NONLINEAR THEORY OF THE INTERACTION BETWEEN ANNULAR ELECTRON BEAM AND AZIMUTHAL SURFACE WAVES

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The possibility of excitation of the extraordinary polarized surface mode, that is eigenmode for cylindrical metal waveguide, partially filled by cold magnetoactive plasma (with concentration n_p), is studied in this paper. This wave propagates along the azimuthal angle, across an external axial steady magnetic field $\vec{B}_0$, and is refers as azimuthal surface wave (ASW). But it is surface type wave only in the region of the plasma cylinder, and in the dielectric (vacuum), that separates ideally conductive waveguide metal wall from the magnetoactive plasma column, this electromagnetic perturbation propagates as a volume wave. Earlier, the results of linear theory of ASW beam excitation were represented in [1]. To simplify digital simulation we have supposed that thickness of vacuum region is rather small ($R_1 - R_2 << R_1$, where R_1 and R_2 are radii of metal wall and plasma cylinder, respectively). Annular electron beam with the concentration n_b rotates around the plasma cylinder in the vacuum region.

To find the set of ordinary differential equations, described nonlinear beam — wave interaction we have used quasi–hydrodynamical plasma model, Maxwell's equations and beam particles motion equations. To simulate electron beam we have utilized macro particles method. Taking into account the inequality $n_b << n_p$ we also have neglected as the beam influence on the dispersion properties of ASW and the effect of self beam field on the electromagnetic field of ASW. Assuming that the dependence of ASW field on time t and azimuthal angle φ is expressed in the form $E, H \propto \exp(i\,m\varphi - i\,\omega\,t)$ (space in $\vec{z}$ direction is supposed to be homogeneous), involving the following boundary conditions:

- tangential component of the electric ASW field is equal to zero on the metal wall;
- tangential components of electric and magnetic ASW field are persistent on the boundary $r = R_1$;
- there are no electric currents on the boundaries $r = R_1, R_2$;

and neglecting any dissipation processes one can derive dispersion equation and equations that govern the nonlinear development of beam instability.

To solve obtained set of ordinary differential equations we use Runge — Kutta method of the fourth order. It is supposed that the beam particles interact with vacuum—plasma and vacuum—metal boundaries according to the mirror reflection model. Results of numerical simulation for the ASW with azimuthal wave number $m = 4$ are represent on Fig. 1—3. For numerical calculations was used such initial parameters: dimensionless wave amplitude $A = 10^{-3}$, the wave phase $\theta = 0$, radial and angle impulses of beam macroparticles $v_i = 0$ and $u_i = z\,R_i\,(\pm 2\%)$ $(z \approx 0.1)$, $n_b\,n_p^{-1} = 10^{-2}$. Initial angle macroparticle distribution was chosen almost homogeneous and initial radial position accidentally placed in the region $R_1 < r < R_2$ ($R_1 - R_2 = 0.1 R_1$).

