

ALFVÉN SURFACE WAVES IN HIGHLY STRUCTURED DUSTY MAGNETOPLASMAS

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Abstract. It is shown that charged dusts or impurity ions carrying a considerable proportion of the negative charge of a structured magnetized plasma can lead to low-frequency electromagnetic surface waves which otherwise do not exist. The waves are of Alfvén type and propagate across the stationary external magnetic field with a frequency below the ion cyclotron but much above the dust cyclotron frequency. The dispersion characteristics of the modes are obtained and applications to space plasmas discussed.

Structured dust-containing plasmas are ubiquitous in space and the laboratory. In particular, they are found in interstellar clouds, cometary tails, planetary rings and atmospheres, solar wind and solar magnetic flux tubes, the Earth's ionosphere and magnetosphere, etc. [1]. The structuring occurs because the dusts, with very large mass and high charge number, are strongly affected by external gravitational, electric, and magnetic fields, so that even relatively weak structuring in these fields can lead to sharp interfaces in a dusty plasma. It is well known that surface waves (SWs) can propagate along the structural boundaries in a plasma [2,3]. Recently it was shown [4,5] that the dust grains, carrying a significant amount of the plasma's negative electric charge, can modify the phase velocity of the SWs and cause an increase of the damping of waves in the plasma. On the other hand, it has been shown [6] that in a cold dust-free plasma, the fields of low-frequency ($\omega < \Omega_i$, where Ω_i is the ion cyclotron frequency) waves propagating across an external magnetic field cannot be localized in the direction perpendicular to the interface since the skin depth significantly exceeds the wavelength. In this Report, we demonstrate that because of the imbalance of the electron and ion densities in a dust-containing plasma, the scale of wave field localization at the interface can be much reduced and become comparable to the wavelength, so that *bona fide* SWs can propagate. That is, magnetoplasma SWs propagating across an external magnetic field, impossible in a dust-free plasma, can exist when sufficient dust grains are present.

For simplicity, we consider a planar interface $x = 0$ between two distinct regions of a dusty plasma. The plasmas are assumed to be cold (low β) with different electron, ion and dust grain densities. The widths of the two regions are assumed to exceed the scale (of order the SW skin depth) of SW field localization in the transverse direction. For convenience, we shall refer to the half-space $x > 0$ as layer 1, and the half-space $x < 0$ as layer 2. The interface between the plasmas is assumed to be sharp, valid if the SW skin depth significantly exceeds the width of the transition layer between the two plasmas. The external magnetic field \mathbf{B}_0 is along the z -axis, and

the waves are assumed to propagate along the y axis. The quantities characterizing the waves are assumed to be in the form $A(\mathbf{r}, t) = A(x) \exp[i(k_y y - \omega t)]$, where k_y is the wave number and ω is the eigenfrequency. We consider low frequency waves satisfying $\Omega_d \ll \omega < \Omega_i$, where $\Omega_d = q_d B_0 / m_d$ is the dust cyclotron frequency. Here, $q_d = -|Z_d e|$ and m_d the (negative) charge and mass of the dust grain, and e is the magnitude of the electron charge. The grain size is much smaller than the electron Debye length and the distance between the plasma particles. In the steady state, we have $en_{i0} - en_{e0} + q_d n_{d0} = 0$, where $n_{\alpha 0}$ is the steady-state number density of the specie $\alpha = i, e, d$ for the ions, electrons and dusts, respectively.

The linearized equations of motion are

$$m_i \frac{\partial \mathbf{v}_i}{\partial t} = e(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}_0), \quad (1)$$

$$0 = \mathbf{E} + \mathbf{v}_e \times \mathbf{B}_0, \quad (2)$$

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_{\alpha 0} \mathbf{v}_\alpha) = 0, \quad (3)$$

where \mathbf{v}_e , \mathbf{v}_i , n_e , and n_i are the electron and ion velocity and density perturbations, and \mathbf{E} and \mathbf{B} are the wave electric and magnetic fields. The electron inertia is neglected, valid for waves with frequencies below the ion cyclotron frequency. The set (1) – (3) are completed by the Maxwell's equations. The displacement current is to be neglected since the phase velocity of the waves is much less than the speed of light.

