# MAGNETORESONANCE IN FREE ELECTRON LASERS WITH ELECTROMAGNETIC AND MAGNETOSTATIC WIGGLER

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**Abstract**

The free electron lasers with electromagnetic and magnetostatic wiggler are considered. It is shown that relativistic effects destroy the magnetoresonance in electromagnetic wiggler and displace it in electrostatic wiggler. Injection of electrons into the one-dimensional magnetostatic wiggler is studied numerically. Considerable reduction in the amplitude of oscillation about the steady-state orbits is found when the axial magnetic field is changed adiabatically.

## 1. Introduction

A constant axial magnetic field $B_0$ in free electron laser (FEL) is employed for the enhancement of the gain. The physical process for this enhancement is based on the increased transverse wiggler velocity $v_w$ by the axial magnetic field. The operative mechanism of FEL may also be based on the stimulated emission due to the pondermotive wave formed by the beating of the radiation and wiggler fields. The electromagnetic wiggler has been investigated [1,2,3] and a resonant enhancement condition has been found when $\omega_c \approx \gamma(\omega + kv_{\parallel})$ where $\omega_c = eB_0/mc$ is the electron cyclotron frequency. In the one-dimensional magnetostatic wiggler the resonance condition becomes $\omega_c \approx \gamma k v_{\parallel}$. Under this condition, beam electrons in one cyclotron period $(2\pi\gamma/\omega_c)$, with relativistic mass, travel one wiggler wavelength $\lambda_w = 2\pi/k$.

The purpose of this investigation is to show that the relativistic effects destroy the magnetoresonance in FEL with electromagnetic wiggler and displace it to $\omega_c \approx v_{\parallel}/c \approx 0$ in FEL with magnetostatic wiggler. Injection with adiabatically tapered wiggler into the region with large transverse velocity is accompanied with large amplitude oscillations about the steady-state orbits. It is shown that these disturbing oscillations can be reduced considerably when the axial magnetic field is also adiabatically tapered.

## 2. Electromagnetic and magnetostatic wiggler

Free electron laser with electromagnetic wiggler consists of a relativistic, monoenergetic and cold, electron beam moving antiparallel to a large amplitude circularly polarized electromagnetic wave which propagates in the negative $z$ direction. This wave is described by

$$\mathbf{B}_w(z,t) = B_w[\hat{\mathbf{x}}cos(kz+\omega t) + \hat{\mathbf{y}}sin(kz+\omega t)]. \quad (1)$$

A uniform and constant magnetic field $B_0\hat{\mathbf{z}}$ is parallel to the beam axis. In order to find electron trajectories the relativistic single particle equation of motion and the Maxwell's equations are employed. The steady-state helical trajectories with constant energy can be found:[3]

$$\mathbf{v} = v_w[\hat{\mathbf{x}}cos(kz+\omega t)+\hat{\mathbf{y}}sin(kz+\omega t)]+v_{\|}\hat{\mathbf{z}}, \quad (2)$$

$$v_w = \frac{\omega_w(\omega+kv_{\|})}{k[\omega_c-\gamma(\omega+kv_{\|})]}, \quad (3)$$

$$v_{\|}^2 + v_w^2 = v_{\|0}^2. \quad (4)$$

A dispersion relation between $\omega$ and $k$ may also be found as follows:

$$\omega^2 - k^2c^2 + \frac{\omega_b^2(\omega+kv_{\|})}{\omega_c-\gamma(\omega+kv_{\|})} = 0. \quad (5)$$

The orbit equation (3) and the dispersion equation (5) are coupled through the conservation equation (4).

In the nonrelativistic limit of transverse velocity ($v_w^2 \ll v_{\|}^2$) $v_w^2$ can be neglected in Eq. (4) giving $v_{\|} = v_{\|0}$. Under this condition Eq. (5) can be written as

$$\omega^2 - k^2c^2 + \frac{\omega_b^2(\omega+kv_{\|0})}{\omega_c-\gamma(\omega+kv_{\|0})} = 0, \quad (6)$$

which is the dispersion relation for the small amplitude wave with the effect of the wiggler amplitude on the dispersion relation neglected. This shows the magnetoresonance at $\omega_c = \gamma(\omega+kv_{\|0})$ reported in Ref. 3.

For the relativistic treatment $v_{\|}$ and $v_w$ should be considered as functions of $\omega$ and $k$, through Eqs. (3) and (4). $v_{\|}$ and $v_w$ can be eliminated from the Eq. (4), using Eqs. (3) and (5), which yields

$$\begin{aligned} &1-\gamma^2\left\{1-\frac{[(k^2c^2-\omega^2)(\omega_c-\gamma\omega)-\omega\omega_b^2]^2}{k^2c^2[\omega_b^2+\gamma(k^2c^2-\omega^2)]^2}\right.\\ &-\frac{\omega_w^2}{k^2c^2}\left[\omega+\frac{(k^2c^2-\omega^2)(\omega_c-\gamma\omega)-\omega\omega_b^2}{\omega_b^2+\gamma(k^2c^2-\omega^2)}\right]^2\\ &\left.\times\left\{\omega_c-\gamma\left[\omega+\frac{(k^2c^2-\omega^2)(\omega_c-\gamma\omega)-\omega\omega_b^2}{\omega_b^2+\gamma(k^2c^2-\omega^2)}\right]\right\}^{-2}\right\}=0. \quad (7)\end{aligned}$$

This is the dispersion relation for the large amplitude electromagnetic waves inside the electron beam. Contrary to the nonrelativistic case there is no asymptotic approach to any resonance line. This result is also been demonstrated by solving Eq. (7) numerically. Asymptotic approach to the resonance line $\omega_c = \gamma(\omega+kv_{\|0})$ with the singularity demonstrates the resonance in the nonrelativistic case. This shows that a proper consideration of relativistic effects, by considering the orbit-dispersion coupling, destroys the magnetoresonance in electromagnetically pumped FEL. In Ref. 3 magnetoresonance has been introduced in the relativistic limit by using Eq. (6) for the dispersion relation.

