)* all the calculations can be made with more complicated models for the potential; *iii)* the detection of the cross section minimum can provide information on $\lambda_D$ and $Z$.

## References

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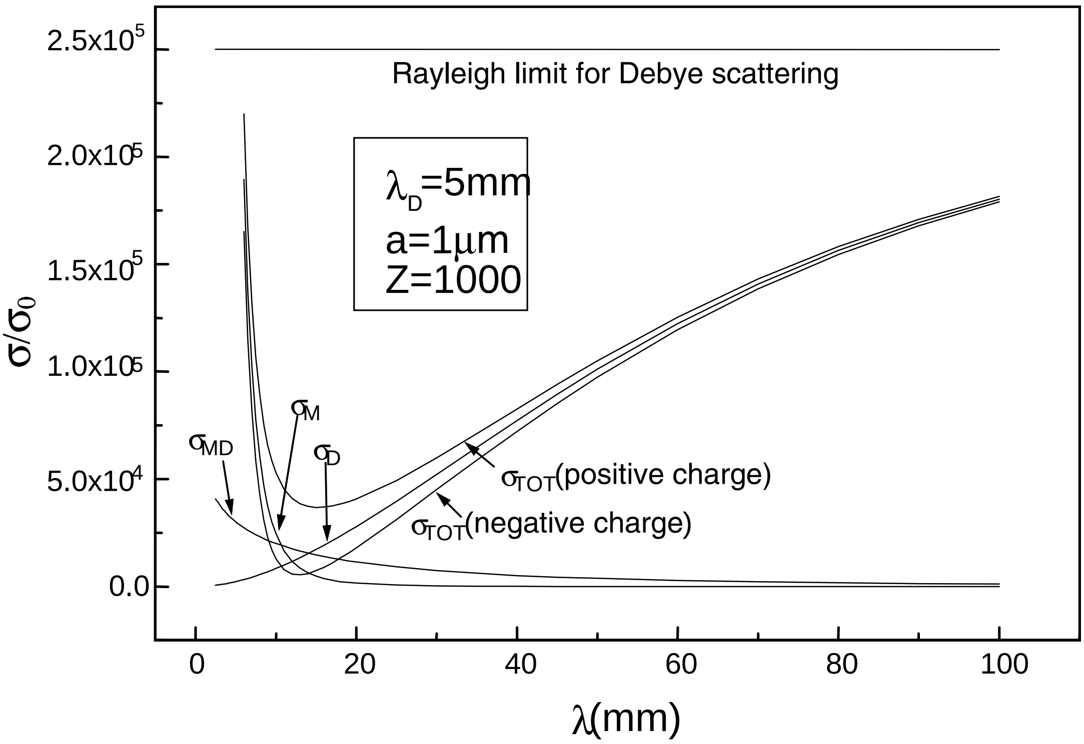


**Figure 1.** Cross sections for Mie and Debye scattering in a typical ionospheric plasma (normalised by the Thomson cross section), as a function of the wavelength. The interference term $\sigma_{MD}$ is also represented and leads to a difference between positively and negatively charged grains in an intermediate regime of $\lambda \sim 10$ mm. The Rayleigh limit for Debye scattering is constant.

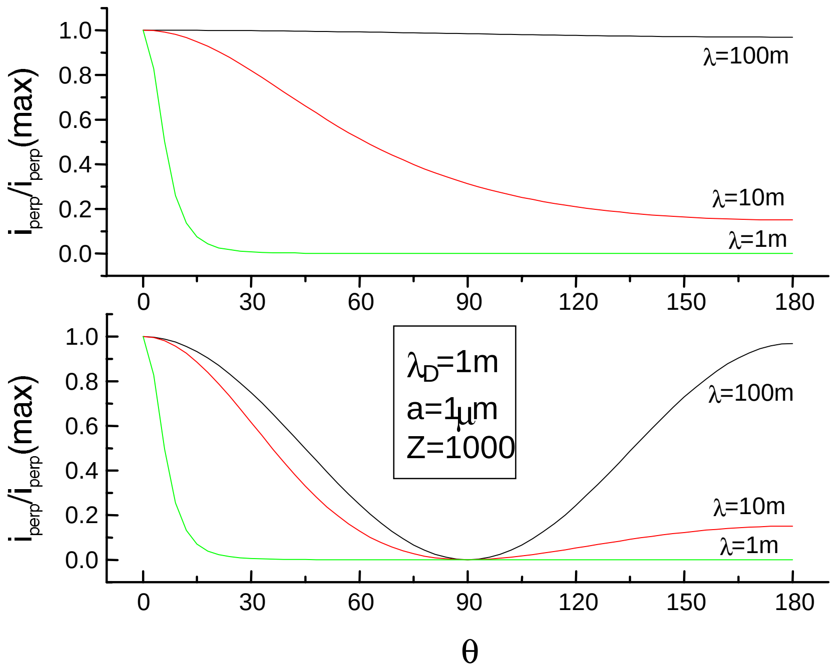


**Figure 2.** Radiant intensity normalised by the maximum value at $\theta$=0. The upper (lower) curves are relative to incident radiation polarised perpendicular (parallel) to the scattering plane (defined by the angle $\phi$). For large values of $\lambda$ the curves are equal to the Rayleigh-Mie case and for $\lambda \sim \lambda_D$ the scattering is strongly forward peaked.