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Nonlinear neoclassical transport in a toroidal plasma with large gradients

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Abstract

The theory of neoclassical transport in an impure, toroidally rotating plasma is extended to allow for steeper pressure and temperature profiles than are usually considered. When the gradients are large, the impurity ions are found to undergo a spontaneous poloidal rearrangement, which reduces their parallel friction with the bulk ions and suppresses the neoclassical ion particle flux. The latter is typically a *non-monotonic* function of the gradients for plasma parameters representative of the tokamak edge. The neoclassical confinement is improved in regions with large gradients, such as the edge pedestal.

If the plasma rotates toroidally, the centrifugal force pushes the impurities to the outboard side of each flux surface. When the gradients are weak, this is found to substantially increase the neoclassical transport, which can then well exceed the conventional Pfirsch-Schlüter value. When the gradients are steep, the transport becomes highly nonlinear in a rotating plasma and is sensitive to the geometry of the magnetic equilibrium. For instance, the ion particle flux changes direction if the toroidal field is reversed, and is *inward* if the ion drift is towards the X-point in single-null magnetic geometry.

Introduction

The conventional theory of neoclassical transport in tokamaks [1,2] is not applicable to regions where the pressure and temperature profiles are very steep, such as the pedestal at the plasma edge. The essential difficulty lies in the use of the expansion parameter $\delta \equiv \rho_\theta / L_\perp$, where ρ_θ is the poloidal ion gyroradius and $L_\perp$ the radial scale length of the density and temperature profiles. Neoclassical theory requires $\delta \ll 1$, and it is fundamentally difficult to construct a tractable transport theory when $\delta = O(1)$. The transport fluxes then depend not only on local gradients but on the entire density and temperature profiles. When δ is infinitesimally small all plasma parameters are approximately constant on flux surfaces, but when δ is made larger poloidal asymmetries become possible. Typically the first plasma parameter to develop a poloidal variation is the density, n_z, of highly charged impurity ions [3], whose poloidal modulation is $\tilde{n}_z / n_z \sim \Delta \equiv \delta \hat{\nu}_{ii} z^2$, where $\hat{\nu}_{ii} \equiv L_{\|} / \lambda_{ii}$ is the collisionality, with λ_{ii} the mean-free path for the bulk ions and $L_{\|}$ the connection length. In the tokamak edge, Δ can easily be of order unity while δ remains small.

In conventional neoclassical theory $\Delta \ll 1$. Here, we adopt a less restrictive ordering,

$$\delta \ll 1, \qquad \Delta = O(1),$$

enabling a non-uniform distribution of impurities over each flux surface. For simplicity, we restrict our attention to the case of a hydrogen plasma with a single species of highly charged ($z \gg 1$) impurity ions of appreciable density, $Z_{\rm eff} - 1 = n_z z^2/n_i = O(1)$. The electrons ($e$) and H ions ($i$) are taken to be collisionless while the impurities are assumed to be collisional, as is typical of a tokamak plasma somewhat inside the last closed flux surface. We also allow for toroidal plasma rotation to produce a poloidal impurity density asymmetry. The impurity Mach number is taken to be of order unity, $M_z^2 = m_z\omega^2 R^2/2T_i = O(1)$, where ω denotes the angular rotation frequency, so that the main (H) ion Mach number is small, $M_i^2 = (m_i/m_z)M_z^2 \ll 1$.

Parallel kinetics

The bulk-ion distribution function is obtained by solving the drift-kinetic equation in the banana regime

$$f_i = f_{i0}\exp\left(-\frac{e\Phi}{T_i} + M_i^2\right) - \frac{Iv_\|}{\Omega_i}\frac{\partial f_{i0}}{\partial\psi} + h_i(H,\mu,\psi,\sigma), \qquad (1)$$

in a conventional way [4]. Here f_{i0} is Maxwellian, $H = m_i v^2/2 + e\Phi$ is the energy, $\Omega_i = eB/m_i$ is the ion cyclotron frequency, and the magnetic field has been written as $\mathbf{B} = I(\psi)\nabla\varphi + \nabla\varphi\times\nabla\psi$, where φ is the toroidal angle and ψ is the poloidal flux. The function h_i depends only on constants of motion, is odd in $\sigma = v_\|/|v_\||$, and vanishes for trapped particles. The electron distribution function is of a similar form to (1).

