Dynamics of Plasma Bunch in Weakly Inhomogeneous Magnetic Field

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Introduction

The dynamics of a plasma bunch in vacuum in the absence of magnetic field is quite predictable. The initially confined plasma bunch expands unlimitedly and cools with time [1]. It appears that a certain configuration of external magnetic field allows the bunch to accelerate and compress as a whole. Such processes are of great interest to astrophysicists and scientists working in the field of plasma fusion. Using the results of a general study of a set of two Vlasov kinetic equations with a self-consistent electric field and external potential forces [2,3] it is possible in some cases to get a detailed description of a two-component plasma bunch in nonstationary weakly inhomogeneous magnetic field of mirror configuration.

1. The physical model

The exactly solvable physical model assumes that there is an axially symmetric bunch of collisionless plasma with two sorts of particles whose distribution functions are described by three-dimensional Vlasov kinetic equations:

$$\frac{\partial f_\alpha}{\partial t} + \left(\mathbf{v},\nabla_\mathbf{r}\right) f_\alpha - \left(\frac{Z_\alpha e}{m_\alpha}\left\{\nabla_\mathbf{r}\varphi + \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}\right\},\nabla_\mathbf{v}\right) f_\alpha + \frac{Z_\alpha e}{m_\alpha c}\left([\mathbf{v}\times\mathbf{B}],\nabla_\mathbf{v}\right) f_\alpha = 0, \tag{1}$$

$$\mathbf{r} = (x,y,z), \quad \mathbf{v} = \left(v_x, v_y, v_z\right), \quad \nabla_\mathbf{r} \equiv \left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right), \quad \nabla_\mathbf{v} \equiv \left(\frac{\partial}{\partial v_x},\frac{\partial}{\partial v_y},\frac{\partial}{\partial v_z}\right),$$

where $Z_\alpha e$ and m_α are charge and mass of particles of sort α, respectively; c is light velocity; $\mathbf{A}(\mathbf{r},t)$ is vector-potential of the axially symmetric non-stationary magnetic field $\mathbf{B}(\mathbf{r},t) = \left[\nabla_\mathbf{r}\times\mathbf{A}\right]$, which is considered given; and $\varphi(\mathbf{r},t)$ is potential of charge-separation electric field, which is found in the course of the solution using the quasi-neutral approximation:

$$\sum_\alpha Z_\alpha n_\alpha(\mathbf{r},t) = 0, \qquad n_a(\mathbf{r},t) \equiv \int f_a(\mathbf{v},\mathbf{r},t) d^3\mathbf{v}. \tag{2}$$

We restrict ourselves to the case of the following configuration of the magnetic field:

$$\mathbf{B}(\mathbf{r},t) = B_0(z,t)\mathbf{e}_z + B_\rho(\rho,z,t)\mathbf{e}_\rho, \quad B_\rho(\rho,z,t) = -\frac{\rho}{2}\frac{\partial B_0}{\partial z}, \quad \rho \equiv \sqrt{x^2+y^2}; \tag{3}$$

where z is coordinate along the axis of symmetry, and $B_0(z,t)$ and $B_\rho(\rho,z,t)$ are axial and radial components of the vector $\mathbf{B}(\mathbf{r},t)$, respectively. If the function $B_0(z,t)$ is weakly dependent on its arguments, i.e.,

$$\left|\frac{1}{\omega_\alpha^2}\frac{\partial\omega_\alpha}{\partial t}\right|, \left|\frac{V_\alpha}{\omega_\alpha^2}\frac{\partial\omega_\alpha}{\partial z}\right|, \left|\frac{l_\rho}{\omega_\alpha}\frac{\partial\omega_\alpha}{\partial z}\right| << 1, \quad \omega_\alpha \equiv \frac{Z_\alpha e}{m_\alpha c}B_0, \quad l_\rho^2(z,t) \equiv \frac{\iint \rho^2 f_\alpha dxdydv_x dv_y}{\iint f_\alpha dxdydv_x dv_y} \tag{4}$$

where $\boldsymbol{V}_\alpha$ is root mean square velocity of particles of sort α, and $l_\rho(z,t)$ is characteristic plasma scale length across to magnetic field, then within adiabatic approximation one can present the distribution functions of particles in the following form:

$$f_\alpha = N_\alpha \Phi_\alpha(t,z,v_z,I_\alpha,J_\alpha), \quad I_\alpha \equiv \frac{m_\alpha\left(v_x^2+v_y^2\right)}{2B_0(z,t)}, \quad J_\alpha \equiv \frac{\omega_\alpha}{2}\left(x^2+y^2\right)+\left(xv_y - yv_x\right). \tag{5}$$