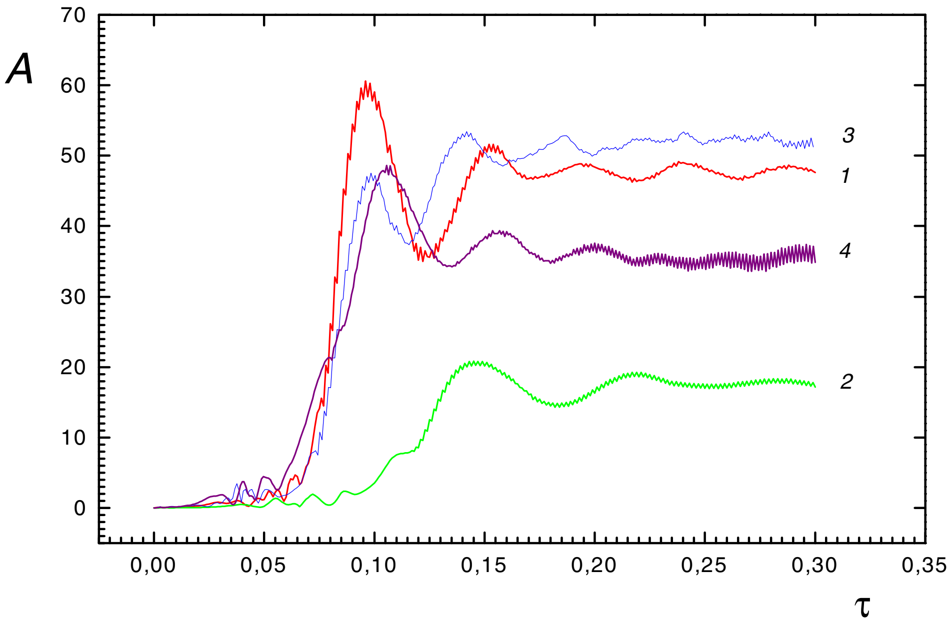

Figure 1. Dependence of the dimensionless wave amplitude A upon dimensionless time t. Curves 1, 2, 3, 4 correspond to the case of $B_o = 100$ *Gss*, $k_{eff} = 0.4$; $B_o = 150$ *Gss*, $k_{eff} = 0.4$; $B_o = 100$ *Gss*, $k_{eff} = 0.3$; $B_o = 100$ *Gss*, $k_{eff} = 0.5$, respectively.

Fig. 1 presents the evolution of the wave amplitude under the different waveguide parameters for the azimutal mode with $m = 4$. At the initial stage of instability beam — wave interaction leads to the exponentially increasing of wave amplitude. Then, when all plasma particles are captured by the wave, the one can see saturation of the amplitude. But it is not a steady state. Captured particles group in bunches and oscillate in potential wells of ASW that leads to the ASW amplitude oscillations (see Fig. 2—3).

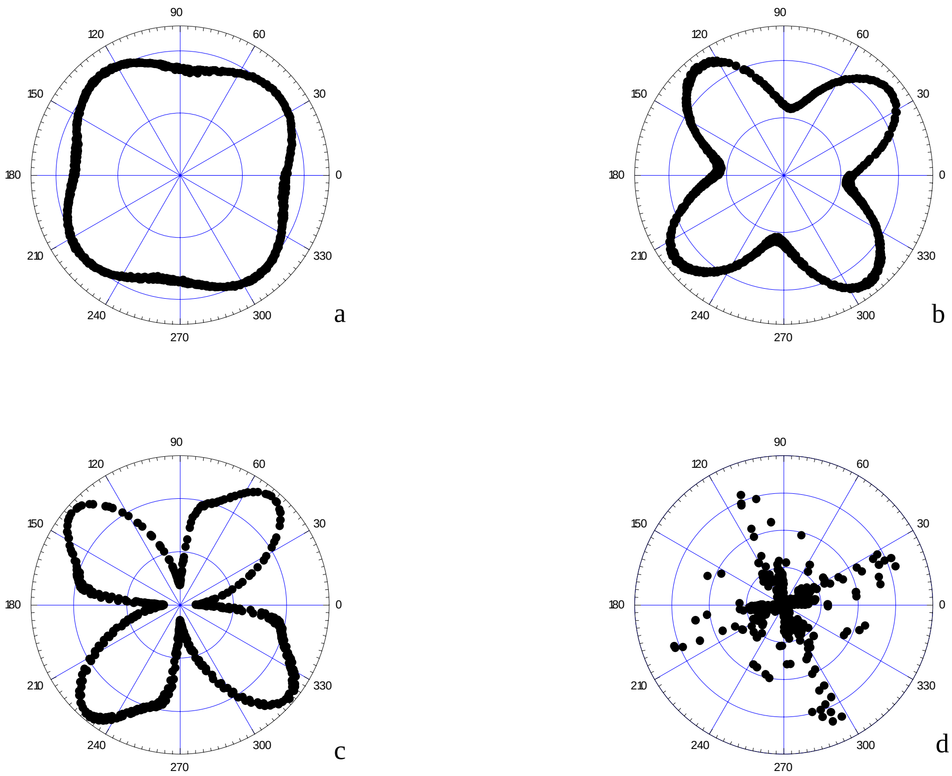

Figure 2. Beam macroparticles distribution in phase space azimuthal impulse – azimuthal angle at the moments of dimensionless time *t:* a) $t = 0.02;$ b) $t = 0.03;$ c) $t = 0.04;$ d) $t = 0.06.$

