

PROPAGATION AND DAMPING OF $m=+1$ AND $m=-1$ HELICON MODES IN AN INHOMOGENEOUS PLASMA COLUMN

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Abstract

The present theoretical-numerical study deals with the helicon wave propagation in a high-density plasma focussing on the effect of the density gradient on the dispersion of the helicon modes. In particular, the helicon wave damping which is intimately related to the rf power absorption and the Poynting flux is investigated. It will be shown that particularly the $m = -1$ mode changes drastically its propagation properties at sufficiently strong density gradients, i.e. the $m = -1$ mode is strongly damped so that it can hardly be excited. These results may explain why, in most cases, the $m = +1$ mode is mainly responsible for sustaining helicon discharges.

1. Introduction

Helicon discharges produced through helical antennae may exhibit a pronounced axial asymmetry of the rf power coupling in that the power is predominantly transferred to the $m = +1$ helicon mode travelling along the plasma column in positive magnetic field direction thereby sustaining an axially asymmetric discharge [1,2]. Till now, however, no satisfactory explanation for this finding has been given. Recently, Sugai et al. [3] verified by launching $m = +1$ and $m = -1$ test waves into a helicon discharge that only the $m = +1$ helicon mode propagates irrespective of the geometry of the antenna. The present theoretical-numerical study is motivated by these findings. In particular, the role of the density gradient for helicon wave propagation and damping is investigated in detail.

2. Theory

The governing equations for helicon wave propagation in a collisional plasma have the following approximate form:

$$\nabla \times \mathbf{E} = -\dot{\mathbf{B}} \quad (1)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} \quad (2)$$

$$\mathbf{E} = \eta \mathbf{j} + \frac{1}{en_0} \mathbf{j} \times \mathbf{B}_0 \quad (3)$$

where n_0 is the equilibrium density, $\eta = \nu_c m_e / (e^2 n_0)$ is the resistivity with the collisional frequency $\nu_c = \nu_{ei} + \nu_{en}$ (here, $\nu_{ei} \gg \nu_{en}$), and the displacement current has been neglected in (2).

First, the propagation of undamped ($\eta = 0$) helicon waves along a non-uniform plasma column is treated ([4]). Applying the wave *ansatz* (e.m. fields $\propto \exp[i(m\theta + k_z z - \omega t)]$) in (1) - (3) and combining these equations, one finally arrives at the wave equation for the inhomogeneous helicon wave guide in the form

$$B_z'' + f(r) B_z' + g(r) B_z = 0, \quad (4)$$

where

$$f(r) = \frac{1}{r} + \frac{2\alpha\alpha'}{k_z^2 - \alpha^2}, \quad g(r) = \alpha^2 - k_z^2 - \frac{m^2}{r^2} + \frac{m\alpha'(k_z^2 + \alpha^2)}{k_z r(k_z^2 - \alpha^2)}, \quad \alpha = \frac{k_0^2 \omega_{pe}^2}{k_z \omega \omega_{ce}}.$$

When helicon wave propagation in a plasma cylinder (radius r_w) is considered, the boundary conditions have to be taken into account. If the walls are perfectly conducting, $\nabla \cdot \mathbf{B} = \mathbf{0}$ leads to

$$B_r(r_w) = m\alpha B_z(r_w) + k_z r_w B'_z(r_w) = 0. \quad (5)$$

To treat the wave equation (4) and find a solution satisfying the boundary condition (5), the shooting method was applied to obtain the wavenumber k_z (denoted in the following as k_{zr}). In a collisional plasma, wave damping has to be taken into account, i.e. $k_z = k_{zr} + ik_{zi}$. The spatial damping decrement k_{zi} can be obtained from the relation between the power flux and the absorbed power density, provided that $k_{zi} < k_{zr}$.