The parallel impurity momentum equation is

$$m_z n_z\left[(\mathbf{V}_z\cdot\nabla)\mathbf{V}_z\right]_\| = -zn_z e\nabla_\|\tilde\Phi - T_i\nabla_\| n_z + R_{zi\|}, \qquad (2)$$

where we have neglected parallel viscosity [4]. The impurity temperature is equilibrated with the bulk ion temperature, T_i, and is therefore constant over the flux surface. The ion-impurity friction force $R_{zi\|}$ can be calculated from (1), and the poloidal electric field $-\nabla_\|\tilde\Phi$ is determined by quasineutrality. Inserting these terms in (2) gives the following equation for the normalised impurity density $n = n_z/\langle n_z\rangle$,

$$(1+\alpha n)\frac{\partial n}{\partial\vartheta} = g\left(n - b^2 + \gamma\left(n - \left\langle nb^2\right\rangle\right)b^2\right) + \frac{\partial M^2}{\partial\vartheta}n - \left\langle n\frac{\partial M^2}{\partial\vartheta}\right\rangle b^2. \qquad (3)$$

Here $b \equiv B/\langle B^2\rangle^{1/2}$, $d\vartheta/d\theta \equiv \langle\mathbf{B}\cdot\nabla\theta\rangle/\mathbf{B}\cdot\nabla\theta$, and $\langle\cdots\rangle$ denotes an average over ϑ. The two most important control parameters in (3) are

$$g = -\frac{m_i n_i I}{e\tau_{iz} n_z\langle\mathbf{B}\cdot\nabla\theta\rangle}\left(\frac{d\ln n_i}{d\psi} - \frac{1}{2}\frac{d\ln T_i}{d\psi}\right) = O(\Delta),$$

where $\tau_{iz} = 3(2\pi)^{3/2}\epsilon_0^2 m_i^{1/2}T_i^{3/2}/n_z z^2 e^4\ln\Lambda = (n_i/n_z z^2)\tau_{ii}$, which measures the steepness of the bulk ion density and temperature profiles, and the modified impurity Mach number

$$M^2 = \frac{m_z\omega^2 R^2}{2T_i}\left(1 - \frac{zm_i}{m_z}\frac{T_e}{T_e+T_i}\right) = O(M_z^2).$$

associated with the toroidal rotation. The impurities are pushed toward the inboard side of the torus when g becomes large, and the neoclassical transport then becomes a strongly nonlinear function of the gradients, as we shall see. It is well known that the impurities are pushed to the outside of the torus by the centrifugal force when their Mach number M is large. The remaining parameters that appear in (3) do not play an important role in the theory. They are defined by $\alpha \equiv \langle Z_{\text{eff}} - 1 \rangle T_e/(T_e + T_i)$ and

$$\gamma \equiv -\frac{3\pi^{1/2}e}{4n_i T_i I B}\left\langle B^2 \right\rangle \left(\frac{d\ln n_i}{d\psi} - \frac{1}{2}\frac{d\ln T_i}{d\psi}\right)^{-1} \int v_{\|} \frac{(2T_i/m_i)^{3/2}}{v^3} h_i \, d^3v \leq O(1).$$

Neoclassical transport

We now proceed to solve (3) in various limits and evaluate the classical and neoclassical particle fluxes $\left\langle \left(\boldsymbol{\Gamma}_i^{\text{cl}} + \boldsymbol{\Gamma}_i^{neo}\right) \cdot \nabla\psi \right\rangle = \left\langle R^2 \nabla\varphi \cdot \left(\mathbf{R}_{iz_\perp} + \mathbf{R}_{iz\|}\right)/eB \right\rangle$.

In a plasma with small inverse aspect ratio, $\epsilon \ll 1$, and circular cross section, (3) can be solved by making the expansions $b^2 = 1 - 2\epsilon\cos\theta + O(\epsilon^2)$, $n = 1 + n_c\cos\theta + n_s\sin\theta + O(\epsilon^2)$, $M^2 = M_0^2(1 + 2\epsilon\cos\theta) + O(\epsilon^2)$. The particle flux then becomes

$$\left\langle \left(\boldsymbol{\Gamma}_i^{\text{cl}} + \boldsymbol{\Gamma}_i^{neo}\right) \cdot \nabla\psi \right\rangle = \frac{\epsilon^2 p_z}{q^3 e}\left(1 + \frac{2\Lambda q^2}{1 + \left(\frac{1+\gamma}{1+\alpha}\right)^2 g^2}\right) g, \tag{4}$$

where the first term is the classical flux and the second term, which contains the factor

$$\Lambda = \left(1 + \frac{M_0^2}{1+\alpha}\right)\left(1 + \frac{1+\gamma}{1+\alpha}M_0^2\right), \tag{5}$$

represents the neoclassical flux. The latter exceeds the former by the Pfirsch-Schlüter factor $2q^2$ when the gradients and the rotation are weak, $g \ll 1$ and $M_0^2 \ll 1$. When either g or M_0 is not small, new and potentially important effects emerge.

