A linear kinetic model for beam driven chirped Alfvén modes

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The dispersion and stability of low frequency Alfvén waves in the presence of an ion beam flowing along the equilibrium magnetic field is considered. We find that the presence of a beam of modest density dramatically alters the dispersion of these waves, producing an upshift in the frequency of the compressional Alfvén wave and a downshift in that of the shear wave as shown in Figures 1 A and B. The handedness of the field

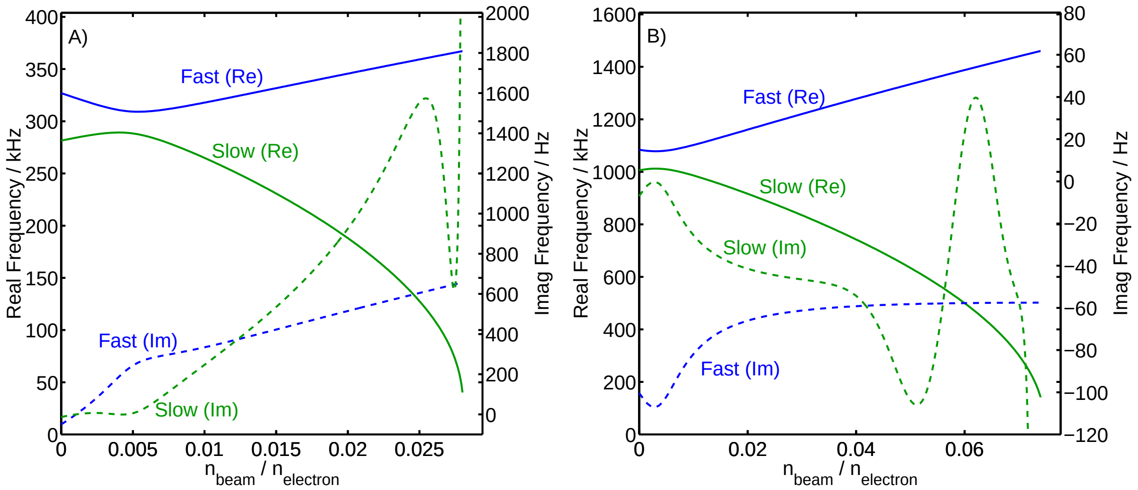

Figure 1: *Frequencies (real and imaginary parts) of fast (compressional) and slow (shear) beam modified Alfvén waves as functions of beam density. Here for a homogeneous plasma. A) Parameters relevant to START.* $\angle(\mathbf{B}^{(0)}, \mathbf{k}) = 20°$, $k = 3$ *rad/m*, $n_e = 5\times10^{19}\,\mathrm{m}^{-3}$, $B^{(0)} = 0.3$ *T*, $T_i = T_e = 200$ *eV*, $v_{\mathrm{Alfven}} = 0.65 \times 10^6\,\mathrm{m/s}$, *Beam: drifting Maxwellian protons*, $T_b = 20$ *keV*, $v_{\mathrm{drift}}/v_{\mathrm{Alfven}} = 3.7$. *B) Parameters relevant to JET.* $\angle(\mathbf{B}^{(0)}, \mathbf{k}) = 20°$, $k = 1$ *rad/m*, $n_e = 6 \times 10^{19}\,\mathrm{m}^{-3}$, $B^{(0)} = 3.4$ *T*, $T_i = T_e = 5$ *keV*, $v_{\mathrm{Alfven}} = 6.8 \times 10^6\,\mathrm{m/s}$, *Beam: drifting Maxwellian deuterons*, $T_b = 5$ *keV*, $v_{\mathrm{drift}}/v_{\mathrm{Alfven}} = 0.55$.

polarisations are reversed for both waves. This is a new and surprising result in basic plasma physics, with relevance to both laboratory and astrophysical plasmas. The beam induced reduction of phase velocity implies that instability of the shear wave is found not just for super–Alfvénic (Figure 1 A) but also for sub–Alfvénic flow velocity (Figure 1 B). For homogeneous plasmas and parameters relevant to START[1] and JET[2] the frequency and phase velocity of the shear wave is reduced to zero for a beam density of a few percent (2–7%) of the electron density. Instability is found at lower beam densities where the phase velocity is above the bulk ion thermal velocity. The instability is reduced when the downshift in the phase velocity allows the wave to undergo bulk ion Landau damping. The dependence of the frequency of the shear wave on the density of the ion beam may provide the basic mechanism explaining the chirping modes observed in a number of beam heated tokamaks[3]. As a significant consequence for fusion plasmas we note that cascades of beam driven chirped Alfvén modes may provide an effective mechanism for direct transfer of energy from beam to bulk ions, somewhat analogous to *alpha–channelling*[4].

