Dynamics of runaway electrons in tokamak disruptions

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Abstract

The kinetic theory of runaway electrons is extended to account for radiation reaction and radial diffusion. It is found that the combined effects of pitch-angle scattering and radiation reaction from synchrotron radiation (caused by gyro-motion) can effectively damp a beam of relativistic electrons. This appears to explain the decay of post-disruption runaway currents in JET. A 3D Monte Carlo code simulating runaway avalanches caused by close Coulomb collisions has been constructed, which includes the effects of radiation reaction and radial diffusion caused by small-scale magnetic perturbations. The latter can prevent runaway avalanches if the transport is sufficiently strong.

Introduction

Runaway electrons produced in tokamak disruptions can be a severe problem since their loss to the first wall may cause localised surface damage. The situation is particularly serious in large tokamaks, where avalanches of runaway electrons can be created by close Coulomb collisions with thermal electrons [1]. It is therefore important to understand the physics behind the generation and loss of runaway electrons. The present paper is devoted to two topics in this area: damping of runaway current by synchrotron radiation, and avalanche mitigation by radial runaway diffusion.

Emission of synchrotron radiation

In JET, a large (about 1 MA) runaway current sometimes persists long after a disruption, showing a smooth decay on a time scale of one or two seconds. This decay cannot be explained by collisional drag alone, and it has been proposed that it could be caused by the emission of synchrotron radiation [2]. The velocity vector of a runaway electron is initially nearly parallel to the magnetic field, but needs only be scattered slightly to acquire Larmor rotation that leads to substantial synchrotron radiation. Since the radiation from a relativistic particle is emitted in a beam centred around the velocity vector, the reaction force is mainly in the parallel direction although it is the perpendicular motion that causes the radiation. Taking the gyro-average of the relativistic Abraham-Lorentz force gives

$$\left\langle \frac{dp}{dt} \right\rangle_{\mathrm{rad}} = -\frac{\sqrt{1+p^{-2}}}{\tau_r}\left(p_\perp^2 + \frac{\rho_0^2}{R^2}p_\parallel^4\right),\tag{1}$$

where $\mathbf{p} = \gamma\mathbf{v}/c$ is the normalised momentum, $\tau_r = 6\pi\epsilon_0(m_e c)^3/e^4 B^2$ is the radiation time scale, and $\rho_0 = m_e c/eB$. The energy loss associated with the curvature of the magnetic field (the term proportional to $p_\parallel^4$) is insignificant in JET.

The kinetic equation for a beam-like $(p_\perp \ll p_\parallel)$ distribution of strongly relativistic electrons experiencing an electric field, Coulomb collisions, and synchrotron radiation reaction is

$$\tau \frac{\partial f}{\partial t} + \left(E - 1 - \sigma p_\perp^2\right) \frac{\partial f}{\partial p_\parallel} = \frac{1+Z}{2p_\perp} \frac{\partial}{\partial p_\perp} \left(p_\perp \frac{\partial f}{\partial p_\perp}\right), \tag{2}$$

where τ is the Coulomb collision time, Z the effective ion charge, $\sigma = \tau/\tau_r$, and $E = -eE_\parallel\tau/m_e c$ the normalised electric field. The latter is induced by the current decay,

$$E = -\frac{l\tau}{I_A} \frac{dI}{dt}. \tag{3}$$

Here $I_A = 4\pi m_e c/\mu_0 e = 0.017$ MA is the Alfvén current, and l is proportional to the plasma inductance $L = \mu_0 lR/2$. As long as the perpendicular momentum remains small, $\sigma p_\perp^2 < E - 1$, the particle is accelerated by the electric field (if $E > 1$), but if pitch-angle scattering (on the right side of the equation) makes $p_\perp$ larger, the radiation reaction force slows down the particle. Equation (2) is separable, and its solution can thus be written as a superposition of eigenfunctions

$$f_{mn\lambda}(x,y,p,t) = e^{-\lambda p - (x^2+y^2)\sqrt{\lambda}/2} H_m\left(x\lambda^{1/4}\right) H_n\left(y\lambda^{1/4}\right) T_{mn}(t),$$

where $p_\perp^2 = [\sigma(1+Z)/2]^{1/2}(x^2+y^2)$, H_n are Hermite polynomials, and the functions T_{mn} are determined by Eq (3). The coefficients in the expansion of f in these eigenfunctions are determined by the initial condition, which is given by the distribution function of avalanche runaway electrons immediately following the disruption [1],

$$f(x,y,p,0) \propto \delta(x)\delta(y)e^{-p/2\ln\Lambda}.$$

Thus solving Eqs (2)-(3) gives the following expression for the post-disruption current

$$I(t)\, e^{\frac{l}{2\ln\Lambda}\frac{I(t)-I_0}{I_A}} = I_0\, g(t)e^{-\left(\frac{1}{2\ln\Lambda}+\beta\right)\frac{t}{\tau}}, \tag{4}$$

$$g(t) = 2\left(\sum_{n=0}^{\infty}\left(-\frac{1}{4}\right)^n \frac{(2n)!}{(n!)^2}e^{-2\beta nt/\tau}\right)^2,$$

where we have assumed that the current density is radially uniform. The exponential on the left-hand side of the first of these equations contains the effect of the plasma inductance, while the first term in the exponential on the right describes collisional drag. The terms containing $\beta = \sqrt{(1+Z)\sigma/\ln\Lambda}$ reflect the combined action of pitch-angle scattering and synchrotron radiation, which is the dominant damping mechanism in JET. Figure 1 shows the calculated runaway current decay, which agrees fairly well with the experimental observations.

