Transport barriers in a model for turbulent equipartition

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Based on the existence of Lagrangian invariants in the presence of turbulence an approach has been suggested for predicting the quasi-steady profiles in tokamak plasmas [1, 2, 3, 4, 5]. The basic assumption is that the turbulent mixing causes the equipartition of these invariants over the accessible phase space, a state denoted Turbulent EquiPartition (TEP) [2].

We present simulations of TEP with self-consistently generated electrostatic turbulence. The density and temperature profiles are allowed to develop under the influence of external heating. When the pressure peaking exceeds the instability threshold $|p'| > |(B^{5/3})'|$, thermal energy can be released by displacing hot and dense fluid parcels to regions with weaker magnetic field, where they expand adiabatically, and the turbulence sets in.

Our model is based on the continuity equation for the electron density and the Braginskii transport equation for the electron temperature accounting for both the $\mathbf{E} \times \mathbf{B}$ and the diamagnetic drifts (cf. Ref. [6]), together with the ion vorticity equation for cold ions. The system of equations is closed by assuming quasi-neutrality.

Assuming that the density n, temperature T and the inhomogeneous magnetic field B deviate only slightly from constant reference levels $\mathcal{N}$, $\mathcal{T}$ and $\mathcal{B}$:

$$n = \mathcal{N}(1 + \tilde{n}(x, y, t)), T = \mathcal{T}(1 + \tilde{T}(x, y, t)), B = \mathcal{B}(1 + \tilde{B}(x, y)), \tag{1}$$

we obtain the model equations for the small quantities $\tilde{n}$, $\tilde{T}$ and $\tilde{B}$:

$$\frac{\partial n}{\partial t} + \{\phi, n - B\} + \{n + T, B\} = 0, \tag{2}$$

$$\frac{\partial T}{\partial t} + \left\{\phi, T - \frac{2}{3}B\right\} + \left\{\frac{2}{3}n + \frac{7}{3}T, B\right\} = 0\,, \tag{3}$$

$$\frac{\partial \nabla^2 \phi}{\partial t} + \left\{\phi, \nabla^2 \phi\right\} + \{n + T, B\} = 0. \tag{4}$$

For convenience we have dropped the tilde. The potential is normalized by $\mathcal{T}/e$, the time by $\omega_{ci}^{-1} = m_i/(e\mathcal{B})$, and the space variables by $\rho = (\mathcal{T}/m_i)^{1/2}/\omega_{ci}$. Note that Eqs. (2)-(4) are scale invariant with respect to multiplying the dependent variables and B with a constant and dividing t by the same constant.

Equations (2)-(3) possess the Lagrangian invariants $l_\pm = \pm\sqrt{5/2}(n - B) + 3T/2 - n$. They are advected by the velocities $\mathbf{v}_\pm = \hat{z} \times \nabla[\phi - n - (1 \pm (5/2)^{1/2})T]$, which are neither fluid nor guiding center velocities, but rather Riemann-like characteristics. Thus,

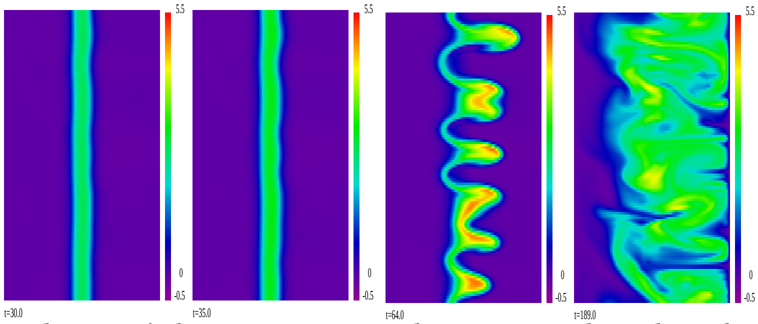

Figure 1: *Time evolution of the temperature. The system is heated in the center region and* $L_x/_y = 0.5$

the TEP profiles can be expected to be given by a spatially homogeneous distribution of $l_\pm$, which implies

$$n - B = const., \quad T - 2B/3 = const., \tag{5}$$

In order to investigate the linear stability we consider a slab model where the equilibrium gradients are in the x-direction. We linearize Eqs. (2)-(4) around the background profiles $n_0(x)$, $T_0(x)$ and $B(x)$ and assume a waveform $exp(i\mathbf{kr} - i\omega t)$ in the local approximation. The dispersion relation reads

