

Global, Two-fluid, Electromagnetic, Nonlinear Simulations of Tokamak Turbulence and Transport Scaling Laws

A. Thyagaraja

*EURATOM/UKAEA Fusion Association, Culham Science Centre,
Abingdon, Oxon OX14 3DB, UK*

1. Introduction: The CUTIE global, nonlinear, electromagnetic turbulence simulation code¹ has been developed further and used to study scalings of global particle and energy confinement in systematic scans of ρ_* , β , ν_* and isotope mass at constant q and geometry. The code results show gyro-Bohm dependence of energy and particle confinement times on ρ_* for the larger values of this parameter. In accordance with ITER, JET and DIII-D data bases, the dependence of the turbulence on ν_* is found to be weak. However, in agreement with previous CUTIE results, there is a significant degradation of confinement with β . Thus the simulations imply $\tau_E \propto \tau_{Bohm} \rho_*^{-1} \beta^{-1.5}$. While the degradation with β is consistent with L-mode scaling, the ITER98H scaling gives a weaker β dependence ($\propto \beta^{-0.5}$). Simulations with flows show substantial improvement in confinement depending on whether the $\mathbf{E} \times \mathbf{B}$ flow shear is increased or decreased by the imposed toroidal flow. Simulations under realistic high β COMPASS-D conditions² with monotonic q profiles indicate stabilization of turbulence in the core where an internal transport barrier in electron and ion temperatures and particle density is predicted to occur. Verification or otherwise of these predictions will be addressed in future experimental campaigns. The simulations suggest generally that turbulent transport in systems where the sources are prescribed (as opposed to gradients) has an intermittent or ‘bursty’ character, and is strongly influenced by transient ‘corrugations’ of the plasma density, pressure and current profiles generated by the nonlinear modal interactions and the self-consistent radial electric fields and bootstrap currents implied by them. The CUTIE code solves the well-known¹ two-fluid, quasi-neutral plasma equations of motion for the variables: $n_e, T_e, T_i, V_{\parallel}$, the potentials, ϕ, ψ , and the vorticity, $\Theta [\equiv \nabla \cdot (n \nabla_{\perp} \phi)]$. Periodic cylinder equilibrium including mode-coupling and destabilizing curvature effects. The equilibrium profiles and the turbulent fluxes are self-consistently co-evolved. Thus, the fields are evolved by:

$$\begin{aligned} \frac{\partial \hat{\Theta}}{\partial t} + i\mathbf{k} \cdot \mathbf{v}_0 \hat{\Theta} + i\bar{k}_{\parallel} \bar{V}_A \rho_s^2 \nabla_{\perp}^2 \hat{\psi} &= \hat{\Sigma}_{\Theta} \\ \frac{\partial \hat{\psi}}{\partial t} + i\mathbf{k} \cdot \mathbf{v}_{e0} \hat{\psi} + i\bar{k}_{\parallel} \bar{V}_A \hat{\phi} &= \hat{\Sigma}_{\psi} \\ \Sigma_{\Theta} &= \bar{V}_A \rho_s \frac{1}{r} \frac{\partial \psi^*}{\partial \theta} \frac{4\pi \rho_s}{cB} \frac{dj_0}{dr} + \bar{V}_{th} \rho_s \frac{1}{r} \frac{\partial(\psi^*, \rho_s^2 \nabla_{\perp}^2 \psi^*)}{\partial(r, \theta)} \\ &+ \bar{V}_{th} \rho_s \left[\frac{1}{r} \frac{\partial(\Theta^*, \phi^*)}{\partial(r, \theta)} + \left(\frac{N^* T_{i0}}{n_0(r) T^*} \right) \frac{1}{r} \frac{\partial(\Theta^*, n^*)}{\partial(r, \theta)} \right] \\ &- \frac{2\bar{V}_{th} \rho_s}{R_0} \left[\frac{\cos \theta}{r} \frac{\partial p^*}{\partial \theta} + \sin \theta \frac{\partial p^*}{\partial r} \right] + \nabla \cdot (D_{\Theta} \nabla \Theta) \end{aligned}$$

$$\begin{aligned} \Sigma_\psi &= \bar{V}_{th}\rho_s \left[\frac{1}{r} \frac{\partial(\psi^*, \phi^*)}{\partial(r, \theta)} - \left(\frac{N^*T_{e0}}{n_0(r)T^*} \right) \frac{1}{r} \frac{\partial(\psi^*, n^*)}{\partial(r, \theta)} \right] \\ &+ \bar{V}_A \left(\frac{N^*T_{e0}}{n_0(r)T^*} \right) \nabla_{\parallel} n^* + \nabla \cdot (D_\psi \nabla \psi) \end{aligned}$$

