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Implications of Torque Balance for the Non-Linear Resistive Wall Mode

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1 Introduction

It is recognised that unstable toroidal plasma equilibria are not, in general, stabilised by a finitely conducting wall [1]. Instead, the instability converts to the 'Resistive Wall Mode' or RWM. Nevertheless, Advanced Tokamak (AT) equilibria call for wall stabilisation in order to produce the high β values required in an economic fusion power plant [2]. Early in the analysis of the RWM it was realised that bulk plasma rotation might have an important effect. If the perturbed plasma is modelled as a rotating current-carrying helical wire, then we would expect suppression of the magnetic flux entering the wall by the classical skin effect. However, a real plasma has more degrees of freedom than a wire and the mode can lock to the wall and eliminate such skin effect. In Ref. [1] it was shown that an ideal RWM in cylindrical geometry did indeed lock to the wall for sub-Alfvénic plasma rotation. In contrast, for a RWM that is resistive in origin, creating a magnetic island at an internal plasma resonance (*e.g.* a tearing mode), the interaction between the island and the wall permits stabilisation of the mode, typically when

$$\Omega \quad \sim \quad \mathcal{O}(1/\tau_W) + \mathcal{O}(1/\tau_L) \ , \tag{1}$$

where Ω is the plasma rotation frequency, τ_L is the tearing layer characteristic time, and τ_W is the wall time. These rotation frequencies are, of course, generally much less than the Alfvén frequency. In the case of AT equilibria, the relevant RWM is the pressure driven toroidal ideal external kink mode. Although, in the absence of the wall, the mode is 'ideal' in nature, toroidicity generates sideband components of the instability that possess resonances in the plasma. There is, then, the possibility that a stabilisation mechanism similar to that leading to eq.(1) exists in the AT. This possibility was first investigated in a paper by Finn [3].

2 A standard form of the Finn model

Ref. [3] modelled a toroidal instability by constructing *cylindrical* equilibria that were ideal MHD unstable to a mode that had an $m = nq$ resonance within the plasma. This requires non-standard current profiles, but models a fundamental ingredient of toroidal RWMs - namely internal plasma resonance [4]. A somewhat generic formulation of the dispersion relation is possible. The second order (Newcomb) ODE that determines the MHD eigenfunction in a cylindrical plasma 'connects' different plasma radial stations in a simple way [1]. The radial stations of interest are the resonance (where a resistive layer forms) and the wall itself, where a simple 'thin shell' response is appropriate [1]. Assuming $\exp(pt)$ time dependence, we find the standard form

$$(p\tau_L)^{5/4} \quad = \quad \frac{1 - \delta p\tau_W}{-\epsilon + p\tau_W} \ . \tag{2}$$

The LHS is the Δ' parameter at the resonance, with τ_L the characteristic resistive layer time. This is related to the wall response, $\Delta'_W = p\tau_W$, as shown on the RHS. In (2) $\epsilon = p_I\tau_A$ is a measure of the ideal growth rate in the absence of a wall ($\tau_W \to 0$, and $\Delta'_{pI} \to -1/(p\tau_A)$). The parameter δ is a measure of the stability of the tearing mode that exists if the wall is perfect ($\tau_W \to \infty$).

3 Stability structures within the standard model

For a non-rotating plasma, eq.(2) with ϵ and δ both positive *always* yields an unstable root, and this is the RWM. To simulate rotation we Doppler-shift $p \rightarrow p - i\Omega$ in the LHS of eq.(2), where Ω is the plasma rotation frequency at the resonance. One then finds that marginal stability can only be achieved, as Ω varies, for

$$0 < \epsilon\delta < 0.04 . \tag{3}$$

The smallness of this region implies stringent conditions on specific equilibria. However, even when eq.(3) is satisfied a stable region is not guaranteed. Analytic and numerical investigation of eq.(2) within the range of eq.(3) revealed that four different topologies exist when the roots are tracked in the complex p plane as Ω is increased. When $\Omega = 0$ there are three physical roots of eq.(2) - an unstable non-rotating mode, the 'RWM', and forward (f) and backward (b) rotating stable 'tearing' modes. When Ω is increased, the f mode always remains stable. The RWM and b modes between them share two marginal points in different ways, and the situation can be summarised in Fig. 1. (In what follows,

