26th EPS Conf. on Contr. Fusion and Plasma Physics, Maastricht, 14 - 18 June 1999 ECA Vol.23J (1999) 497 - 500

Single-Particle Theory for Cross-Field Plasma Acceleration and Potential Formation in a Magnetized Plasma

R. Sugaya

Department of Physics, Faculty of Science, Ehime University
Matsuyama 790-8577, Japan

Abstract. Single-particle theory was developed for understanding of plasma acceleration along and across a magnetic field and electric field across a magnetic field induced by almost perpendicularly propagating electrostatic waves.

1. INTRODUCTION

Single-particle theory was developed for understanding of fluctuation-induced plasma acceleration along and across a magnetic field. It was verified based on the kinetic equation for a cold or warm particle beam that plasma particles can be accelerated to the k-direction by the electrostatic waves propagating almost perpendicularly in the homogeneous magnetized plasma immersed in the uniform cross-field electric field. Assuming that the generalized Ohm's law $\boldsymbol{E}_0 + \boldsymbol{v}_d \times \boldsymbol{B}_0/c = 0$ holds simultaneously it can be shown that the intense cross-field electric field $\boldsymbol{E}_0$ is generated by the dynamo effect of the perpendicular particle drift. Parallel and perpendicular particle acceleration results from Landau or cyclotron damping of electrostatic waves (quasilinear interaction with particles). Namely the same results as those shown by the previously derived quasilinear transport equations [1, 2] were obtained. They were derived from Vlasov-Maxwell equations by perturbation theory [3-5]. We can understand easily the physical mechanism of particle acceleration by electrostatic waves and the simultaneously generated cross-field electric field.

2. KINETIC EQUATION

In order to understand fluctuation-induced particle acceleration along and across a magnetic field, we treat a warm particle beam influenced by the electrostatic waves propagating almost perpendicularly in the homogeneous magnetized plasma immersed in the uniform cross-field electric field. The kinetic equation for a warm particle beam moving obliquely to the magnetic field with a velocity v becomes

$$m_s \mathrm{d}\boldsymbol{v}/\mathrm{d}t = e_s(\boldsymbol{E}_0 + \Sigma_k \boldsymbol{E}_k) + e_s \boldsymbol{v} \times \boldsymbol{B}_0/c \quad . \qquad (1)$$

It is assumed that the generated electric field $\boldsymbol{E}_0$ is determined by the following generalized

Ohm's law:

$$\boldsymbol{E}_o + \boldsymbol{v}_d \times \boldsymbol{B}_o / c = 0 \quad . \tag{2}$$

Here,

$$\boldsymbol{v} = \boldsymbol{v}_d + \boldsymbol{v}_z + v_t(\cos\omega_{cs}t, \sigma_s \sin\omega_{cs}t, 0) + \Sigma_k \boldsymbol{v}_k \quad ,$$

$$\boldsymbol{r} = \boldsymbol{r}_o + \mathrm{V}dt + \boldsymbol{v}_z t + (v_t/\omega_{cs})(\sin\omega_{cs}t, -\sigma_s\cos\omega_{cs}t, 0) + \Sigma_k \boldsymbol{r}_k \quad ,$$

$\boldsymbol{E}_k = (\boldsymbol{k}/k)E_k$, $\omega_{cs} = |e_s|B_o/m_s c$, $\sigma_s = e_s/|e_s|$, $\boldsymbol{v}_d = (v_d, 0, 0)$, $\boldsymbol{v}_z = (0, 0, v_z)$, $\boldsymbol{E}_o =$ $(0, E_o, 0)$, $\boldsymbol{B}_o = (0, 0, B_o)$, $E_o = (v_d/c)B_o$, $\boldsymbol{r}_k$, v_k , $\boldsymbol{E}_k \propto \exp[i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega_k t)]$, $k = (k_r, 0, k_z)$ and the subscript **s** designates the species of particle beam.

The first- and second-order kinetic equations are given by

$$m_s d\boldsymbol{v}_k/dt = e_s \boldsymbol{E}_k + e_s \boldsymbol{v}_k \times \boldsymbol{B}_o/c \quad , \tag{3}$$

$$m_s d\boldsymbol{v}''/dt = e_s \boldsymbol{E}_o'' + \Sigma_k e_s(\boldsymbol{r}_k \cdot V)\boldsymbol{E}_k^* + e_s \boldsymbol{v}'' \times \boldsymbol{B}_o/c \quad . \tag{4}$$

The symbol " means the second-order quantity. Then the solutions are obtained as

$$m_s d\boldsymbol{v}''/dt = \Sigma_k i\boldsymbol{k}(e_s^2/m_s k^2)|E_k|^2 \Sigma_r A_r \quad , \tag{5}$$

$$\tfrac{1}{2} m_s dv^2/dt = \Sigma_k i(e_s^2/m_s k^2)|E_k|^2 \Sigma_r ((k_z v_z + k_r v_d + r\omega_{cs})A_r + B_r + C_r) \quad , \tag{6}$$

$$\boldsymbol{E}_o'' + \boldsymbol{v}'' \times \boldsymbol{B}/c = 0 \quad , \tag{7}$$

where the summation with respect to the integer r is carried out on $r = -\infty, \infty$, and

$$A_r = \{k_r^2/((\omega_{dk} - r\omega_{cs})^2 - \omega_{cs}^2) + k_z^2/(\omega_{dk} - r\omega_{cs})^2\} J_r^2(\mu_k) \quad ,$$

$$B_r = k_r^2(\omega_{dk} - r\omega_{cs}) J_r^2(\mu_k)/((\omega_{dk} - r\omega_{cs})^2 - \omega_{cs}^2) \quad ,$$

$$C_r = k_z^2 J_r^2(\mu_k)/(\omega_{dk} - r\omega_{cs}) \quad ,$$

$$\omega_{dk} = \omega_k - k_r v_d - k_z v_z \quad , \qquad \mu_k = \quad k_r v_r/\omega_{cs} \quad . \tag{8}$$