Radial diffusion

We now turn our attention to the production of runaway electrons by close Coulomb collisions, and ask the question whether sufficiently strong radial diffusion can interrupt
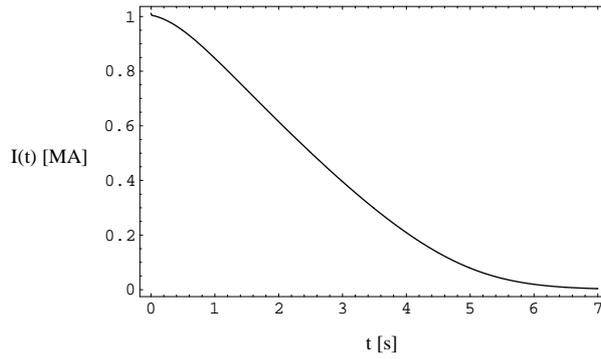
Figure 1: *Runaway current vs time as calculated from Eq (4) for JET parameters.*

a runaway avalanche. At high energies, the runaways obey the kinetic equation

$$\tau \frac{\partial f}{\partial t} + (E - 1) \frac{\partial f}{\partial p_\|} = \frac{\tau}{r} \frac{\partial}{\partial r} r D(p_\|) \frac{\partial f}{\partial r}, \tag{5}$$

where D is the diffusion coefficient and pitch-angle scattering has been ignored. The source of fast electrons from close collisions can also be neglected for large p [1]. Instead, the avalanche mechanism can be accounted for by a boundary condition at low $p_\|$, say at $p_\| = p_*$,

$$(E - 1) f(p_*, r, t) = \gamma_r \tau n_r(r, t) = \gamma_r \tau \int_{p_*}^{\infty} f(p, r, t) \, dp,$$

which provides a flux of electrons into the high-energy region. This flux is proportional to the number of existing runaways and to the growth rate γ_r calculated in Ref [1].

Integrating the kinetic equation (5) whilst retaining only the lowest-order radial (Bessel) eigenfunction, gives an integral equation for the runaway density

$$n_r(t) = \int_0^{\infty} n_r(t - t') K(t') dt', \tag{6}$$

$$K(t') = \gamma_r \exp \left[-k^2 \int_0^{t'} D(p_* + (E - 1)s) ds \right],$$

where $k = 2.40/a$ and a is the minor radius of the plasma column. This equation has a clear physical interpretation. The runaways at time t are created as a result of close collisions between thermal electrons and runaways originating from earlier times $t - t'$. If there were no diffusion, the latter would have contributed a number $\int \gamma_r n_r(t - t') dt'$ of new runaways. Since some of them have diffused out of the plasma their contribution to new runaways is reduced and is instead described by $K(t')$. Equation (6) provides the means to calculate the avalanche growth rate in a situation where the runaways are subject to radial diffusion but become progressively better confined as they are accelerated by the electric field. The exact dependence $D(p_\|)$ of the diffusion coefficient on the momentum is sensitive to the mode structure of the magnetic fluctuations. Regardless of its absolute magnitude, D generally decreases so rapidly with $p_\|$ that $K(t')$ remains finite as $t' \to \infty$ [3,4]. Equation (6) thus always predicts a positive growth of the runaway population. However, the energy loss from synchrotron radiation (1) limits the energy, and hence the confinement, of the runaways. Thus, diffusion of fast electrons slows down the growth of the runaway avalanche, and halts it altogether if the diffusion coefficient becomes large

enough. (Synchrotron radiation is important to resolve this limit but is unlikely to be significant during a disruption since the electric field is then very large.) For practical purposes it is sufficient to reduce the growth rate to a level where there is not enough induced electric field to achieve significant avalanche growth. The amplitude of the magnetic fluctuations required for this depends sensitively on their mode structure.

Monte Carlo simulation

In order to verify these ideas, we have constructed a three-dimensional Monte Carlo code that numerically solves the orbit-average of the kinetic equation for runaway electrons in toroidal geometry. The effects of collisional drag, pitch-angle scattering, parallel electric field, synchrotron radiation reaction, and radial diffusion are all included in the simulation. A source of new runaways is provided by the inclusion of the leading-order term in the quantum mechanical Møller scattering formula for close collisions between runaway electrons and thermal ones [5].

When there is no radial diffusion, the calculated growth rate of runaway avalanches agrees well with Ref 1. When enough radial diffusion is added, so that the runaway confinement time becomes comparable with the growth rate, the latter falls noticeably. In agreement with the analytical results derived above, if there is no synchrotron radiation the growth rate approaches zero asymptotically, but always stays positive, as the diffusion coefficient increases. However, when the radiation reaction force is included in the calculation, the growth rate vanishes for some finite (but large) D_0, see Fig 2. In either case, enough radial diffusion effectively prevents the runaway avalanche.
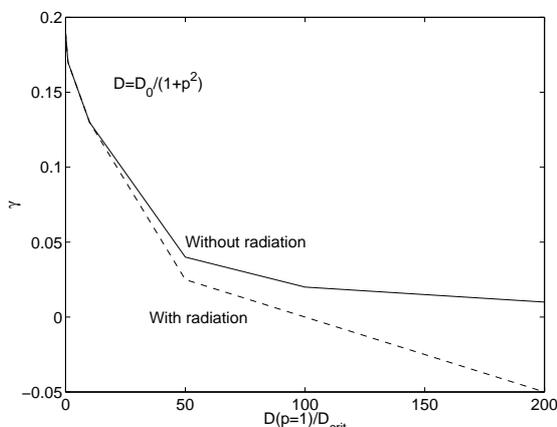
Figure 2: *Runaway avalanche growth rate vs $D(p = 1)$ for a model diffusion coefficient $D(p) = D_0/(1 + p^2)$ and $E = 10$. Strong diffusion reduces the avalanche growth rate and, combined with synchrotron radiation, prevents the avalanche altogether.*

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