By setting $\omega = 0$, Eq. (1) reduces to the one-dimensional helical magnetostatic wiggler and the steady state solution for the electron motion, Eq. (3), reduces to

$$v_w = \frac{\omega_w v_{\parallel}}{\omega_c - \gamma k v_{\parallel}}. \tag{8}$$

The axial velocity $v_{\parallel}$ can be found in the nonrelativistic limit,

$$v_{\parallel}^2 = v_{\parallel 0}^2 - \frac{\omega_w^2 v_{\parallel 0}^2}{(\omega_c - \gamma k v_{\parallel 0})^2}, \tag{9}$$

with the asymptotic approach $\omega_c \rightarrow \gamma k v_{\parallel 0}$ representing the magnetoresonance.

For the relativistic treatment, $v_w$ from Eq. (8) is substituted in Eq. (4) and solved for $v_{\parallel}$. Numerical calculations has shown that the resonance where located at $\omega_c = \gamma k v_{\parallel 0}$ in the nonrelativistic treatment is displaced to the origin at $\omega_c \approx v_{\parallel}/c \approx 0$. The reason is that the lowest possible value for $\omega_c - \gamma k v_{\parallel}$ corresponds to the highest value of $v_w$ which is $v_{\parallel 0}$. This makes $v_{\parallel} \approx 0$ and Eqs. (4) and (8) are satisfied for the resonance at $\omega_c \approx v_{\parallel}/c \approx 0$.

## 3. Injection with tapered wiggler and axial magnetic field

In order to study the behavior of electrons near or far from the steady-state orbits, the single particle equation of motion is written in a frame rotating with wiggler field. To inject the electrons into the wiggler field the wiggler amplitude $B_w$ is adiabatically increased as follows:

$$B_w(z) = \begin{cases} B_w sin^2(\frac{kz}{4N_w}), & 0 \leq z \leq N_w\lambda_w \\ B_w, & N_w\lambda_w < z. \end{cases} \tag{10}$$

As the wiggler amplitude $B_w(z)$ is increased in the transitional region, electrons loose some of their initial axial velocity $v_3$ which results in a decline in their average value. The average absolute values for $v_1$ is increased and for $v_2$ is zero. For $z > N_w\lambda_w$, velocity components for electron motion oscillate about their steady-state values. When the electrons are injected near the region of so called magnetoresonance all velocity components $v_1$, $v_2$, and $v_3$ exhibit strong oscillations [4].

The large amplitude fluctuation does not allow the electrons to maintain their phase with respect to the radiation. To overcome this problem, and reduce the oscillation amplitude, electrons can be injected into the Group II orbits with large $v_{\parallel}$ over $N_w$ wiggler periods using adiabatically tapered wiggler field. At this point since electrons have small transverse velocity they will have small amplitude oscillations about their steady-state orbits. Next, from $z = N_w\lambda_w$ the axial magnetic field will be adiabatically decreased over $N_0$ wiggler periods as

$$B_0(z) = \begin{cases} B_0, & z \leq N_w\lambda_w \\ B_0[1 - \alpha sin^2\frac{k(z-N_w\lambda_w)}{4N_0}], & N_w\lambda_w \leq z \leq (N_w + N_0)\lambda_w \\ B_0(1-\alpha), & z > (N_w + N_0)\lambda_w, \end{cases} \tag{11}$$

where $\alpha < 1$ is a constant. For $z > (N_w + N_0)\lambda_w$ electrons are transported to the region of weaker axial magnetic field with small axial and large transverse velocities. Numerical computation shows that the tapered axial magnetic field decreases the unwanted oscillations in the region of large transverse velocity by a factor of ten. To test this method experimentally windings of the solenoid for generating the axial magnetic field may be modified for the tapering [5].

## References


[1] V.L. Granatstein, P. Sprangle, R.K. Parker, J.A. Pasour, M. Herndon, and S.P. Schlesinger: Phys. Rev. A **14**, 1194 (1976).

[2] V.L. Granatstein, S.P. Schlesinger, M. Herndon, R.K. Parker, and J.A. Pasour: Appl. Phys. Lett. **30**, 384 (1977).

[3] H.P. Freund, R.A. Kehs, and V. L. Granatstein: Phys. Rev. A **34**, 2007 (1986).

[4] H.P. Freund and T.M. Antonsen, Jr.: Principles of Free-electron Lasers. Chapman and Hall, London, 1992, pp. 165-167.

[5] M.E. Conde and G. Bekefi: Phys. Rev. Lett. **67**, 3082 (1991).