Dynamics of transport barriers in toroidal confinement systems

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Abstract Various models have been suggested for the formation of H-mode and internal transport barriers in tokamaks and other toroidal plasma confinement systems. Some of these involve a feedback mechanism whereby an increase in the plasma pressure gradient stabilises the turbulence responsible for transport and allows a further increase in pressure gradient. The transport coefficients are, effectively, nonlinear functions of the gradient. The effect of such mechanisms is to introduce terms which can lead to propagation and growth as well as spreading of perturbations, suggesting that transport barriers may require an additional mechanism to stabilise their radial position. The experimental evidence indicates that the magnetic field structure provides the 'anchor' for transport barriers.

Introduction

The understanding and control of transport barriers in toroidal magnetic confinement systems is important for improving their performance and reducing their cost. In addition to the H-mode barrier at the plasma boundary, various internal barriers have been recently identified and studied. These seem to occur at specific radial locations, often in the neighbourhood of rational magnetic surfaces [1,2] suggesting that the local reduction in transport coefficients defining the barrier is mediated by the structure of the magnetic field. An alternative and popular paradigm for barrier formation, with strong experimental and theoretical support, regards the barrier as a self-organised structure in which the reduction in transport is due to the stabilisation of turbulence by local shear in the **ExB** drift velocity, the latter maintained by the barrier itself. In this type of model, the magnetic field structure plays only a minor or insignificant role and there is no reason why a barrier should not be formed at any radius under suitable conditions, or why it should remain stationary at a fixed radius [3]. Indeed, some of the simpler models which have been proposed for barrier formation would also lead to convection of the barrier, because the radius where the drift velocity shear is a maximum is not generally at its centre, where the transport coefficients should be a minimum.

This paper explores some of the consequences of the dependence of the diffusion coefficient in a transport barrier on the first and second spatial differentials of the plasma parameters. In particular, the effects on the propagation of perturbations are examined in the approximation that the perturbations are small and that there is no time dependence in the functional relationship between the diffusion coefficient and the plasma parameters. Even in this simple case, it is clear that perturbations such as a sudden cooling, due for example to deliberate or accidental injection of solid material, can exhibit a more complex response than a simple diffusive spreading of the perturbation, reminiscent of those seen experimentally and sometimes ascribed to 'non-local' transport [4,5].

Diffusion of small perturbations

The diffusion equation is examined in the slab approximation with a diffusion coefficient

which is a function of the first and second differentials of the independent variable, representing a range of possible models for the physics of transport barrier formation.

$$\frac{\partial y}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial y}{\partial x}\right)$$

where $D = D(y', y'')$, the primes indicating differentiation w.r.t. x. Now put $y = \bar{y} + \tilde{y}$ and $D = \bar{D} + \tilde{D}$, where bars indicate 'equilibrium' values and tildes small perturbations about the equilibrium. The perturbation on D is given by,

$$\tilde{D} = \frac{\partial D}{\partial y'}\tilde{y}' + \frac{\partial D}{\partial y''}\tilde{y}'' .$$

Substitution in the diffusion equation, with cancellation of the equilibrium terms then gives,

$$\frac{\partial \tilde{y}}{\partial t} = \frac{\partial \tilde{y}}{\partial x}\left(\frac{\partial \bar{D}}{\partial x} + \frac{\partial \bar{y}}{\partial x}\frac{\partial^2 D}{\partial x\partial y'} + \frac{\partial^2 \bar{y}}{\partial x^2}\frac{\partial D}{\partial y'}\right) + \frac{\partial^2 \tilde{y}}{\partial x^2}\left(\bar{D} + \frac{\partial \bar{y}}{\partial x}\frac{\partial D}{\partial y'} + \frac{\partial \bar{y}}{\partial x}\frac{\partial^2 D}{\partial x\partial y''} + \frac{\partial^2 \bar{y}}{\partial x^2}\frac{\partial D}{\partial y''}\right)$$

$$+ \frac{\partial^3 \tilde{y}}{\partial x^3}\frac{\partial \bar{y}}{\partial x}\frac{\partial D}{\partial y''} \tag{1}$$

or, more simply,

$$\frac{\partial \tilde{y}}{\partial t} = A\frac{\partial \tilde{y}}{\partial x} + B\frac{\partial^2 \tilde{y}}{\partial x^2} + C\frac{\partial^3 \tilde{y}}{\partial x^3} ,$$

where A, B and C are slowly varying functions of x.

Equation (1) is solved by putting $\tilde{y} = \hat{y}(t)\,\exp jkx$, which on substitution gives,

$$\frac{d\hat{y}}{dt} = \left(jkA - k^2 B - jk^3 C\right)\hat{y} = \gamma\ (k)\hat{y} ,$$

which has the solution,

$$\hat{y} = \hat{y}_0 \exp\gamma\ t ,$$

where $\hat{y}_0$ is the value at $t = 0$. Evidently the full solution can be constructed from a Fourier sum of such terms. The results presented below have used the FFT routines in MATLAB [6] on a spatial grid of 256 points. The initial perturbation is Gaussian.

Model results

A difficulty with many models of transport barriers is that they rarely provide a complete solution of the diffusion equation. In some cases, the spatial form of the equilibrium is of main interest [7,8] and in others the focus is on the time dependence, with plasma parameters averaged over the assumed width of the barrier [9,10,11].