>From (1) – (3) and the Maxwell's equations, one can obtain for B_{zj} , E_{yj} , and E_{xj}

$$\frac{\partial^2 B_{zj}}{\partial x^2} - \kappa_j^2 B_{zj} = 0, \quad (4)$$

$$E_{yj} = \frac{-iV_{Aj}}{\omega_{pi}(j)\Omega_i\sigma_j} \left(\frac{\partial B_{zj}}{\partial x} + k_y \xi_j \frac{\Omega_i}{\omega} B_{zj} \right), \quad (5)$$

$$E_{xj} = -\frac{i}{k_y} \frac{\partial E_{yj}}{\partial x} - \frac{\omega}{k_y V_{Aj}} \frac{\Omega_i}{\omega_{pi}(j)} B_{zj}, \quad (6)$$

where $\kappa_j^2 = k_y^2 - \omega \Omega_i^2 \sigma_j / V_{Aj}^2$, $V_{Aj} = c \Omega_i / \omega_{pi}(j)$ is the Alfvén velocity, $\omega_{pi}(j)$ is the ion plasma frequency, $\sigma_j = \omega(1 - \Omega_i^2 \xi_j^2 / \omega^2) / (\Omega_i^2 - \omega^2)$, $\xi_j = 1 - \eta_j(1 - \omega^2 / \Omega_i^2)$, and $\eta_j = n_{e0}(j) / n_{i0}(j)$ characterizes the charge imbalance in a dust-containing plasma which reduces to unity in the absence of dust grains. Here, $j = 1, 2$ corresponds to the plasma layer considered.

We look for solutions of (4) in the form

$$B_{zj} = A_j \exp[(-1)^j \kappa_j x], \quad (7)$$

$$E_{yj} = \frac{-iV_{Aj}}{\omega_{pi}(j)\Omega_i\sigma_j} \left[(-1)^j \kappa_j + k_y \frac{\Omega_i}{\omega} \xi_j \right] A_j \exp[(-1)^j \kappa_j x], \quad (8)$$

which describe SW fields decaying away (in both regions) from the interface. Using the boundary condition that the tangential components of the wave electric and magnetic fields at the interface $x = 0$ are continuous, we obtain

$$\frac{n_{i0}(1)\sigma_1}{n_{i0}(2)\sigma_2} \left(\kappa_2 + k_y \frac{\Omega_i}{\omega} \xi_2 \right) = -\kappa_1 + k_y \frac{\Omega_i}{\omega} \xi_1, \quad (9)$$

which is the dispersion relation of the SW.

The plasma is highly structured when the number densities n_{j0} or the composition of the bordering plasmas differ significantly. From (9), we see that the SW properties are mainly determined by the more rarefied of the two plasmas. For $n_{i0}(2) \gg n_{i0}(1)$ (but the plasma in layer 1 not too rarefied), the general dispersion relation (9) can be approximated by

$$-\kappa_1 + k_y \Omega_i \xi_1 / \omega = 0, \quad (10)$$

and the solution is given by $k_y = \omega \Omega_i / V_{A1} (\Omega_i^2 - \omega^2)^{1/2}$, or

$$\omega = \frac{k_y V_A}{(1 + k_y^2 c^2 / \omega_{pi}^2)^{1/2}}, \quad (11)$$

where we have used the expression for κ_1 . The latter can be expressed as

$$\kappa_1 = \frac{\Omega_i}{V_{A1}} \frac{\Omega_i \xi_1}{(\Omega_i^2 - \omega^2)^{1/2}}, \quad (12)$$

where ω is given by (11).

We note that the mode considered here differs significantly from the usual compressional and kinetic Alfvén SWs, which have dispersion relations of the form $\omega \approx \pm \sqrt{2} k_z V_A$. The dispersion relation (10) describes Alfvén-type SWs at the interface of a structured plasma. For very low frequencies ($\omega^2 \ll \Omega_i^2$, but still satisfying $\omega \gg \Omega_d$), the wave number and the inverse skin depth can be approximated by $k_y = \omega / V_A$ and $\kappa_1 = \Omega_i (1 - \eta_1) / V_A$, respectively.

It is of interest to note that in the absence of the dust particles ($\eta_1 = 1$), the dispersion relation (10) yields $\kappa_1 = (\omega / \Omega_i) k_y$. Thus, the low-frequency ($\omega \ll \Omega_i$) wave fields are not localized at the interface. Oblique (to \mathbf{B}_0) SWs propagation at the dusty plasma-vacuum interface was considered in Ref. [4]. By setting $s = k_z / k_y$ (where k_z is the wavenumber along the z axis) in (37) of the latter paper to zero, one finds that no SW can propagate across \mathbf{B}_0 .

