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MARFE and helical radiation-cooling instabilities in tokamaks.

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Abstract

The application of linear stability theory to helical modes driven by impurity radiation, at the edge of tokamaks, is studied. Growth rates and the spatial structure of helical perturbations have been obtained and a stabilising effect due to the diamagnetic heat flow (Righi-Leduc) has been found. It therefore addresses the issue as to why in experiments only axisymmetric MARFEs are generally observed.

Interactions between the plasma particles and the walls of tokamaks produce impurities which contaminate the plasma. Contamination makes it more difficult to achieve net fusion energy conditions, because:

A) Impurities in the core of the plasma, particularly medium/high-Z elements, generate strong line and bremsstrahlung radiation [1], so reducing the central temperature and hence the fusion power output P_F, potentially preventing ignition [2].

B) Low-Z impurities such as oxygen and carbon, generate line radiation causing cooling which can also lead to shrinking of the current profile which can destabilise tearing modes and eventually trigger a disruption [3,4,5]. The radiated power increases as $n_e n_Z$, where n_e and n_Z are the electron and impurity number densities respectively, so that density limits arise. At lower densities another instability is also observed the multifaceted asymmetric radiation from the edge (MARFE). MARFES are poloidally localised, toroidally symmetric ($m = 1, n = 0$) radiation bands whereas the detached plasma instability is poloidally symmetric ($m = n = 0$). It has been a puzzle why these radiation bands do not appear in a helical form i.e. having $m, n \neq 0$ (following the helical magnetic field geometry). In this letter we present a theoretical model describing these phenomena and find an explanation of the helical mode stabilisation by introducing the Righi-Leduc heat flow.

The Braginskii fluid equations [6] for which a more accurate set of transport coefficiants is given by Epperlein and Haines [7] describing, density n_a, parallel fluid velocity $u_{||a}$ (assumed to be the dominant velocity), and the temperatures T_a, for electrons and ions, are used to study radiative instabilities,

$$\frac{3}{2}\frac{\partial(n_a T_a)}{\partial t} + \nabla\cdot\left(\frac{5}{2}n_a T_a u_{||a}\right) + \nabla\cdot q_a = H_a - n_a n_Z L(T_a) + Q_a \quad (1)$$

$$n_a m_a \frac{\partial u_{||a}}{\partial t} + n_a m_a u_{||a}\frac{\partial u_{||a}}{\partial s} + \frac{\partial(n_a T_a)}{\partial s} = \pm n_a e E_{||} \quad (2)$$

$$\frac{\partial n_a}{\partial t} + \frac{\partial}{\partial s}(n u_{||a}) = 0 \quad (3)$$

where the heat flow q is given for $\omega_a \tau_a \gg 1$ by,

$$\bar{q}_e = -\kappa_{||e}\nabla_{||}T_e - \kappa_{\perp e}\nabla_{\perp}T_e - \frac{5}{2}\frac{n_e T_e}{m_e \omega_{ce}}\hat{b}\times\nabla T_e \quad (4)$$

$$\bar{q}_i = -\kappa_{||i}\nabla_{||}T_i - \kappa_{\perp i}\nabla_{\perp}T_i + \frac{5}{2}\frac{n_i T_i}{m_i \omega_{ci}}\hat{b}\times\nabla T_i \quad (5)$$

ω_{ce} and ω_{ci} are cyclotron frequencies of electrons and ions and H_e, H_i their sources of heat. Q_i is the heat exchange between electrons and ions due to their collisions and is equal to $Q_i = -Q_e = \frac{3}{2}\frac{n(T_e-T_i)}{\tau_{ex}}$. $\nabla_{||} = \frac{\partial}{\partial s}$ and $\nabla_{\perp}$ are the gradients along and across the magnetic field respectively. $\hat{b}$ is the unit vector along the magnetic field and the first, second and third terms of eqns (4) and (5) include the parallel, perpendicular and diamagnetic heat flow (Righi-Leduc in the collissionless limit) respectively. The diamagnetic heat flow has opposite sign for electrons and ions and is eliminated when the heat equations of the two species are added (if $T_e = T_i$ and all ions are hydrogenic). We assume that the impurity number density is very small and has a radial profile given by $n_e \simeq n_i = n = n_o(1-r^2/a^2)+ n_{edge}$, where n_o and n_{edge} are the densities at the core and edge respectively, and $T_e \simeq T_i = T$. The sink term $n_e n_Z L(T)$, where $L(T) = L_\circ \frac{T}{T_L}\exp(-\frac{T}{T_L})$, is the radiation loss from the impurities and is used in the electron heat equation because electrons excite impurity ions which emit line radiation after their de-excitation. Adding the heat equations for electrons and ions and considering the temperature time independent ($T = T(r)$) we derive an equilibrium equation where the divergence of the perpendicular heat flow balances impurity radiation in a thin layer at the edge where any heat source is negligible. This equilibrium is then perturbed to check for it's stability.
Linearising the electron and ion continuity, momentum and energy equations we derive an eigenvalue equation for the spatial structure of the perturbed temperature $\tilde{T}(r)\exp(\gamma t + im\theta - in\phi)$:

$$\begin{aligned}\frac{\partial^2\tilde{T}}{\partial r^2} &+ \left[\frac{1}{n_\circ}\frac{\partial n_\circ}{\partial r} + \frac{1}{r} - \frac{1}{T_\circ}\frac{\partial T_\circ}{\partial r}S\right]\frac{\partial\tilde{T}}{\partial r} \\ &+ \left[\left(\frac{1}{T_\circ}\frac{\partial T_\circ}{\partial r}\right)^2 S - \frac{1}{rT_\circ}\frac{\partial T_\circ}{\partial r}S - \frac{1}{n_\circ T_\circ}\frac{\partial T_\circ}{\partial r}\frac{\partial n_\circ}{\partial r}S - \frac{1}{T_\circ}\frac{\partial^2 T_\circ}{\partial r^2}S\right. \\ &- \frac{1}{n_\circ D}\kappa_{||e}k_{||}^2 - \frac{n_{Z\circ}}{D}\left.\frac{\partial L}{\partial T}\right|_{T\circ} + 2Sn_{Z\circ}\frac{L}{T_\circ} - \frac{m^2}{r^2} - \frac{3\gamma}{D}\frac{\gamma^2 + \frac{5}{3}c_s^2k_{||}^2}{\gamma^2 + c_s^2k_{||}^2} \\ &+ \left. l \times i\frac{m}{r}B_\phi\frac{\kappa_{\Lambda_{Z\circ}}}{n_\circ DB}\left[\frac{1}{B(r)}\frac{dB(r)}{dr} - \frac{S}{T_\circ}\frac{dT_\circ}{dr} - \frac{1}{n_{Z\circ}}\frac{dn_{Z\circ}}{dr}\right](Z^2-1)\right]\tilde{T} \\ &= 0 \end{aligned} \tag{6}$$

where S and $k_{||}$ are given by,

$$S = \frac{c_s^2k_{||}^2}{\gamma^2 + c_s^2k_{||}^2} \tag{7}$$

$$k_{||} = \frac{m - nq}{Rq} \tag{8}$$

In this equation the two driving terms of the instability are distinguished: The term containing $\frac{\partial L}{\partial T}$ is the thermal effect and is destabilising when $\frac{\partial L}{\partial T} < 0$ i.e. where a local drop in temperature would increase the radiation loss; the term containing $\frac{L}{T}$ is the condensation effect. It arises from the density perturbation and is destabilising when $k_{||} \neq 0$. Parallel heat conduction is stabilising for $k_{||} \neq 0$.
The last term on the left hand side is the linearised divergence of the diamagnetic term, $\nabla\cdot\tilde{q}_\Lambda$, and l is an artificial factor which we vary from zero to one, i.e. from no diamagnetic

heat flow to its full value. The diamagnetic heat flows, $q_\Lambda = \frac{5nT}{2m\omega_c}(\hat{b} \times \nabla T)$, of electrons and ions exactly cancel when the heat equations of electrons and ions are added if $T_e = T_i$ and $Z = 1$. So we have included in the study a single species impurity and use the quasineutrality condition $n_e = n_i + Zn_Z$ where Z is the charge of the impurity ions.