For the results presented here, the underlying equilibrium, if present, has not been calculated. However, we can make some deductions from the behaviour of small perturbations. In the case of a stable diffusive equilibrium, perturbations would be expected to decay monotonically on the diffusive time scale [5]. A tendency to propagate during decay would indicate that the 'equilibrium' is not positionally stable and a tendency to grow would indicate an unstable equilibrium.

output from pdsolv1 with gamma = j*k - k.^2

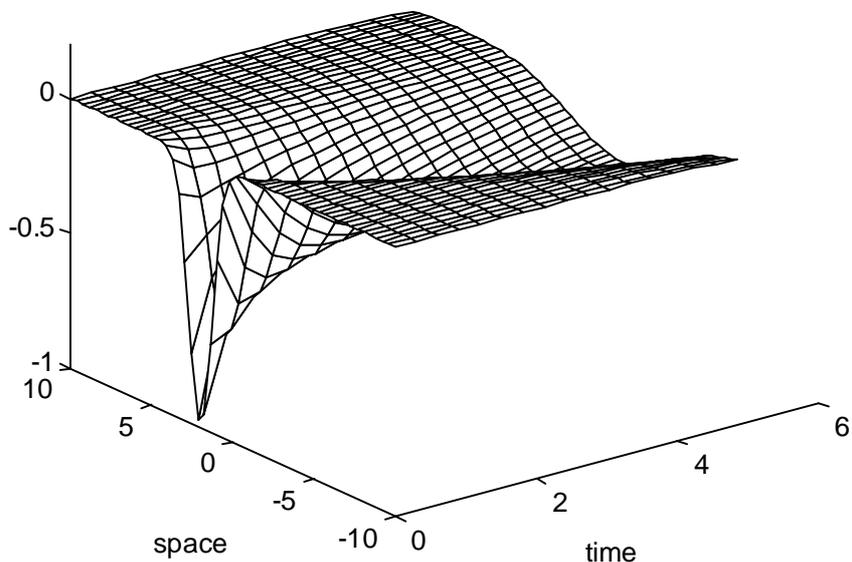*Figure 1. Response to small perturbation with A=1, B=1.*

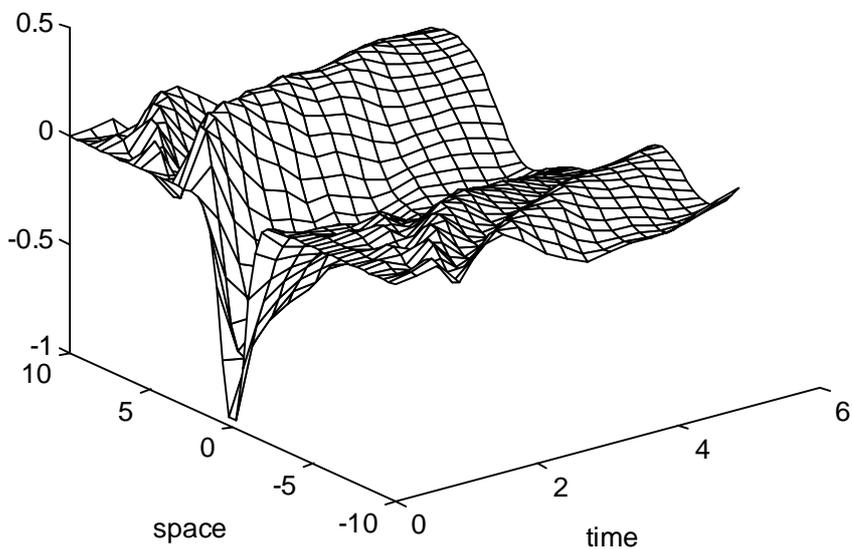output from pdsolv1 with gamma = -0.2*k.^2 - j*k.^3

Figure 2. Response to small perturbation with B=0.2, C=1.

Two examples are illustrated in figures 1. and 2. above. In the first, D is a function of the first differential only, as in the model of Hinton [8], where the form used is,

$$D = D_{NC} + \frac{D_a}{1 + \lambda_0 y'^4} \ ,$$

where D represents the thermal conductivity, D_{NC} being the neoclassical value and D_a the anomalous value due to turbulence, and y is the temperature. In the H-mode, where the turbulence is suppressed by a large temperature gradient, we would have,

$$A = \frac{16 D_a}{\lambda_0 y'^4} \frac{y''}{y'} \ , \ B = D_{NC} - \frac{3 D_a}{\lambda_0 y'^4} \ , \ C = 0 \ .$$

Clearly there is a range of y' where $B < 0$ corresponding to instability, but stability is regained when y' is sufficiently large. However, as shown by figure 1, a finite value of A would then lead to convection of a perturbation, suggesting positional instability.

A second example is the case where turbulence is stabilised by shear in the **ExB** drift. Assuming that momentum losses are high and the fluid velocity correspondingly small, this leads to a dependence of D on the second differential of the pressure, which can be regarded as the dependent variable in (1). We now have $A = 0$, B, $C \neq 0$. A particular case is illustrated in figure 2. The response is rather complex due to the added importance of high k-values in the Fourier sum. There is no obvious tendency to regain equilibrium on a short time scale; on the contrary there is a growing and travelling response in the opposite sense to the initial perturbation, a feature which resembles the 'non-local' transport effects following sudden cooling of the plasma boundary in experiments [4,5].

Conclusions

The effects of a dependence of the transport coefficient on the first and second spatial differentials of the plasma parameters, such as are required in various models of transport barriers, has been examined. The linearised response to small perturbations typically indicates that transport barriers based on such models would be subject to temporal and/or spatial instability. Non-linearities intrinsic to the models or those associated with boundary conditions may provide the stability observed experimentally. An alternative possibility, with some support from experiment, is that the magnetic field structure provides the necessary anchor.

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