Furthermore, we see from (10) that only solutions with positive phase velocity ω / k_y can appear. That is, SWs can only propagate across the external magnetic field in the positive y direction. This reflects the nonreciprocal nature of SW in magnetized plasmas. In fact, for SW propagating at an angle to the external magnetic field, nonreciprocity is the strongest for the case considered here, namely with $\mathbf{k} \perp \mathbf{B}_0$. On the other hand, the ordinary Alfvén SWs [3] which propagate parallel or nearly parallel to \mathbf{B}_0 , are bi-directional.

From (10) we note that in the frequency range corresponding to the Alfvén-type SWs the presence of dust grains does not strongly affect the wavelength (11) and its phase velocity. On the other hand, the dusts do affect the skin depth (12) of the wave electromagnetic field at the interface. In fact, in the absence of the dusts, low-frequency SWs propagating perpendicular to \mathbf{B}_0 do not exist since the skin depth in this case can significantly exceed the wavelength, so that the perturbation fields are not localized at the interface. Physically, this occurs because of a magnetic field-induced gyrotropy of the medium, and is analogous to the problem of SWs in the Voigt geometry [6]. Plasma gyrotropy is governed by the ratio $\epsilon_{xy} / \epsilon_{xx}$ (for the present geometry), where ϵ_{xx} and $i\epsilon_{xy}$ are components of the dielectric tensor of the magnetoplasma. For low frequencies, we have $\epsilon_{xy} \rightarrow 0$ because the electron and ion Hall currents are equal for $\omega \ll \Omega_i$. Since $\kappa_1 / k_y = \epsilon_{xy} / \epsilon_{xx}$, the waves are not localized.

In the presence of dusts, which can account for a significant part of the negative charge of

the plasma, the mismatch of the electron and ion number densities leads to a difference in their respective Hall currents [7]. In this case, ϵ_{xy} is finite and the ratio $\epsilon_{xy}/\epsilon_{xx}$ is no longer small. The skin depth is thus reduced and can be comparable or even less than the wavelength. That is, the dust particles can lead to a localization of the electromagnetic field near the interface. The increase (compared with the dust-free case) of the SW skin depth can be characterized by

$$\gamma = \kappa_1/\kappa_1^{(0)} = \Omega_i^2 \xi_1 / \omega^2, \quad (13)$$

where $\kappa_1^{(0)}$ is the inverse skin depth in the absence of the dusts, and can be obtained from (12) by setting $\eta_1 = 1$. One can show that the presence of dust particles leads to a significant decrease of the SW skin depth, and hence improves the localization of the wave electromagnetic field at the interface. Similar conclusions can be drawn if the plasma contains a large proportion of impurity ions or is sufficiently non-neutral.

Another interesting physical feature of the SWs is that of local Alfvén resonance [7]. To briefly discuss this problem, we refine the sharp boundary model by inserting a transition layer $-a < x < a$ in which $n_{0i} = n_{0i}(x)$, where $n_{0i}(x)$ is a suitable density profile. The SW phase velocity in the region 1 ($x > a$) can be obtained from (11). We see that the condition $V_{ph}(a) = V_A(x)$ for local Alfvén resonance is realized at $x = x_r$ where x_r satisfies $[n_{0i}(x_r) - n_{0i}(a)]/n_{0i}(x_r) = \omega^2/\Omega_i^2$, so that for low-frequency SWs in a dusty plasma the resonance point is very close to the low density side of the transition layer. Near the local Alfvén resonance there is a significant increase of the perpendicular component (E_x) of the wave electric field.

To conclude, we emphasize that the existence of Alfvén-like cross-field SWs is possible only in the presence of charged dust particles. Thus, the appearance of such low-frequency waves in a highly-structured plasma may indicate the presence of charged impurities. Furthermore, details of the SW structure, in particular the skin depth, can also yield information on the number density of the dust particles. Since highly structured magnetized plasmas are common in space plasmas, observation of low-frequency Alfvén-like modes or resonances near the discontinuities can provide valuable information on the formation, evolution, and physical properties of the structures.

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