$$ck^2 \left[c^2 + \frac{10}{3}cB' + \frac{5}{3}(B')^2\right] + cB'(n_0' + T_0' - \frac{5}{3}B') + \frac{5}{3}(B')^2(n_0' - B') = 0 \tag{6}$$

where we have introduced $c = \omega/k_y$ and the prime denotes differentiation with respect to x. The long wavelength solution is $c^2 \approx -B'(n_0' + T_0' - \frac{5}{3}B')/k^2$. This is recognized as a special case of the RTI. It is unstable for

$$B'(n_0' + T_0' - \frac{5}{3}B') > 0. \tag{7}$$

Assuming that the magnetic field is decreasing with increasing x, i.e., $B' < 0$, the instability condition becomes $(n_0 + T_0 - 5B/3)' < 0$. Solutions of Eq. (6) shows that there is a finite wavenumber cut-off corresponding to $k \approx \rho^{-1}$ (in dimensional units) for the RTI in this model, contrary to models for RTI where the magnetic field inhomogeneity is represented by an "artificial gravity". We observe that the TEP-profiles are marginally stable.

We solved the equations (2)-(4) numerically in a two-dimensional domain using a finite difference code. Dissipative terms of the form $\mu\nabla^2 f$, where f denotes n, T or $\nabla^2\phi$, were added to the right hand side of each of the equations (2 - 4) for numerical convenience. The domain was periodic in y with length L_y and bounded in x with length L_x. We considered several different situations, and observed a clear tendency for the plasma to relax toward the TEP profiles. Here we present a typical case, where the turbulence is driven by a distributed heat source in a region near $x = 0$, while the temperature at both boundaries is fixed to zero. The density profile was initially flat $n = 0$. The magnetic field variation is $B \sim 1/(3.5 + x)$. In Fig. 1 we show the evolution of the temperature. The system heats up and a RTI sets in, but only develops to the right of the heat source initially, in accordance with the stability criterion (7). Large scale convective rolls are seen to develop. Fig. 2 shows the evolution of the temperature and density profiles. They both approach the TEP profiles in the quasi-stationary limit. Note also that the temperature

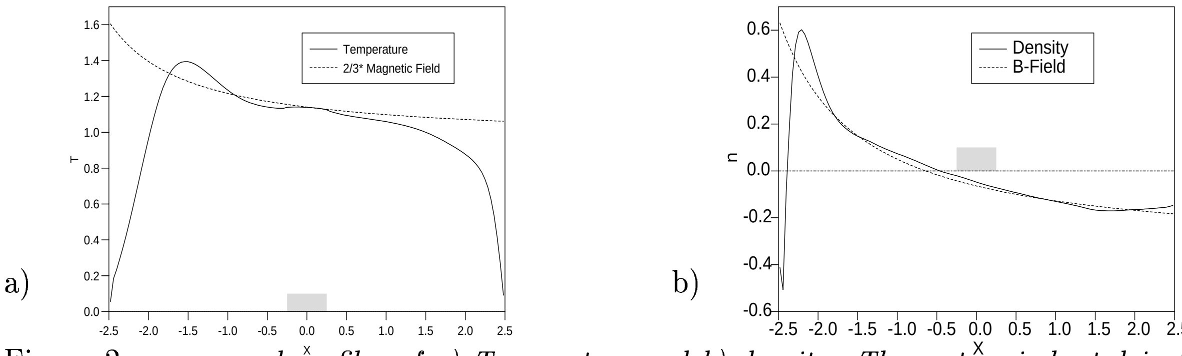

Figure 2: *y averaged profiles of a) Temperature and b) density. The system is heated in the hatched area. Aspect ratio is* $L_x/L_y = 0.5$

maximum is to the left of the heat source, which is marked by the shaded patch in Fig. 2. Thus, there is an up-gradient heat flux in the region $-1.5 > x > -0.2$.

If, however, the aspect ratio $\epsilon = L_x/L_y$ falls below a value of 0.5 the turbulence self-organizes into a poloidal (y-direction) shear flow. This poloidal shear flow hinders the turbulent redistribution of heat and consequently there is a transition from a turbulence dominated transport regime to a diffusion dominated transport regime. This manifests in the temperature profiles through a much steeper gradient in regions of shear, while in regions with large enough fluctuations the rather flat TEP profile prevails (see Fig. 3).