The time-averaged energy flux in a wave guide of cross section A is equal to the component of the Poynting vector $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ along the wave guide axis integrated over A yielding

$$P = \int_A \langle \mathbf{S} \cdot d\mathbf{A} \rangle_t = \frac{1}{4} \int_A (\hat{\mathbf{E}} \times \hat{\mathbf{H}}^* + c. c.) \cdot d\mathbf{A} = \frac{\omega}{2\mu_0 k_{zr}} \int_A |\hat{\mathbf{B}}_{\perp}|^2 dA. \quad (6)$$

The absorption of the wave power is given by the time-averaged scalar product of the current density and the electric field. Hence, the absorbed power density becomes

$$Q = \langle \mathbf{j} \cdot \mathbf{E} \rangle_t = \frac{\eta}{2} |\hat{\mathbf{j}}|^2 = \frac{\eta}{2\mu_0^2} |\nabla \times \hat{\mathbf{B}}|^2, \quad (7)$$

It is easy to obtain the expression for the spatial damping decrement k_{zi} . The wave fields \mathbf{E} and \mathbf{H} are damped as $\exp(-k_{zi}z)$ so that the axial dependence of the power flux is given by $P(z) = P(z=0) \exp(-2k_{zi}z)$. The power decay per length is now equal to the absorbed power per length [5],

$$\int_A Q dA = 2k_{zi} P, \quad (8)$$

and the damping decrement becomes

$$k_{zi} = \frac{1}{2} \int_A \eta |\hat{\mathbf{j}}|^2 dA \Big/ \int_A |\hat{\mathbf{B}}_{\perp}|^2 dA. \quad (9)$$

3. Results

The computations have been carried out for a high density plasma column immersed in a strong magnetic field. The following standard parameters have been taken: $n_e = 10^{20} \text{ m}^{-3}$, $T_e = 4 \text{ eV}$, $B_0 = 0.1 \text{ T}$, $r_p = 2 \text{ cm}$, $f = 25 \text{ MHz}$, $P_{rf} = 1 \text{ kW}$. For the inhomogeneous calculations, Gaussian profiles have been chosen so that only one parameter, the profile width w , has to be varied: $n_e = \left\{ n_0^{(w)} \exp(-(r/w)^2), \text{ if } r \leq r_p, \text{ 0 if } r > r_p \right\}$. $n_0^{(w)}$ was related to the homogeneous case and normalized by keeping the electron number over the cross section constant.