In addition, the codes solves the electron continuity, ion parallel momentum and the two energy equations. Detailed definitions of various symbols and descriptions of the solution procedure can be found in Ref. 1. The fluctuating potentials, $\delta\phi, \delta\psi$ are in Gaussian units with nondimensional forms: $\phi^* = \frac{e\delta\phi}{T}, \psi^* = \frac{\delta\psi}{B_0\rho_s\beta^{1/2}}$. Aliasing at high k is avoided by a turbulent viscosity, $\nu_{turb} \simeq V_{thi}R(\delta j^2 + \delta\Theta^2)$.

2. Global confinement scaling studies: The code has been applied to study global confinement (τ_E/τ_{Bohm}) as a function of ρ_*, β, ν_*, A and Ma , where $A = m_i/m_e$ and Ma is the flow Mach number of any imposed toroidal plasma flows. It includes electromagnetic effects measured by the ‘drift Alfvén’ parameter, $\Delta_A \equiv \frac{\omega_*}{\omega_A} = \frac{\omega_*}{V_A k_{\parallel}} \propto \beta^{1/2} \left(\frac{\rho_s k_{\perp}}{L_n k_{\parallel}} \right)$. Fixing $Z_{eff}, R/a, q_a$, the parameters, ρ_*, ν_*, β, A have been separately varied. Conditions studied: $\beta \simeq 0.8\%, \beta_N \simeq 1.55, \nu_* \simeq 0.14 - 0.28, \rho_* \simeq 0.05 - 0.015, q_a = 3.5, R/a = 3, Ma = 0$. The following results were obtained: τ_p, τ_E have gyroBohm dependence on ρ_* , except for the smallest value ($=0.015$) tried. There is little or very weak ν_* dependence (consistent with the ITER scaling, $\nu_*^{-0.11}$). A scan over β shows degradation with β in both particle and energy confinement: $\frac{\tau_p}{\tau_{GB}} \simeq \beta^{-1}$ when β_N is varied from 1.55 to 2. For these conditions, $\frac{\tau_E}{\tau_{GB}} \simeq \beta^{-3/2}$. This is like L-Mode but different from JET, DIII-D databases in H-Mode. A favourable isotope effect is found when $\rho_*, \beta, \nu_*, q_a, Ma$ are fixed and A is varied. In many cases, at high β_N , the (2, 1) mode is prominent (cf. Ref. 2) and grows algebraically with time. CUTIE does not have the full stabilising Glasser/Shafraanov shift effects of toroidal equilibrium geometry and may overestimate confinement degradation with β . Rotation effects were studied by imposing a specified toroidal flow velocity, $V_\zeta(r) = MaV_{thi}$, in terms of Ma . The resultant $E_r = \frac{1}{en_{i0}} \frac{dp_{i0}}{dr} + V_\zeta(r) \frac{B_\theta}{c}$. It is found that changing from $Ma = 0.5$ to -0.5 makes a difference to the radial electric field shear and the confinement. Confinement times virtually double when the shearing rate is enhanced by flow. The effect also works at higher $\beta_N (= 2)$, tending to stabilize the (2, 1) mode.

3. Corrugated profiles and meso-scale dynamics of transport barriers: The turbulent fluxes are *quadratic* in the amplitudes which implies, in addition to the time and magnetic-surface averaged flux components which are usually described in terms of an ‘effective transport matrix’, the existence of ‘corrugated’ components. These drive relatively rapid (‘mesoscale’) spatio-temporal variations in the density, temperature and current density profiles. This transient flux effect of turbulence on the $m = n = 0$ plasma properties such as the radial electric field and plasma current density has important feed-back effects on the growth, saturation and spectral distributions of the turbulence. Recent calculations with CUTIE have simulated internal transport barriers (ITBs) which have been seen on several machines. In a simulation of COMPASS-D in ECH heated conditions², transport barriers form ‘spontaneously’ near the $q = 4/3$ surface ($q_0 \simeq 1.1$).