The question that arises, is for the stabilization of the RTI in presence of a background poloidal flow. First, we derive the dispersion relation accounting for a shear as before. In a local approximation we get as a condition for **instability** in the long wave-length limit:

$$\left(V''_{poloidal}\right)^2 + 4k^2 B' \left(\frac{5}{3}B' - n'_0 - T'_0\right) < 0. \tag{8}$$

From that we deduce, that a curvature in the poloidal flow profile stabilizes the RTI. However, the situation is more complex. The problem is strongly related to the problem of the stability of a stratified shear flow in neutral fluids. Usually this problem is treated from a different limit. A stable stratification is subjected to a shear flow and the stability of the system with respect to global modes is considered. These global modes cannot be found with the local analysis we presented above, like e. g. the classical result for a Kevin-Helmholtz instability (KHI) cannot be reproduced in the limit $B' = 0$. The equation governing stability for our system is the Taylor-Goldstein equation [7] which in the long wave limit reads as:

$$\frac{d^2}{dx^2}\Phi_0 + \left[-k_x^2 + \frac{U''_0}{c - U_0} + \frac{N^2}{(c - U_0)^2}\right]\Phi_0 = 0 \tag{9}$$

with the "buoyancy frequency" $N^2 = B'(5/3B' - n'_0 - T'_0)$. Then the Miles-Howard-Theorem [8] guarantees stability for

$$\frac{N^2}{(U')^2} > \frac{1}{4} \tag{10}$$

This result implies, that a added shear flow is always unstable if the stratification alone is already unstable. This is clearly cannot explain the stabilization observed in our numerical

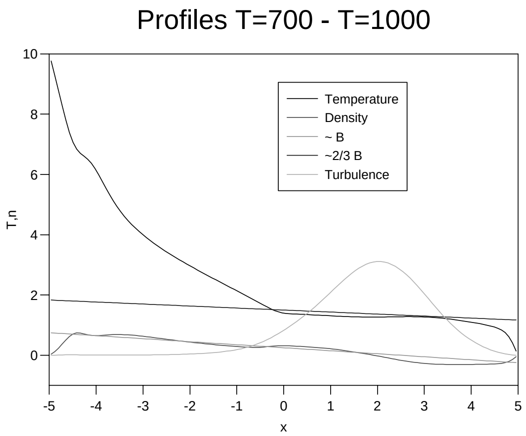

Figure 3: *Density and temperature profile in the presence of a background shear flow. A steep diffusion dominated region for the temperature is seen to the left, where the system is heatet. In the region with sufficient turbulence a flat temperature profile prevails.*

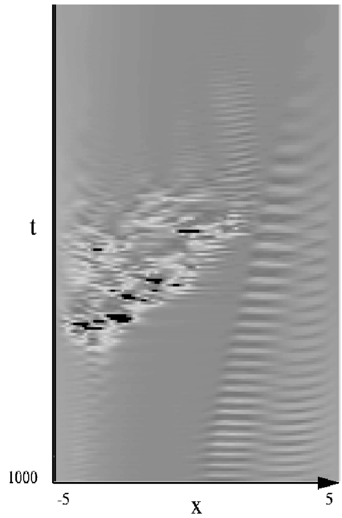

Figure 4: *Heat transport over time and radial postion in the presence of a stabilizing poloidal flow. A large transport event is exited around* $t = 1400$ *and spreads with a finite velocity outwards.*

results. Therefor more attention has to be focused on the question of the stabilization of RTI type of instabilities via shear flows.

However, when in the numerical simulation the self consistently generated shear flow stabilizes the RTI and a steep temperature gradient builds up, burst like events occur in the flow (see Fig. 4.), which locally raise the level of turbulence and flatten out the observed profile locally again.

In conclusion, we verified that the nonlinear evolution of pressure driven electrostatic flute modes in a system with sources and sinks leads to a quasi-equilibrium with density and temperature profiles as predicted by the TEP, i.e., with the Lagrangian invariants n/B and $T/(2B/3)$ roughly constant. The profiles are changed towards diffusion dominated ones, when a self-organized shear flow stabilizes the instability and the fluctuation level is reduced. As for the stability of this stratified-shear flow different approaches lead as of yet to inconsistent results.

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