Dispersion curves for the beam modified slow Alfvén mode are given in Figure 2. We find the largest reduction in phase velocity for propagation near parallel to the magnetic field, while greatest instability occurs with a finite perpendicular component of the wavevector ($k_x > 0$).

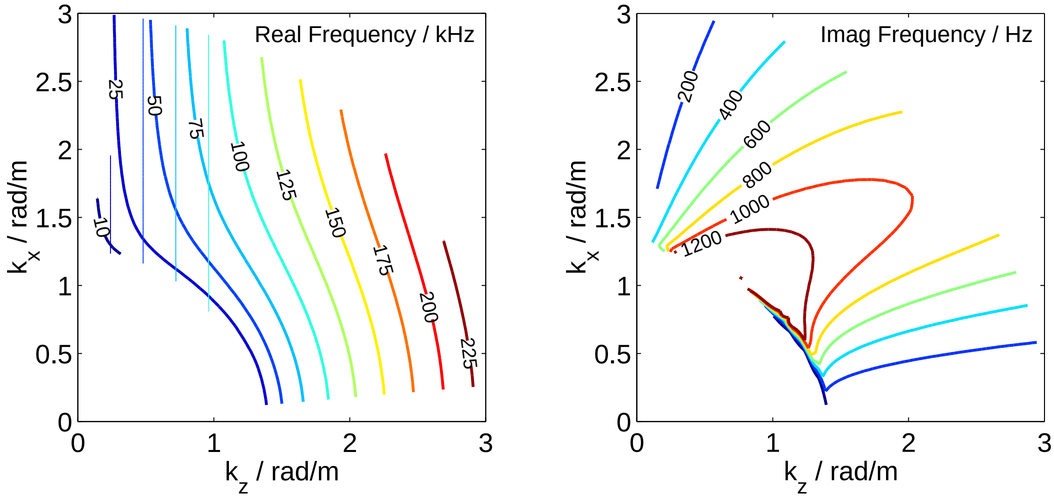

Figure 2: *Frequency contours (real and imaginary parts) of the slow (shear) beam modified Alfvén wave in* (k_z, k_x) *space. Here* $\mathbf{B}^{(0)} || \hat{\mathbf{z}}$. *Parameters as in Figure 1 A with* $n_b/n_e = 1.5\%$. *For reference the thin contours for 25 to 100 kHz show the dispersion in the absence of a beam.*

To elucidate the mechanism for this dramatic modification of the real dispersion we consider the dielectric responses of the bulk ion and electrons, and the beam ions. With numerical verification that the thermal effects play only a minor role for the real dispersion, we limit our analytic discussion to a cold plasma.

The currents associated with the cold beam ions take the form:

$$\mathbf{j} = qn^{(0)}\mathbf{v} + n\mathbf{v}^{(0)} \ , \quad (\partial_t + \mathbf{v}^{(0)} \cdot \nabla)\mathbf{v} = \frac{q}{m}(\mathbf{v} \times \mathbf{B}^{(0)} + \mathbf{E} + \mathbf{v}^{(0)} \times \mathbf{B}) \tag{1}$$

where $\mathbf{v}^{(0)}$ is the unperturbed velocity of the beam ions and n the first order density perturbation. The current contributions due to density perturbations and those due to velocity perturbations in the x direction driven by electric fields in the z direction are in the present case small. Ignoring these currents the susceptibility takes the simple form

$$\boldsymbol{\chi} \approx \left(\frac{\omega_{pb}}{\omega}\right)^2 \begin{pmatrix} \omega'^2/\Gamma_b & i\omega_{cb}\omega'/\Gamma_b & 0 \\ -i\omega_{cb}\omega'/\Gamma_b & \omega'^2/\Gamma_b & 0 \\ 0 & 0 & -\omega/\omega' \end{pmatrix} \tag{2}$$

Here $\Gamma_b = \omega_{cb}^2 - \omega'^2$ and $\omega' = \omega - \mathbf{k} \cdot \mathbf{v}^{(0)}$. The beam current fluctuations are driven both by the electric field and by the magnetic field fluctuations through the term $\mathbf{v}^{(0)} \times \mathbf{B}$ on the right side of equation (1). The former dominates when $\mathbf{v}^{(0)} < \omega/k_z$ while the drive by the magnetic fluctuations dominates when $\mathbf{v}^{(0)} > \omega/k_z$.