Both density and temperatures appear to develop sharp gradients close to the resonant point and this implies an increase in the local bootstrap current as well as the radial electric field shear. In turn, these quantities react on the turbulence. The q profiles tend to flatten locally and the turbulence is suppressed in a region close to the rational surface. Some simulation results are illustrated in Figs.1a-d for centrally heated (ECH) plasmas in typical COMPASS-D conditions: $R_0 = 0.55\text{m}$, $a = 0.2\text{m}$, $\langle n_e \rangle \simeq 10^{19}\text{m}^{-3}$, $B_\phi = 1.1\text{T}$, $I_p \simeq 112\text{kA}$, $q_0 = 1.14$, $q_{95} = 3.54$, $P_{ECH} = 1\text{MW}$ radially distributed like $\exp -(r/a)^2$, $\rho_* = 0.05$, $\beta = 1.9\%$, $\beta_N \simeq 3$. Both T_e, T_i also show strong barriers near $q = 1.33$, with T_e gradients of order 1 keV/m at the ITB. Movies have been made of the transient dynamics of the process. They suggest that close to rational values of q , reduced magnetic shear and increased electric drift shear have a synergistic effect on the turbulence and introduce profile corrugations which in turn support the barriers. These findings also apply to TEXT-U³ and RTP⁴ conditions. Simulations for the TEXT-U conditions (ECH heated discharge with a $q = 1$ surface) are illustrated in Figs. 2a,2b. The electron temperature and density (not shown) develop ‘ears’ near the $q = 1$ barrier(Fig.2a). There is a slowly growing $m = 1$ ‘snake’ (cf. Ref.3) which actually occurs just within the barrier(Fig. 2b).

4. Conclusions: The results presented suggest that CUTIE simulations are in qualitative agreement with some of the remarkable results obtained by the RTP group⁴, and the observations in TEXT-U³. They indicate interesting structural features in the turbulence: there appears to be a strong inverse cascade to relatively long wavelength, low n ballooning type modes in addition to gross modes like ‘snakes’, which form near mode rational radii. The mode dynamics (rotation, oscillations) is often complex and involves locking, periodic relaxation oscillations and bursts of intermittent, high frequency turbulence. The frequency and wave number spectra are computed and have been analysed. They reveal a wealth of fascinating detail, many of which resemble experimental observations in JET and other machines. It is therefore reasonable to conclude that the dynamical system modelled by CUTIE is sufficiently rich to capture several interesting features of actual tokamaks. In particular, it seems important to recognize that features like the ‘ears’ in the profiles observed in RTP and TEXT-U are indicative of the complex nonlinear, nonlocal interactions between rapidly varying components of the turbulent fluxes and the plasma profiles. Oscillatory, highly sheared radial electric fields and bootstrap currents appear to play key roles in the evolution of transport barriers as simulated by CUTIE and possibly also in real tokamaks.

Acknowledgements: This work was funded jointly by the UK Dept. of Trade and Industry and Euratom.

References

- ¹ A. Thyagaraja, C.N. Lashmore-Davies, W. Han and W-H. Yang, *Proc. 24th EPS Conf. on Contr. Fusion and Plasma Phys, Berchtesgaden*, **21A**, p. 277 (1997) and references therein.
- ² C.D. Warrick *et al*, *Proc. 25th EPS. Conf on Contr. Fusion and Plasma Phys. Prague*, **22C**, p. 1418 (1998) and Ref. 2 therein.
- ³ F. Porcelli, E. Rossi, G. Cima and A. Wootton, *Phys. Rev. Lett.*, **82**, p. 1458 (1999).
- ⁴ N.Lopes Cardozo *et al*, *Physica*, **20**, p. 169 (1998) and *Electron Transport Barriers in Tokamak Plasmas*, M.R. de Baar, Ph.D Thesis (Technical Univ. Eindhoven, 1999).