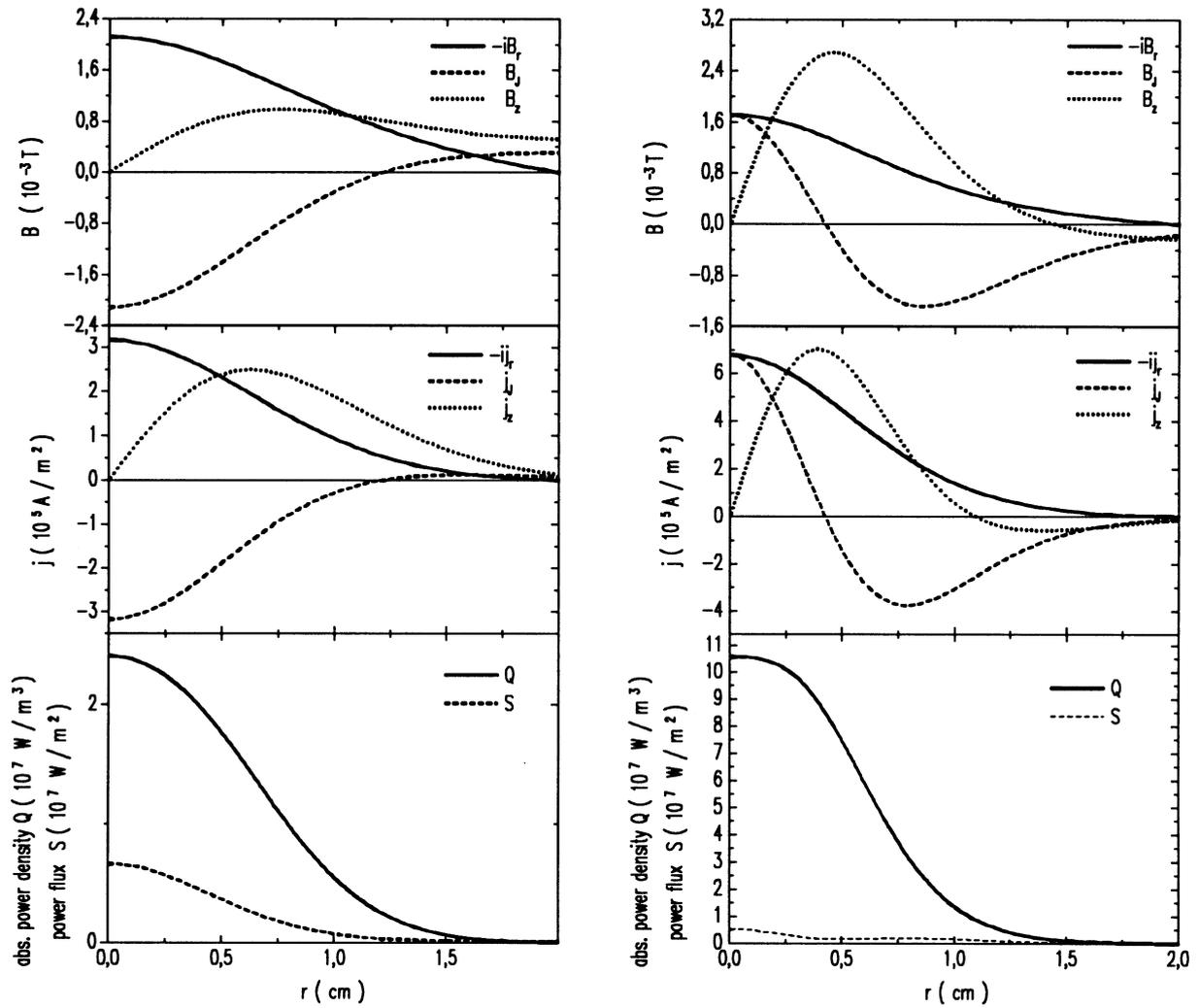


Fig. 1. Radial profiles of rf magnetic fields, current densities, absorbed power density and power flux for $m = +1$ (left) and $m = -1$ (right).- Gaussian profile with $w = 1.5cm$

Fig. 1 shows the comparison of the computations for the $m = +1$ and $m = -1$ modes in the non-uniform case. The magnetic field and the current density profiles resemble those presented by Chen [4]. The power flux of the $m = +1$ mode turns out to be more concentrated on the axis than that of the $m = -1$ mode while the absorbed power density is maximum on the axis in both cases. In the homogeneous plasma (not shown here), however, the $m = -1$ reveals a minimum there whereas the $m = +1$ peaks again on the axis. The maximum power absorption on the plasma axis may be one of the reasons for the effectiveness of helicon discharges.

In Fig. 2 it is shown, how the width of the density profile affects the field and absorbed power profiles of the $m = -1$ mode. With narrowing of the density profiles the field (and current) profiles become narrow as well and the amplitudes increase. Simultaneously, the absorbed power density increases strongly whereby the total rf energy flux was kept constant. According to (9), the spatial damping decrement scaling with the ratio of the absorbed power and the energy flux increases strongly. As can be seen in Fig. 3., this holds for both modes. The damping is however stronger in case of the $m = -1$ mode, particularly, for the parameters of Fig.3 (bottom) which are nearly similar to those of the high-density helicon discharge [6] (with $w \approx 1.2 cm$). The damping length of the $m = -1$ mode is then only a few centimeters so that this mode

can hardly be excited. These results are confirmed by computations with the parameters of the helicon discharge [3] differing considerably from the present plasma parameters, but yield similar results.