Here, J_r is the Bessel function of the rth order. The first and second equations show the temporal evolution of the second-order drift velocity and kinetic energy of the particle beam, and the third equation the second-order cross-field electric field caused by the change of the cross-field drift velocity $(E_0'' = (v_d''/c)B_0)$.

3. TRANSPORT EQUATIOS

In order to take the linear singularrity of $\omega_{dk} - r\omega_{cs} = 0$, we adopt the following relation

$$\mathrm{Im}[1/(\omega_{dk} - r\omega_{cs})] = -\pi\delta(k_z v_z + k_r v_d - \omega_k + r\omega_{cs}) \quad . \tag{9}$$

Thus the transport equations for the energy and momentum densities of the particle beams become

$$\mathrm{d}U_s/\mathrm{d}t = -\Sigma_k 2\gamma_{sk} U_k \quad , \tag{10}$$

$$\mathrm{d}P_s/\mathrm{d}t = -\quad \Sigma_k (2k/\omega_k)\gamma_{sk} U_k \quad , \tag{11}$$

where $U_k = (1/8\pi)|\partial(\varepsilon_k'\omega_k)/\partial\omega_k||E_k|^2$ is the wave energy density, kU_k/ω_k is the wave momentum density $U_s = \int \mathrm{d}v \tfrac{1}{2} n_s m_s v^2 g_s$ and $P_s = \int \mathrm{d}v n_s m_s v g_s$ are the energy and momentun densities of the particle beam of species s , respectively, g_s is the velocity distribution function of particle beam of species s , $\varepsilon_k = \varepsilon_k' + i\varepsilon_k''$ is the dielectric constant, and the linear damping rate ascribed to the particle beam of species s is given by

$$\gamma_{sk} = (\pi\omega_{ps}^2/k^2(\partial\varepsilon_k'/\partial\omega_k))\Sigma_r \int \mathrm{d}v\delta(k_z v_z + k_r v_d - \omega_k + r\omega_{cs})J_r^2(\mu_k)U_r(k)g_{so}$$

$$= \quad -\varepsilon_k''/(\partial\varepsilon_k'/\partial\omega_k) \quad , \tag{12}$$

where $U_r(k) = k_z\partial/\partial v_z + (r\omega_{cs}/v_r)\partial/\partial v_r$, and $\omega_{ps}^2 = 4\pi n_s e_s^2/m_s$, g_{so} is thevelocity distribution function expressed in the displaced cylindrical coordinate.

When the wave energy density U_k *is* governed by

$$\mathrm{d}U_k/\mathrm{d}t = 2\gamma_k U_k \quad , \tag{13}$$

with the linear damping rate of $\gamma_k = \Sigma_s \gamma_{sk}$, we find immediately the following conservation laws for the total energy and momentum densities of waves and particle beams:

$$(\mathrm{d}/\mathrm{d}t)(\Sigma_k U_k + \Sigma_s U_s) = 0 \quad , \tag{14}$$

$$(\mathrm{d}/\mathrm{d}t)(\Sigma_k k U_k/\omega_k + \Sigma_s P_s) = 0 \tag{15}$$

4. CONCLUSION

Particle acceleration and transport along and across the magnetic field have been investigated based on the single-particle theory. The transport equations can be derived from this theory and show that the electrostatic waves can accelerate the plasma particles to the k-direction via Landau and cyclotron damping. The dynamo effect of perpendicular particle drift generates the cross-field electric field. This particle transport can explain the fluctuation-induced anomalous particle transport occurring in sawtooth crashes or edge plasmas in tokamaks and the perpendicular ion acceleration in tokamaks and space plasmas.

This work was supported partially by a Grant-in-Aid for Scientific Research from the Japanese Ministry of Education, Sports and Culture.

References

[1] R. Sugaya, in *Proceedings of the Seventh European Fusion Theory Conference, Julich, Germany, 1997,* Vol. **1,** p. *207.*
[2] R. Sugaya, in *Proceedings of International Congress on Plasma Physics combined with the 25th EPS Conference on Controlled Fusion and Plasma Physics, Czech, 1998,* Vol. **22C,** p. 209.
[3] R. Sugaya, J. Phys. Soc. Jpn. 60,518 (1991); **60, 3367** (1991); **63, 1775** (1994).
[4] R. Sugaya, J. Plasma Phys. 56,193 (1996).
[5] R. Sugaya, Phys. Plasmas **1, 2768** (1994); R. Sugaya *et al.,* Phys. Plasmas **3, 3485** (1996).