Here N_α is the number of particles of sort α, I_α is an adiabatic invariant corresponding to conservation of the magnetic moment of particles, J_α is generalised angular momentum of a charged particle in the magnetic field, and functions Φ_α satisfy the following equations:

$$\frac{\partial\Phi_\alpha}{\partial t} + v_z\frac{\partial\Phi_\alpha}{\partial z} - \frac{Z_\alpha e}{m_\alpha}\frac{\partial\varphi}{\partial z}\frac{\partial\Phi_\alpha}{\partial v_z} - \frac{1}{2}\frac{\partial\omega_\alpha}{\partial z}\left(\frac{v_x^2+v_y^2}{\omega_\alpha}\right)\frac{\partial\Phi_\alpha}{\partial v_z} = 0 \tag{6}$$

In particular, when the dependence of the initial distribution functions Φ_α on transversal variables is presented by a delta-function of the magnetic moment I_α, i.e.

$$\Phi_\alpha(t=0) = \Phi_\alpha^0(z,v_z)\delta\left(I_\alpha - I_\alpha^0\right), \qquad I_\alpha^0 = const, \tag{7}$$

the integration of Eqs. (6) over the transverse coordinates and velocities results in the following equations:

$$\frac{\partial F_\alpha}{\partial t} + v_z\frac{\partial F_\alpha}{\partial z} - \frac{Z_\alpha e}{m_\alpha}\frac{\partial\varphi}{\partial z}\frac{\partial F_\alpha}{\partial v_z} - \frac{g_\alpha}{m_\alpha}\frac{\partial B_0}{\partial z}\frac{\partial F_\alpha}{\partial v_z} = 0, \quad F_\alpha(t,z,v_z) \equiv \iint \Phi_\alpha dxdydv_x dv_y, \tag{8}$$

where in the case (7) $g_\alpha \equiv \frac{m_\alpha}{2B_0 F_\alpha}\iint\left(v_x^2+v_y^2\right)\Phi_\alpha dxdydv_x dv_y = I_\alpha^0$. Eqs. (8) are similar to one-dimensional kinetic Vlasov equations with effective external potentials $U_\alpha^{eff} \equiv g_\alpha B_0(z,t)$.

2. Dynamics of the two-component collisionless plasma bunch in weakly inhomogeneous potential external fields

A detailed analysis of equations similar to Eqs. (8) in a general three-dimensional case $f_\alpha = f_\alpha(\mathbf{v},\mathbf{r},t)$, $U_\alpha = U_\alpha(\mathbf{r},t)$ [2,3] demonstrates that since the characteristic scale lengths of the plasma bunch are much smaller than the scales of inhomogeneity of the potential of external forces (so that the latter can be described by a square form of coordinates) the equations for the coordinates of the center of mass of the plasma,

$R_k(t) \equiv N_\alpha^{-1} \iint r_k f_\alpha \prod_{k=1}^{3} dr_k dv_k$, and for the temporal dependence of the plasma scale length, $l_k(t) \equiv \left[N_\alpha^{-1} \iint (r_k - R_k(t))^2 f_\alpha \prod_{k=1}^{3} dr_k dv_k \right]^{1/2}$, can be easily obtained:

$$\frac{d^2}{dt^2} R_k = a_k , \; a_k \equiv - \frac{1}{M} \left. \frac{\partial U}{\partial r_k} \right|_{r_k = R_k} , \; U(\mathbf{r},t) \equiv \sum_\alpha N_\alpha U_\alpha(\mathbf{r},t), \; M \equiv \sum_\alpha N_\alpha m_\alpha , \tag{9}$$

$$l_k^3(t) \frac{d^2 l_k}{dt^2} + b_{kk}(t) l_k^4(t) = \left[\frac{2W_k}{M} - \left(\frac{dl_k}{dt} \right)^2 \right] l_k^2(t) = \sum_\alpha \frac{m_\alpha N_\alpha}{M} V_{k\alpha}^2(t) l_k^2(t) = const , \tag{10}$$

$$b_{ij} \equiv \frac{1}{M} \left. \frac{\partial^2 U}{\partial r_i \partial r_j} \right|_{r_i = R_i, r_j = R_j} , \; W_k \equiv \frac{1}{2N_\alpha} \sum_\alpha m_\alpha \iint \left(v_k - \frac{dR_k}{dt} \right)^2 f_\alpha \prod_{k=1}^{3} dr_k dv_k ,$$

where W_k are average energies of a particle's thermal motion along corresponding directions in the reference frame related to the center of mass of the plasma bunch.

