Effect of Edge Convection on the H-Mode

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1. Introduction

This paper describes a theoretical model of convection in the tokamak edge plasma and studies the effect of convection on the H-mode temperature pedestal and transport barrier. Here we are concerned with the role of steady-state convection driven by poloidal and toroidal symmetry-breaking of the equilibrium (e.g. by applied potentials or by heat sources and sinks), not with turbulence-driven convection. This work is motivated by the following experimental observations: (i) steady-state convective flows have been measured in the edge and SOL of several tokamaks, caused by spatially localized disturbances such as rf antennas and gas puffing; (ii) JET data [1-4] shows that under certain conditions H-mode properties such as the ratio of τ_p/τ_E, the temperature pedestal height, and the ELM amplitude and repetition rate can be significantly different for ICRF H-modes than for NBI H-modes; (iii) there is an interesting parallel between the effects of ICRF [1-4] and gas puffing [5, 6] on the H-mode temperature pedestals, ELM amplitudes and frequencies in JET; and (iv) recent measurements [7] on Alcator C-MOD showing that convection may be responsible for a significant fraction of the energy transport across the separatrix.

The theoretical approach described in this paper is valid for either the case of applied potential or temperature perturbations, but the specific analysis considered here is the one relevant to ICRF experiments, in which the self-biasing effect of the antennas drives an rf sheath potential at the antenna and gives rise to a rectified (DC) **E**×**B** convection in the SOL and edge plasmas [1, 2]. The SOL physics is modeled here by a boundary condition (BC): an applied spatial potential modulation at the separatrix which drives the **E**×**B** flow in the edge plasma. The main result of this paper is that strong edge convection can nonlinearly modify both the electron temperature and radial electric field profiles in the edge in ways that would be expected to affect the H-mode pedestal and transport barrier.

2. The Model

The calculation is based on the following reduced set of Braginskii equations for the electrostatic potential Φ and electron temperature T:

$$\frac{c^2}{B^2} n m_i \frac{d}{dt} \nabla_\perp^2 \Phi = \nabla_\| J_\| \ , \tag{1}$$

$$\eta J_\| = -\nabla_\| \Phi + \frac{\alpha}{e} \nabla_\| T_e \ , \tag{2}$$

$$\frac{3}{2} n \frac{dT_e}{dt} - \nabla_\| \kappa_{\|e} \nabla_\| T_e - \nabla_\perp \kappa_{\perp e} \nabla_\perp T_e = 0, \tag{3}$$

where $\alpha = 1.71$, $d/dt = \partial/\partial t + \mathbf{v}_E \cdot \nabla$, $\mathbf{v}_E = (c/B)\, \mathbf{b} \times \nabla_\perp \Phi$, $n = n_e = n_i$ is the particle density, m_i is the ion mass, $T = T_e >> T_i$, $\Omega_i = eB/m_i c$ is the ion cyclotron frequency, η is the electrical resistivity, and $\kappa_\|$ and $\kappa_\perp$ are the parallel and perpendicular electron heat conductivies. The collisional Braginskii description is valid when the mean free path λ_{ii} satisfies $\lambda_{ii} < L_{||} \sim qR$. In writing Eqs. (1) - (3), we neglect $v_{||i}$ and T_i effects, treat n as a constant, and retain only the **E**×**B** nonlinearity to simplify the model. The neglect of the density evolution equation is made only for convenience and is not expected to change the conclusions reached here. The terms proportional to $J_\parallel$ neglected in Eq. (3) are small. Radiation sink terms may be important in Eq. (3) when thermal perturbations are driving the convection, but they are not important for the case of convection driven by an externally applied potential perturbation considered here. The result of these approximations is a 2-field model which, despite its simplicity, has a rich and interesting nonlinear behavior.

The unperturbed H-mode equilibrium is treated as one-dimensional in the radial coordinate x and we consider the effect of convection induced by the coupled (zero-frequency) perturbations $\Phi_1(x,y,z)$ and $T_1(x,y,z)$, where z is the coordinate along **B**, and y is in the $\mathbf{e}_z\times\mathbf{e}_x$ direction perpendicular to **B**. Each quantity Q is expanded in powers of the perturbations, $Q = Q_0 + \varepsilon\, Q_1 + \varepsilon^2 Q_2$ ..., where $Q_0(x)$ is the equilibrium quantity in the absence of convection, $Q_1 = Q_1(x)\exp(ik_y y + ik_\parallel z)$ is the perturbation, and $Q_2(x)$ is the second-order surface-averaged nonlinear modification of the equilibrium. Equations (1) - (3) are linearized and made dimensionless in terms of the following quantities: $\bar{\Phi} = e\Phi_0/T_s$, $\tilde{\Phi} = e\Phi_1/T_s$, $\bar{T} = T_0/T_s$, $\tilde{T} = T_1/T_s$, $\bar{v} = v_0/c_s$, $\tilde{v} = v_1/c_s$, $\chi = (T_s/n_s\eta\, e^2)$, $\chi_\perp = (2\kappa_\perp/3n_s)$ and $\chi_\parallel = (2\kappa_\parallel/3n_s)$, where n_s, T_s and $c_s = (T_s/m_i)^{1/2}$ are constants, taken to be the separatrix values. Treating the transport coefficients as constants, the linearized set of equations can be put in the form