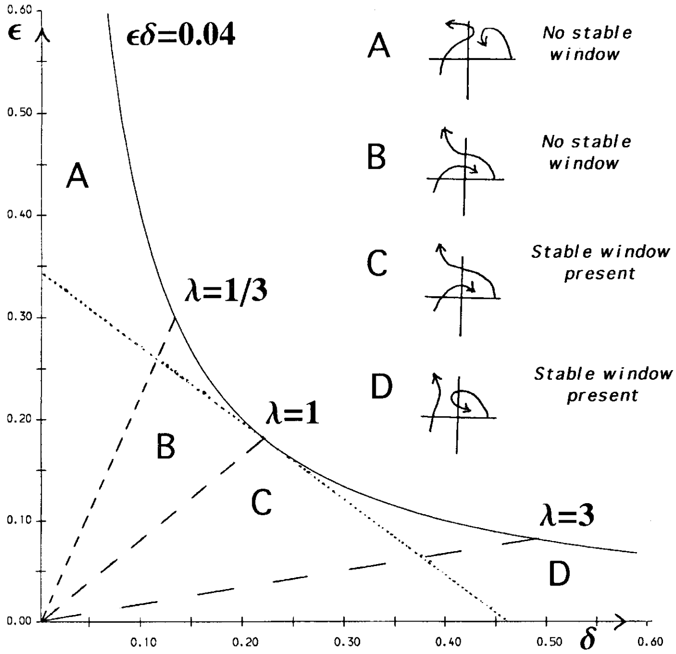

Figure 1: $\epsilon - \delta$ *space and topology changes*

disregard for the time being the two dashed lines of Fig. 1, labelled $\lambda(\equiv \tau_L/\tau_W) = 1/3$ and 3, as we will discuss the case $\lambda = 1$).

The area given by eq.(3) divides into four regions A-D. They are demarcated by topology changes in complex p space, as shown schematically in the top right hand side of the figure. In region A the RWM is never unstable while the b mode undergoes a window of instability. A topology change occurs across the dotted line as region B is entered. Here, the b mode is destabilised *before* the RWM stabilises, so neither region A or B possesses a stability window. In region C, however, the b mode is destabilised *after* the RWM stabilises, so C displays a stability window in Ω. Another topology change occurs as we move into region D. Here, the RWM itself displays a stability window while the b mode remains stable. Finally, we remark that as λ is made smaller, the region in which a stability window exists expands (relative to the plasma, the wall is becoming a better conductor).

4 Torque balance and stability windows

So far we have assumed that Ω is a given quantity. However, as the RWM grows, it will exert a torque across the resonant layer [5] which has the general form

$$\mathcal{T}_{em} \sim (\delta b_r)^2 \mathcal{I}(\Delta') , \quad (4)$$

where δb_r is the perturbed radial magnetic induction, and $\mathcal{I}$ denotes imaginary part. Because of conservation of total torque on the system, we can choose to evaluate eq.(4) either at the tearing layer, or at the wall, and we choose the latter. This torque has to be balanced against a viscous plasma torque and a given driving torque. So, if Ω_0 is the rotation frequency of an unperturbed plasma equilibrium and Ω is the actual bulk plasma rotation (in the presence of an RWM), the viscous torque will be proportional to the difference between them. Accordingly, a simple steady state model of torque balance is

$$\Omega_0 - \Omega = C\omega . \quad (5)$$

Here, ω is the *mode* rotation frequency, and the quantity C encapsulates the ratio of the electromagnetic and viscous torques [6]. We consider a case in region D of Fig. 1, where a stability window occurs for $\Omega_1 = 2.1 < \Omega_2 = 4.5$. Solving eq.(2) for $\omega(\Omega)$ we then use eq.(5) to construct traces of $\Omega(C)$ for various Ω_0, and this is shown in Fig. 2. We see immediately that despite the apparent simplicity of eq.(5), there is multi-valued