These oscillations can be damped due to violation of wave — particle synchronism caused by the dependence of the period of captured beam particles phase oscillations upon transverse coordinate r. Increase of the applied external magnetic field value leads to the increasing of the time required for the growth of wave amplitude upon initial value to its maximum (curves 1 and 2 on Fig. 1). Such effect caused also by the decreasing of parameter $n_b\, n_p^{-1}$. As it was mentioned in [1] there is a region of maximum increment values for beam ASW instability upon effective wavenumber k_{eff}. It approximately corresponds to $k_{eff} \approx 0.4$. The results of numerical simulation confirm the predictions of work [1]. Really, amplitude growth was slow down when we chose $k_{eff} \approx 0.3$ and $k_{eff} \approx 0.5$ (curves 1, 3, 4 on Fig. 1). Curve marked by the number 1 relates to the mentioned case when $k_{eff} \equiv m c R_1^{-1} \Omega_e = 0.4$.

Changing of the azimuthal wave number m sign, that determined propagation direction of ASW, leads to failure of instability.

Fig. 2, 3 present the evolution of particle distribution in coordinate (angle and radial coordinates) and in phase (angle's coordinate and impulse) spaces. We have chosen the most characteristic time moments when initial uniform particle distribution (in coordinate and phase spaces it looks like continuous approximately nonmodified ring) begins its modification (Fig. 2a, 3a), begins to broke at the bunches (Fig. 2b, c; 3b,c) and then begins to transform into four (according to the azimuthal mode wave number $m = 4$) real bunches (Fig. 2d, 3d) (in phase space this process is similar on creation of four formations like spokes).

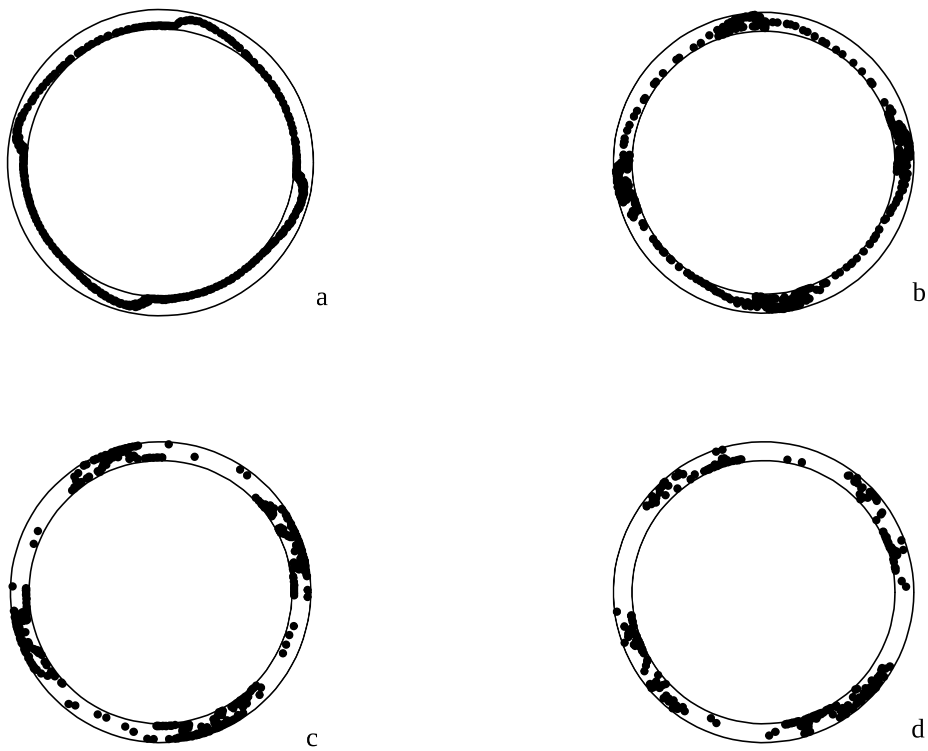

Figure 3. Beam macroparticles distribution in coordinate space at the moments of dimensionless time *t:* a) *t = 0.04;* b) *t = 0.05;* c) *t = 0.06;* d) *t = 0.07.*

Reference

[1] Girka V.A., Girka I.A., Olefir V.P. and Tkachenko V.I.: "Generation of electromagnetic waves by annular relativistic electron beam." Pis'ma v Zhurnal Technicheskoi Fiziki **17**(1), 87–91 (1991) (*in Russian*)