According to Eqs. (9,10) the motion of the center of mass of the plasma is governed by the gradient of the potential $U(\mathbf{r},t)$ at the point $r_k = R_k(t)$. The spatial structure of the bunch depends only on the values of the second derivatives $\partial^2 U / \partial r_k^2$ at the same point. Therefore, acceleration of the plasma bunch and its spatial confinement can be controlled independently.

Applying the obtained results to the case of the weakly inhomogeneous magnetic field described by Eqs. (8), it can be seen that the bunch accelerates as a whole in the opposite direction to the gradient of the function $B_0(z,t)$. At the same time the equation for the longitudinal plasma scale length $l_z(t)$ suggests that the plasma can be confined within a finite region of space provided that the second derivative of $B_0(z,t)$ is positive:

$$b_z(t) \equiv \gamma \left. \frac{\partial^2 B_0}{\partial z^2} \right|_{z=Z(t)} > 0; \qquad \gamma \equiv \frac{\sum_\alpha N_\alpha g_\alpha}{M} . \tag{11}$$

If the inequality (11) is valid, the plasma does not expand unlimitedly with time and the energy of its particles' thermal motion with respect the center of mass stays finite during the process of acceleration, while the total kinetic energy of the plasma bunch can increase unlimitedly.

An exact self-similar solution of Eqs. (8) obtained in quasi-neutral approximation [2,3] is:

$$|Z_\alpha| F_\alpha = F\left(\left(\frac{z - R_z(t)}{l_z(t)} \right)^2 + \left(\frac{v_z - u_z}{V_{z\alpha}(t)} \right)^2 \right), \qquad u_z = \frac{dR_z}{dt} + \frac{z - R_z(t)}{l_z(t)} \frac{dl_z}{dt} , \tag{12}$$

$$V_{z\alpha}(t) l_z(t) = const\,, \quad e\varphi(z,t) = -\left(\sum_\alpha \frac{1}{M_\alpha}\right)^{-1} \sum_\alpha \frac{1}{Z_\alpha N_\alpha}\left[\frac{V_{z\alpha}^2(t)}{2 l_z^2(t)}\left(z - R_z(t)\right)^2 - \frac{U_\alpha^{eff}}{m_\alpha}\right]$$

3. Examples

It is remarkable that there exists a stable acceleration regime of a plasma bunch with a constant longitudinal scale length $l_z^2 = \frac{2W_z}{Mb_z} = const$. Such solution exists when the magnetic field obeys the following relationship:

$$B_0(z,t) = B_1(t) + \frac{b_z}{2\gamma}\left[\left(z - Z_0(t)\right) - \frac{1}{b_z}\frac{d^2 Z_0}{dt^2}\right]^2, \quad b_z = const\,, \tag{13}$$

where $B_1(t)$ is an arbitrary function and $Z_0(t)$ describes the desirable law of motion of the center of mass of the plasma bunch: $Z(t) = Z_0(t)$.

Another interesting example of the obtained solution is compression of the plasma bunch in an inhomogeneous magnetic field that increases with time:

$$B_0(z,t) = \frac{1}{2\gamma} b_z(t) z^2 + B_2(t)\,, \quad b_z(t) > 0\,, \tag{14}$$

where $b_z(t)$ and $B_2(t)$ are slowly increasing functions of time. In this case the compression of the plasma bunch is described by adiabatic laws:

$$B_2 l_\rho^2 = const\,, \quad W_\rho / B_2(t) = const\,, \qquad W_\rho \equiv \sum_\alpha \frac{m_\alpha}{2} \frac{\iint \left(v_x^2 + v_y^2\right) f_\alpha dx dy dv_x dv_y}{\iint f_\alpha dx dy dv_x dv_y}$$

$$l_z^2 \sqrt{b_z} = const\,, \qquad W_z / \sqrt{b_z(t)} = const\,. \tag{15}$$

Note that the ratio $B_0(l_z, t)/B_0(0,t)$ becomes smaller with time. So the approximation of the magnetic field, $B_0(z,t)$, by a square dependence upon z remains valid during the process of compression.

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