First, if the gradients are weak but the rotation is significant, i.e., if $g \ll 1$ and $M_0 = O(1)$, the neoclassical flux is increased by the factor (5) over the usual Pfirsch-Schlüter result [5]. The diffusion coefficient thus becomes $D = (1 + 2\Lambda q^2)D_{cl}$, where $D_{cl} = T_i/m_i\Omega_i^2\tau_{iz}$ is the classical diffusion coefficient and $2q^2 D_{cl}$ the Pfirsch-Schlüter diffusion coefficient. The enhancement factor Λ can be quite large if M_0 exceeds unity, as is frequently the case for heavy impurities [6].

The second conclusion to draw from Eq (4) is that if the pressure or temperature gradient becomes sufficiently steep ($g \gg 1$) the neoclassical flux is suppressed since the denominator in the second term of Eq (4) depends quadratically on g. (The dependence of γ on g is typically quite weak and is unimportant in this context.) Classical transport then dominates, and the total flux is a non-monotonic function of the gradients [4]. Figure 1 shows the particle fluxes as functions of g. Conventional transport theory only covers the lower left corner of this figure. Note that the total flux (solid line) depends on the gradients in a way characteristic of bifurcating systems.

We now consider in more detail the limit $g \gg 1$, where the neoclassical transport tends to be suppressed. Expanding the solution of Eq (3) in g^{-1} gives

$$n = \frac{\gamma}{(1 - \langle(1+\gamma b^2)^{-1}\rangle)}\frac{b^2}{1+\gamma b^2} + O\left(g^{-1}\right),$$

indicating that the impurities are pushed toward the inboard side of the torus. The neoclassical cross-field particle flux now becomes

$$\langle \boldsymbol{\Gamma}^{neo} \cdot \nabla\psi \rangle = -\frac{I\langle p_z\rangle \langle \mathbf{B}\cdot\nabla\theta\rangle \gamma}{e\langle B^2\rangle (1 - \langle(1+\gamma b^2)^{-1}\rangle)}\left\langle \frac{\partial M^2/\partial\vartheta}{1+\gamma b^2}\right\rangle.$$

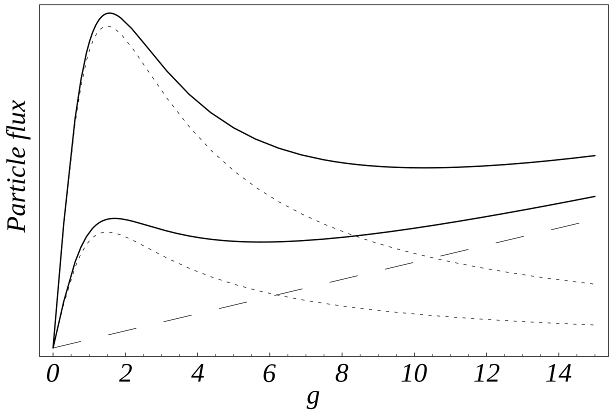

Figure 1: *Ion particle fluxes versus normalized gradient g in a large-aspect-ratio tokamak with circular cross section, $\epsilon \ll 1$, $\alpha = 0.5$. The dashed line is the classical flux, the dotted lines are neoclassical fluxes, and the solid lines represent the sum of classical and neoclassical fluxes. The lower pair of dotted and solid lines are for vanishing toroidal rotation, $M_0^2 = 0$, and the upper pair for impurity Mach number $M_0^2 = 1$. Note that the neoclassical fluxes are enhanced by finite rotation and suppressed by large gradients.*

This flux vanishes unless the rotation is finite and the equilibrium is up-down asymmetric. The residual transport, which for instance could occur in a steep edge transport barrier, has a number of surprising properties. It can be either inward or outward, and it depends on the geometry of the magnetic field in a non-trivial way.

For instance, consider the particularly simple limit $\gamma \ll 1$, $\epsilon \ll 1$, $n_z z^2 \ll n_i$. The flux then becomes

$$\langle \boldsymbol{\Gamma}^{neo} \cdot \nabla\psi \rangle = 0.33 \frac{f_c I \langle p_z \rangle}{e \langle B^2 \rangle^2 \left(\frac{d \ln n_i}{d \ln T_i} - \frac{1}{2} \right)} \left\langle B^2 \mathbf{B} \cdot \nabla M^2 \right\rangle .$$

and is thus independent of the collision frequency although it is caused by Coulomb collisions. Remarkably, it is proportional to $I = RB_\varphi$ and therefore changes sign if the toroidal field is reversed. If the density profile is at least half as steep as the temperature profile, which is normally the case in the tokamak edge, and the magnetic field has a single X-point below the midplane, the flux is inward if $\mathbf{B} \times \nabla B$ is downward, and vice versa. Thus, if the ion ∇B-drift is toward the X-point, which is experimentally favorable for attaining the high-confinement H-mode, the neoclassical bulk-ion particle flux is inward, and the impurities (whose flux is opposite to that of the main ions) are prevented from entering the plasma core.

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