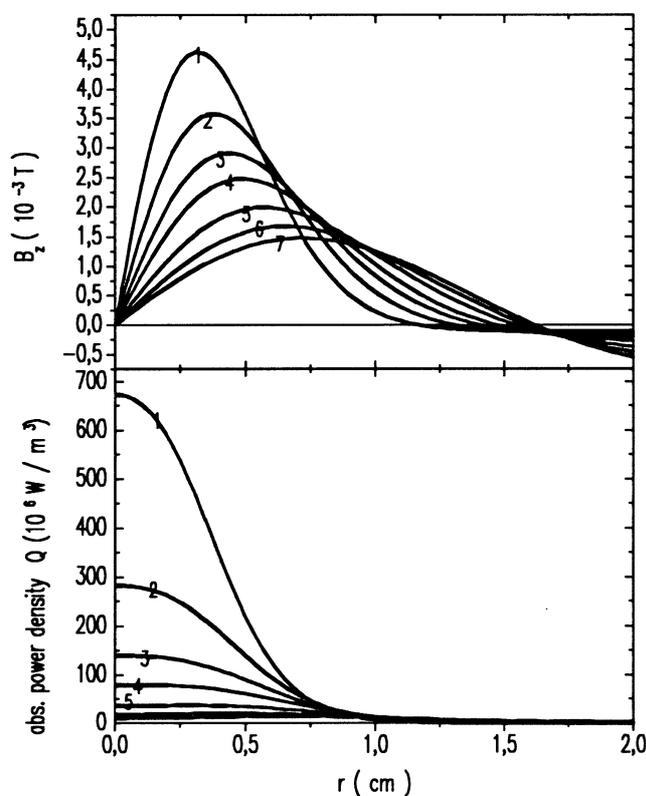


Fig. 2. Axial rf magnetic field and absorbed power density profiles for different density profiles.- curves 1 - 7: $w = 1.0, 1.2, 1.4, 1.6, 2.0, 2.6, 3.8\text{cm}$

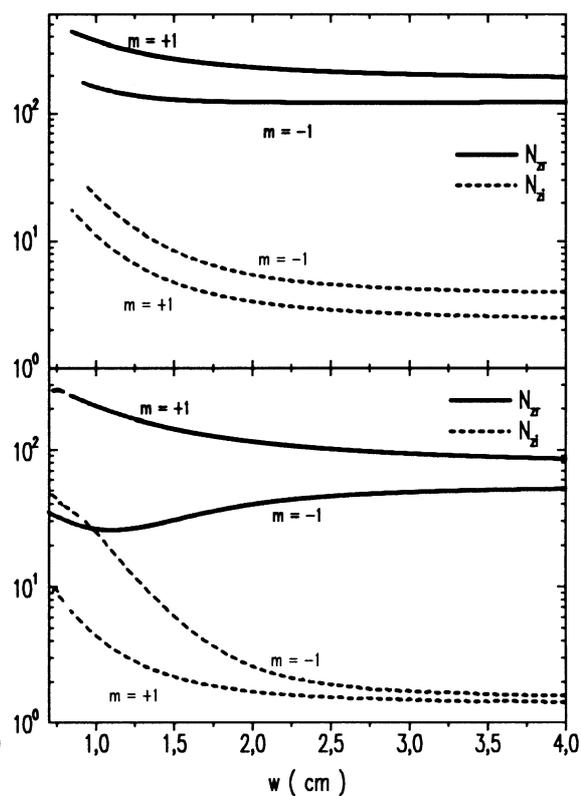


Fig. 3. Axial refractive index vs. width of density profile.- $n_{e0} = 5 \times 10^{19} \text{m}^{-3}$ (top), $n_{e0} = 2 \times 10^{19} \text{m}^{-3}$ (bottom)

4. Conclusions

The present numerical study has shown that particularly the $m = -1$ helicon wave is strongly affected by a sufficiently steep density gradient in that it is strongly damped. However, to generate and sustain a helicon discharge, the wave damping must not be too strong so that the $m = +1$ mode can be more easily excited. Another fact may favour the coupling to this mode: The antenna wavenumber spectrum must fit to the helicon wave dispersion. Because the $m = +1$ mode has the shorter wavelength as compared to $m = -1$ mode, it is most likely that the antenna couples better to the $m = +1$ mode.

References

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