The dominant motion for bulk ions and electrons is the $\mathbf{E} \times \mathbf{B}^{(0)}/B^{(0)2}$ drift. Currents associated with these drifts are, however, in opposite directions, and thus cancel. This is reflected in their combined susceptibility which takes the form

$$\boldsymbol{\chi}^{(e+i)} \approx \begin{pmatrix} \omega_{pi}^2/\Gamma_i & -id & 0 \\ id & \omega_{pi}^2/\Gamma_i & 0 \\ 0 & 0 & -\omega_{pe}^2/\omega^2 \end{pmatrix} \ ; \quad d = \frac{\omega_{pi}^2}{\omega_{ci}\omega}\left(\frac{n_e - n_i}{n_e} - \frac{\omega^2}{\omega_{ci}^2}\right) \ . \tag{3}$$

where $\Gamma_i = \omega_{ci}^2 - \omega^2$. The first term in d is due to the $\mathbf{E}\times\mathbf{B}^{(0)}/B^{(0)2}$ drift in the case where the densities are not equal because of the presence of beam ions. The remaining terms are due to induced accelerations which, combined with the finite and different inertias of ions and electrons, give rise to cross $\mathbf{B}^{(0)}$ drifts. To see this we expand $\mathbf{v}$ in the small parameter ω/ω_c. Retaining only the first three terms, we find

$$\mathrm{J}^{(3)} = \frac{\varepsilon_0\omega_p^2}{\omega_c}(\mathbf{R} + \frac{i\omega}{\omega_c}\mathbf{R}^2 - \frac{\omega^2}{\omega_c^2}\mathbf{R}^3)\mathbf{E}\ ; \quad \mathbf{R} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\ . \tag{4}$$

The first term is the $\mathbf{E}\times\mathbf{B}^{(0)}/B^{(0)2}$ drift which cancels in the limit of equal electron and ion densities. The second term gives us the diagonal terms to lowest order in ω/ω_c. The temporal variation of the $\mathbf{E}\times\mathbf{B}^{(0)}/B^{(0)2}$ drift gives rise to this term through the associated acceleration and consequent cross $\mathbf{B}^{(0)}$ drift. We refer to this as the second order cross $\mathbf{B}^{(0)}$ drift. The third term gives off diagonal elements associated with the second term in d. This term is the third order cross $\mathbf{B}^{(0)}$ drift stemming from the time varying nature of the second order drift.

Using the fact that $|\chi_{33}| \gg 1$ the dispersion relation takes the simple form

$$(N^2 - \varepsilon_{11})(N_z^2 - \varepsilon_{11}) + \chi_{12}^2 = 0\ . \tag{5}$$

Referring to Figures 1 A and B we note the following features: at zero beam density there is a finite frequency difference between the fast and the slow wave even in the limit of propagation parallel to the static magnetic field. Considering the reduced dispersion relation (5) this can be attributed to the χ_{12} term, which in the limit of no beam density, consists only of the second term in d. This is the third order drift term discussed in connection with expression (4). For the left hand polarised slow wave the second and third order drift currents are in phase, giving a larger total current than for the right handed fast wave, for which the two currents are out of phase. The larger current gives rise to a larger magnetic field, consistent with the lower frequency.

With the appearance of a beam ion population a new set of currents appear, which in the case of super–Alfvénic beam velocity is primarily due to the $\mathbf{v}^{(0)}\times\mathbf{B}$ interaction. For low beam densities it is primarily the currents associated with the off diagonal elements (1,2 and 2,1) which are significant. They have the opposite sign of the third order drift current and thus reduce the total current for the slow wave and increase the current for the fast wave. The observed approach in frequencies is the result. At sufficient beam density the fluctuating beam current becomes greater than the third order drift current and as a consequence the handedness of both the fast and the slow waves reverse, and the frequency difference increases with further increases in beam density. Here the first term in d, associated with decreasing bulk ion density, is not significant.