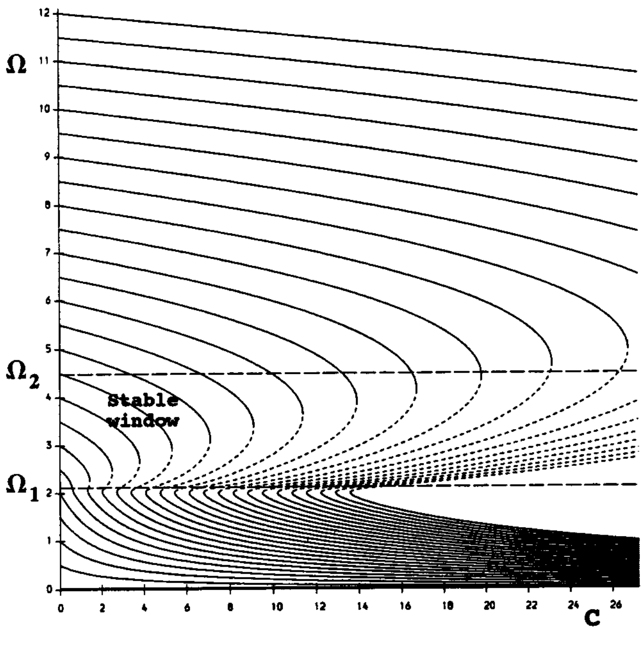

Figure 2: *Path of the RWM in $\Omega - C$ space*

structure in the result. This multi-valuedness gives rise to the so-called 'forbidden' bands of plasma rotation [7]. If, for instance, the system is started off with a rotation frequency less than Ω_1, then the RWM is unstable and as the mode grows C will increase while Ω monotonically decreases, asymptoting to zero. If, of course, the system is started off with $\Omega_1 < \Omega < \Omega_2$, then the system is stable and would remain in the window. If the system is initiated with Ω somewhat greater than Ω_2, then the RWM is again unstable, leading to growth in C. Once more this will lead to a reduction in Ω, but now this growth will be arrested when Ω drops to the top of the stability window at Ω_2. In this case we see that, *at the expense of a saturated island being present*, the effective stability window has been enlarged (the top boundary of the window has moved from 4.5 to ~ 8, as explained below). The last possibility in this regime is that which occurs when the initial Ω defines a curve in Fig. 2 for which the top 'knee' is above Ω_2 (this happens whenever the initial Ω is greater than ~ 8 (not shown)). Again, the RWM is unstable, C will increase and Ω decrease until the knee is encountered. At this point, it is well known that the system does not follow the re-entrant track of the curve (the dotted sections of Fig. 2) as this is unstable [7]. Instead, it is forced to migrate vertically down the graph to meet up with the lower branch of the curve. Further, this point (for the ϵ and δ chosen) is beneath the stability window - so that C continues to increase and the island asymptotically locks to the wall. Other possible cases occur in region C, and this will be reported in Ref. [6].

Conclusions and Discussion

We have investigated a non-linear model of the rotational stabilisation of the Resistive Wall Mode (RWM). The system is a qualitative model for the actual toroidal external kink mode that is relevant in AT scenarios. The rotation frequencies required for RWM stabilisation are of order the inverse wall, or resistive layer, time. Other theory on RWM stabilisation requires much higher values, typically a few percent of the Alfvén frequency. However, the model shows that the parameter regime in which stabilisation can take place is small. Essentially, the plasma has to be only slightly ideal unstable in the absence of a wall, and slightly tearing stable were the wall perfect. Within this small parameter regime, stability windows in Ω_0 can be considerably extended, but only at the expense of the growth of a magnetic island, which may degrade confinement. It would appear, then, that rotational stabilisation of the RWM may not be a reliable option.

A possibility for stabilising all RWMs was suggested in Ref. [8]. Here a rotating secondary wall was envisaged (simulating a flowing lithium blanket). The RWM clearly cannot lock to both walls simultaneously, and RWM stabilisation occurred at $\Omega \sim \mathcal{O}(1/\tau_{W1}) + \mathcal{O}(1/\tau_{W2})$, so the required rotation rate is determined largely by the inverse time constant of the least conducting wall. (Essentially, this is the same effect that led to eq.(1).) It was found that the requirement on the *position* of the secondary rotating wall was that it should be within the marginal point of the most unstable mode that exists with no wall. Recently, this idea has been further developed in Ref. [9] where it was shown that a network of conductors external to the plasma could be practicably configured so as to simulate a 'fake' rotating wall. This scheme, then, would appear to be more power plant relevant than the approach of inducing bulk plasma rotation.

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