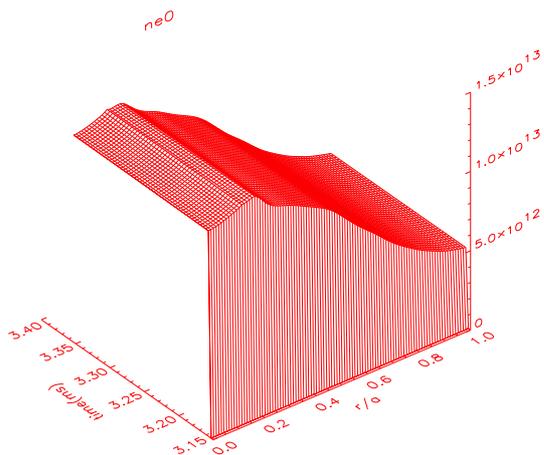


Fig.1a $n_{e0}(r, t)$, for COMPASS-D

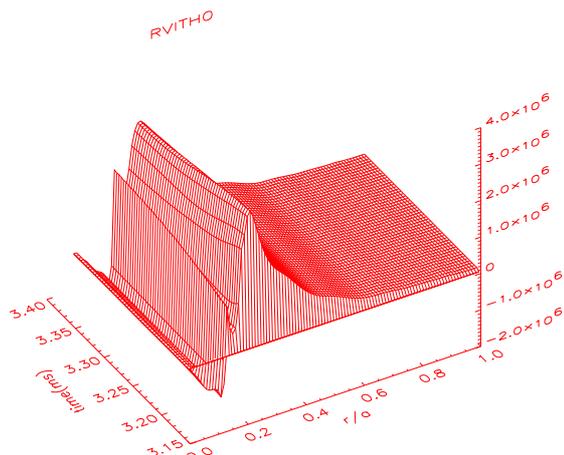


Fig.1b $cE_r(r, t)/B$ for COMPASS-D

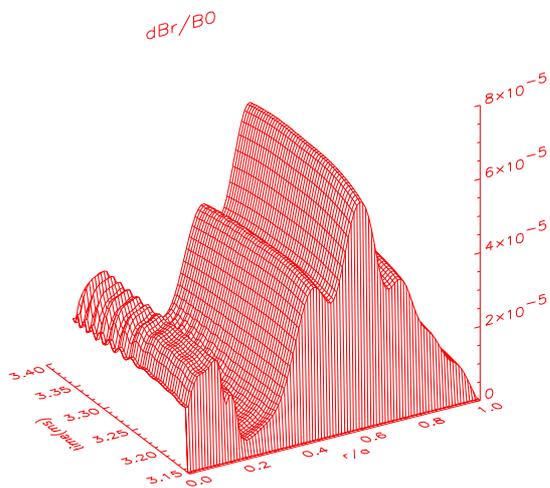


Fig.1c $\delta B_r(r, t)/B(rms)$

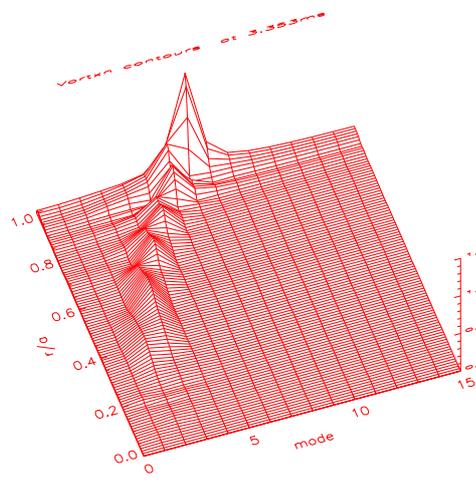


Fig.1d Enstrophy spectrum(3.35 ms)

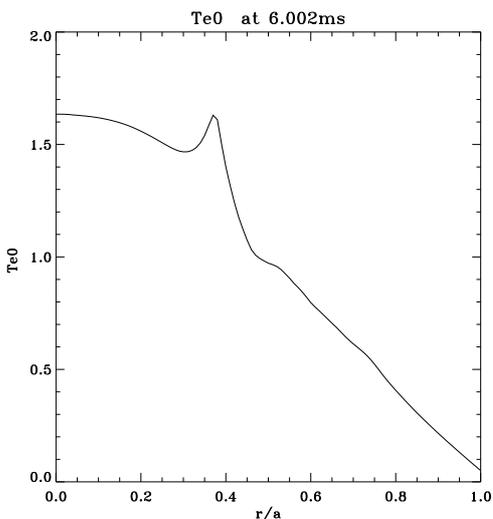


Fig.2a. T_e , for TEXT-U (Note 'ears')

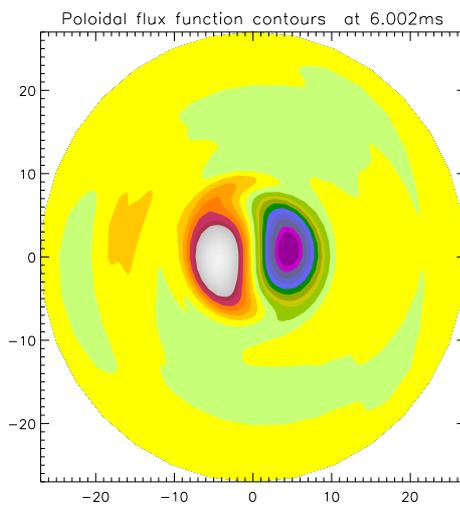


Fig.2b. ψ^* , for TEXT-U ($m=1$ 'snake')