$$i\,[\,\omega_E\,\rho_s^2\,\nabla_\perp^2\,\tilde{\Phi} - \rho_s^2\,\omega_E''\,\tilde{\Phi}] = k_\parallel^2\,\chi\,(\tilde{\Phi} - \alpha\,\tilde{T})\,, \tag{4}$$

$$i\,[\omega_E\,\tilde{T} - \omega_{*T}\,\tilde{\Phi}\,] = \chi_\perp\nabla_\perp^2\tilde{T} - k_\parallel^2\,\chi_\parallel\tilde{T}\,, \tag{5}$$

where a prime denotes d/dx, $\rho_s = c_s/\Omega_i$, $\omega_E = k_y\,v_{Ey} = k_y c_s\rho_s\,\nabla_x\bar{\Phi}$, and $\omega_{*T} = k_y\,v_{*Ty} = k_y c_s\rho_s\,\nabla_x\bar{T}$. Note that the perturbations $\tilde{\Phi}$ and $\tilde{T}$ are coupled by the thermoelectric force in the vorticity equation (4) and by the ω_{*T} drift in the temperature equation (5). We assume that $k_\parallel$ is of order $k_\parallel \sim 1/qR$, where q is the safety factor and R is the major radius, and the linearized Eqs. (4) and (5) are expanded in the parameter $\delta = (\rho_s/L_\perp)^2\,(\omega_E/k_\parallel^2\chi) << 1$ for typical edge plasma parameters. To order δ^0, the LHS of Eq. (4) vanishes, $J_\parallel = 0$, and $\tilde{\Phi} = \alpha\,\tilde{T}$ in this order. Thus, there is a symmetry between perturbations in Φ and T in this model due to the thermoelectric force.

In the nonlinear analysis, the **E**×**B** nonlinearities on the LHS of Eqs. (1) and (3) couple the perturbations quadratically to produce a net surface-averaged modification of the underlying equilibrium. We substitute the linear solutions for $\tilde{\Phi}$ and $\tilde{T}$ to order δ into the nonlinear terms and carry out the spatial averaging to obtain the following set of modified equilibrium equations for the vorticity and the electron temperature:

$$\frac{\partial}{\partial t}\rho_s^2\,\nabla_\perp^2\,\bar{\Phi} + \mu_\theta\,\rho_s^2\,\nabla_\perp^2\,(\bar{\Phi} - \bar{\Phi}_b) = \frac{k_y c_s\rho_s^3}{2\chi_\perp}\nabla_x\,[\,|\tilde{\Phi}|^2\,(\omega_E - \alpha\,\omega_{*T})\,]\,, \tag{6}$$

$$\frac{\partial}{\partial t}\bar{T} - \chi_\perp\nabla_\perp^2\,(\bar{T} - \bar{T}_b) = -\frac{k_y c_s\rho_s^3}{2\alpha k_\parallel^2\chi\,\chi_\perp}\nabla_x\,[\,|\tilde{\Phi}|^2\,(k_\parallel^2\,\chi_\parallel\,\omega_E - \chi_\perp\omega_E'')\,]\,. \tag{7}$$

A neoclassical damping term was included in Eq. (6) to provide a steady state solution. The coefficient is given by $\mu_\theta \approx (v_i/qR)^2\,\nu_{ii}^{-1}$ in the Braginskii regime, where v_i is the ion thermal velocity and ν_{ii} is the ion-ion collision frequency. The functions $\bar{\Phi}_b(x)$ and $\bar{T}_b(x)$ are inputs to the present theory and denote the steady state solution in the absence of the nonlinear terms; these functions can be regarded as source terms representing the other physical effects not explicitly considered in our model, such as turbulent generation of **E**×**B** flow shear. Equations (6) and (7) must be supplemented by an equation to determine the radial penetration of $\tilde{\Phi}(x)$ into the edge plasma for given BCs; this equation is obtained by using the lowest order result $\tilde{\Phi} = \alpha \tilde{T}$ in Eq. (5). Neglecting the drift terms, one finds that a potential modulation imposed at the separatrix will exponentially decay in the edge plasma with a scale length $L_0 = (\chi_\perp/k_\|^2\chi)^{1/2}$. It can be shown that the neglect of cubic and higher nonlinearities in Eqs. (6) and (7) is valid when $\tilde{\Phi} << \tilde{\Phi}_c^2 \equiv 2\chi_\perp^2 / k_y^2 c_s^2 \rho_s^4$.