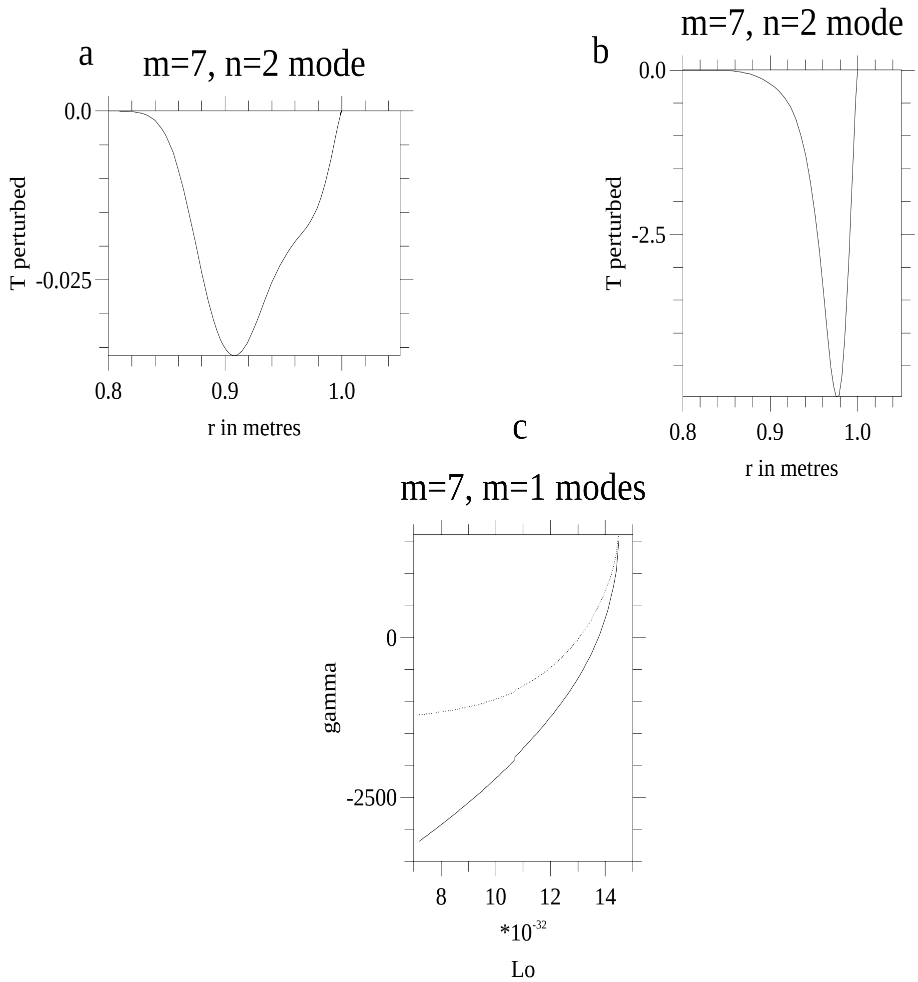

Figure 1: Linear stability analysis of helical perturbations for $l = 0$, i.e. no Righi-Leduc heat flow. In figures a and b we plot the eigenfunctions for the minimum and maximum radiation respectively for the range of $L_\circ$ indicated in c. Fig c plots the algebraic growth rates versus the radiation coefficient $L_\circ$, showing that helical modes (dotted line) are more unstable than axisummetric MARFES (solid line).

The stability of modes with $m \neq 0$, $n \neq 0$ is investigated. A parabolic profile is used for the safety factor,

$$q(r) = q_\circ \left(1 + 2\frac{r^2}{a^2}\right) \tag{9}$$

where a is the minor plasma radius. With $q_\circ = 1.34$, we solve eqn (6) in an annulus

$0.8 < r/a < 1.0$ where $3 < q(r) < 4$. We have chosen to study the mode $m = 7$, $n = 2$ and initially take $l = 0$, i.e. no Righi-Leduc heat flow. The results are shown in Figure 1. The main result of this is that this 7/2 helical MARFE is always more unstable than the $n = 0$ MARFE and should appear before the radiation limit of the latter. Drake et al [8] also investigated helical radiative instabilities and also found that they are unstable in the tokamak edge for high poloidal mode numbers. The diamagnetic heat flow was not included in their calculations.

Figure 1 shows that the helical mode is more unstable and should occur at a lower impurity threshold than the MARFE. However helical modes are not normally observed in experiments. In this letter we show that the reason for this inconsistency is that the diamagnetic heat flow was not included in the earlier theory. By varying l we find that the growth rates of helical modes are reduced by the diamagnetic heat flow and that for $l = 1$ the helical mode is damped for all $L_\circ$. This stabilisation is induced by the linearised diamagnetic heat flow $\nabla \cdot \tilde{q}_\Lambda = i\frac{m}{r}\frac{\kappa_\Lambda z}{B} f(r)(Z^2 - 1)\tilde{T}$ which causes the mode to propagate, i.e. $\gamma \to \gamma + i\omega$ and pure growth is modified to overstability. Indeed any diamagnetic heat flow, regardless of sign, has the potential to affect the mode since it enters eqn (6) as the only imaginary term. Because of its (the $\nabla \cdot \tilde{q}_\Lambda$) dependence on the poloidal mode number,m, the frequency ω of the wave can exceed the sound frequency, $c_s k_{||}$, for the helical mode ($m = 7, n = 2$) changing the sign of the condensation term $S\frac{L}{T}$, and making helical modes more stable. This is not true for $m = 1, n = 0$ MARFES where because the sound frequency dominates in the expression for S keeping it around unity ($S \simeq 1$), the condensation term is always destabilising in this case. The importance of the factor S in the condensation term in eqn (6) has been demonstrated by artificially fixing S equal to one for the helical mode ($m = 7, n = 2$), which then becomes unstable. The diamagnetic heat flow $q_\Lambda = \frac{5}{2}\frac{1}{e}\left(Z - \frac{1}{Z}\right)\frac{n_Z T}{B^2}(B \times \nabla T)$ is a non entropy producing term and exactly cancels for e and i (for equal temperatures and $Z = 1$) so impurity ions are included in the equations. The term in $\frac{1}{Z}$ arises from the impurity diamagnetic heat flow but the term in Z is the net electron ($n_e - n_i$) term which actually dominates. In conclusion helical modes are found to be more stable than axisymmetric $m = 1$ MARFES when the diamagnetic heat flow is retained and this agrees with the fact that helical Marfe-like structures are not normally observed in the outer plasma.

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