It is noteworthy that for near parallel propagation the fast and slow modes almost become degenerate. This situation may be worth further study in the context of fluctuations near marginal stability. Near the degeneracy point both waves have near linear polarisations, the slow wave having magnetic and bulk plasma fluctuations in the y direction while those of the fast wave are in the x direction ($\mathbf{k}$ is in the $x-z$ plane).

At sub–Alfvénic beam velocities the beam current fluctuations are primarily driven directly by the electric field. For these parameters we also find near degeneracy of the fast and slow modes for finite beam density. But unlike the super–Alfvénic case discussed above, here it is the unbalanced $\mathbf{E} \times \mathbf{B}^{(0)}/B^{(0)2}$ drift represented by the first term in d which is the dominant beam density dependent term for low beam densities. Its increase with beam density causes the initial approach of frequencies, then change of handedness and finally diverging frequencies with increasing beam density. While the fast wave was the most unstable mode near the degeneracy point for the super–Alvénic beam, here the roles are reversed in this respect.

Turning now to inhomogeneous plasmas, we note that the field distribution of the slow beam modified Alfvén mode can be localised around a beam with finite width as seen in Figure 3 A. This is due to the reduction in phase velocity in the presence of the beam, making the beam act as a dielectric wave guide. Inhomogeneity in the magnetic field

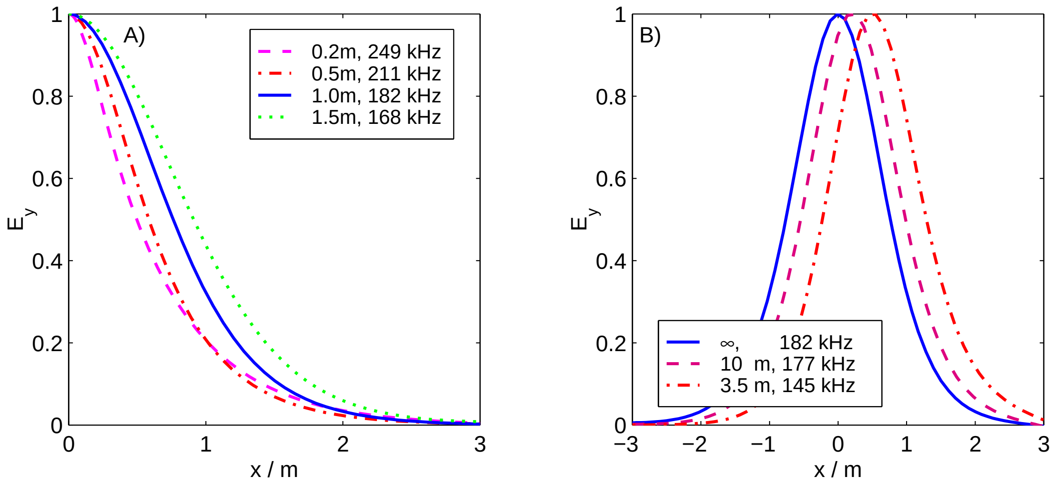

Figure 3: Spatial distribution of E_y for inhomogeneous beam density and in B) also inhomogeneous magnetic field. Parameters as in Figure 1 A with $n_b = n_{b0} \times \exp\{-x^2/(2 * \sigma_b^2)\}$, $n_{b0}/n_e = 2.5\%$. In A) the beam width σ_b is varied between 0.2 m and 1.5 m. In B) $\sigma_b = 1.0$ m, while $B^{(0)} = B_0 R_0/(R_0 + x)$ and R_0 is varied between ∞ ($B^{(0)} = B_0$) and 3.5 m. $B_0 = 0.3$ T.

strength shifts the field distribution toward the low magnetic field side as demonstrated in Figure 3 B. Also this can be understood by simple consideration of the phase velocity which reduces with the magnetic field strength. Shearing the static magnetic field does not change the field distribution qualitatively. The dependence on beam density shown in Figures 1 A and B is retained qualitatively in inhomogeneous plasmas. Solutions have been found for START assuming a perfectly conducting wall boundary. The dramatic dependence of the frequency on beam density of these modes, and strong interaction with both beam ions and bulk ions imply that they may significantly impact on the dynamics of beam heated plasmas. They may also provide an explanation for the observed chirped modes in START[3].

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