3. Analytic Solution and Boundary Conditions

A complete analysis of the problem would entail numerical solution of the steady-state solutions of Eqs. (5) - (7) with appropriate BCs, which has not yet been done. However, an interesting analytic solution has been obtained in the the limit of strong convection ($\tilde{\Phi} >> \tilde{\Phi}_c$), in which the nonlinear terms dominate the equilibrium equations (6) and (7). The important point to note from this solution is that *in the strong convection limit the vorticity and temperature equations impose constraints relating the equilibrium flux-surface-averaged E_x and T profiles* which are not present in the usual H-mode:

$$\omega_E = \alpha\, \omega_{*T} \ , \qquad (8a)$$

$$L_0^2\, \omega_E'' - \omega_E = 0 \ . \qquad (8b)$$

Equation (8b) implies that the equilibrium profile $E_x(x)$ varies on the same scale L_0 as the convection, and Eq. (8a) then relates the sign of E_x to the sign of $\nabla_x T$. Equation (8a) also justifies the neglect of the drift terms on the LHS of Eq. (5). This 3rd order system has been solved subject to the following BCs: $T(x_m) = T_m$, $T(x_s) = T_s$, and $E_x(x_s) = E_s$, where x_s denotes the separatrix position and x_m denotes the interior point where $\tilde{\Phi} = \tilde{\Phi}_c$. The temperature T_m is set by core and edge physics outside the convective layer, T_s is set by the atomic physics in the SOL, and E_s is determined by the sheath physics in the SOL. Previous simulations [2] of the SOL sheath physics show E_s can have either sign, depending on the antenna voltage and the antenna-plasma separation. Other means of biasing, such as probes [8], can also give both signs of E_s. An analytic solution of Eqs. (8) for these BCs shows that a variety of behaviors are possible as E_s is varied for fixed $\Delta T = T_m - T_s$, including regimes of nonlinear cooling or heating of the convective layer and unphysical regimes [$T < 0$ or non-monotonic T(x)]. The most experimentally-relevant regime is obtained for moderate E_s. In this case, the convection cools the edge by flattening the T profile near the separatrix, and the constraint (8a) forces $E_x > 0$ and reduces the shear E_x' in the convective layer; here E_x has the opposite sign from the normal H-mode and is in the direction to increase ion losses. Thus, our model suggests that strong convection can significantly modify the H-mode pedestal and transport barrier region by modifying the plasma profiles near the separatrix.

4. Summary and Discussion

For collisional edge plasmas described by the Braginkii equations, we have derived a set of nonlinear model equations describing the interaction of steady-state **E**×**B** convection with the edge plasma electric field and electron temperature. The convection can be driven by a spatial modulation of either the equilibrium edge potential Φ (e.g. due to ICRF-driven sheath effects) or the edge T (e.g. due to gas puffing). Application to the former case was illustrated here. These zero-frequency perturbations satisfy $\tilde{\Phi} = \alpha \tilde{T}$ to lowest order, because of the thermoelectric force. The quadratic interaction between $\tilde{\Phi}$ and $\tilde{T}$ give nonlinear terms in the flux-surface-averaged vorticity and electron temperature equations which can modify the equilibrium profiles in the convective layer. An analytic solution of the nonlinear equations in the limit of strong convection ($1 << \tilde{\Phi}_c << \tilde{\Phi} << \tilde{\Phi}_c{}^2$) and for reasonable choices of the BCs yields additional constraints on the profiles T(x) and $E_x(x)$. This solution implies that the convection can produce significant cooling, a reversal in the sign of E_x, and a reduction in the **E**×**B** shear in the edge plasma inside the separatrix.

Combined with the SOL model of rf-driven convection in Ref. [2], this work provides a mechanism to explain the experimental dependence of the ICRF H-mode on the JET A1 antenna phasing [1, 2]. It may also be relevant to the other experimental observations described in Sec. 1. Convective cooling can reduce the temperature pedestal, increase the edge plasma resistivity, and thereby change the character of the MHD modes producing ELMs; the modified **E**×**B** shear affects the edge turbulence and the global confinement, if the convection penetrates into the transport barrier region; also, the sign reversal in E_x should reduce the ion confinement in the convective layer. For example, the H-mode biasing experiments on Textor [8] showed a clear asymmetry between H-modes produced with positive and negative E_x: comparable τ_E in the two cases, but the ratio of τ_p/τ_E was about three times lower for $E_x > 0$. A similar reduction in τ_p was obtained in the "low particle confinement" H-modes on JET [9] and the "Enhanced D_α" H-modes on C-Mod [10]. In future work, we will extend this model to include non-rf-driven convection, density evolution and particle transport, and examine in more detail the relation of the theory to these experiments.

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