26^{th} EPS Conf. on Contr. Fusion and Plasma Physics, Maastricht, 14 - 18 June 1999 ECA Vol.23J (1999) 549 - 552

Density and Temperature Gradient Driven Drift Waves

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The control of energy and particle confinement in fusion plasma devices is strongly connected to the understanding of the nature of anomalous transport. Hence a major task is to correlate results from theoretical and numerical models with experimental data on transport (see e.g. [1]). In order to obtain better accordance between the numerical models and the experimental results ever more elaborate models are developed. However, these models have a tendency to become complicated and a distinction between different physical and geometrical effects becomes difficult. It may thus be worthwhile to consider the problems from a more basic perspective and investigate more simple models, so-called *toy models*, such as the Hasegawa-Mima [2] and the Hasegawa-Wakatani models [3].
The Hasegawa-Wakatani model describes electrostatic resistive drift wave turbulence driven by a density gradient. The model consists of two coupled nonlinear partial differential equations in the perturbations in the electrostatic potential and the density. The model has been investigated in both two and three dimensional geometries in e.g. [4–7]. However, significant electron temperature fluctuations have been observed experimentally and should be included in drift wave models in order to obtain a wider range of validity than the Hasegawa-Wakatani model. Hence, a new model is proposed which describes the evolution of the fluctuations in the potential, density and electron temperature, which are driven by a density and an electron temperature background gradient.
Maintaining the relatively simple scope of the Hasegawa-Wakatani model, the new model is derived from the ion vorticity equation, the electron continuity equation and the generalised Ohm's law, now including electron temperature perturbations. Finally, the electron temperature dynamics is included by adding the Braginskii equation for electron temperature fluctuations [8]. The shape of the background gradients of the density and the electron temperature are

$$n_0 = n_0(x) = N_0 \mathrm{e}^{-\frac{x}{L_n}} \quad \text{and} \quad T_{e0} = T_{e0}(x) = T_0 \mathrm{e}^{-\frac{x}{L_T}}$$

The equations of the model are of the order ε^2 ($\frac{n_1}{n_0} \sim \frac{e\phi}{T_{e0}} \sim \frac{T_{e1}}{T_{e0}} \sim \frac{\rho_s}{L_n} \sim \varepsilon \ll 1$) and are simplified and normalised to the order unity by using the following dimensionless variables:

$$\tilde{x} = \frac{x}{\rho_s}, \quad \tilde{y} = \frac{y}{\rho_s}, \quad \tilde{z} = \frac{z}{L_{||}}, \quad \tilde{t} = t\omega_{ci}\frac{\rho_s}{L_n}, \quad \tilde{n} = \frac{n_1}{n_0}\frac{L_n}{\rho_s}, \quad \tilde{\phi} = \frac{e\phi}{T_{e0}}\frac{L_n}{\rho_s}, \text{ and } \tilde{T} = \frac{T_{e1}}{T_{e0}}\frac{L_n}{\rho_s}$$

The model equations are thus (where the tildes have been omitted):

$$(\frac{\partial}{\partial t} + \mathbf{v}_E \cdot \nabla_\perp) n + \frac{\partial \phi}{\partial y} = \mathcal{C}\frac{\partial^2}{\partial z^2}(n + T - \phi) + \nu \mathcal{D}^{2p} n \tag{1}$$

$$(\frac{\partial}{\partial t} + \mathbf{v}_E \cdot \nabla_\perp)(\nabla_\perp^2 \phi) = \mathcal{C}\frac{\partial^2}{\partial z^2}(n + T - \phi) + \nu \mathcal{D}^{2p+2} \phi \tag{2}$$

$$(\frac{\partial}{\partial t} + \mathbf{v}_E \cdot \nabla_\perp) T + \frac{L_n}{L_T}\frac{\partial \phi}{\partial y} - \zeta \frac{\partial^2 T}{\partial z^2} = \chi \mathcal{C}\frac{\partial^2}{\partial z^2}(n + T - \phi) + \nu \mathcal{D}^{2p} T \tag{3}$$

where $\chi = \frac{2}{3}0.71$, and $\mathcal{C}$ and ζ are defined as

$$\mathcal{C} \equiv \frac{T_{e0}L_n}{\eta e^2 n_0 \omega_{ci}\rho_s L_{||}^2} \quad \text{and} \quad \zeta = \frac{2}{3}\frac{\kappa_{||}L_n}{n_0\omega_{ci}L_{||}\rho_s} = \frac{2}{3}\frac{\kappa_{||}\eta e^2}{T_{e0}}\mathcal{C}$$

where the parallel thermal conductivity, $\kappa_{||}$, is given by: $\kappa_{||} = 3.16 n_e T_{e0}\tau_e/m_e$. In each equation a hyperviscosity term is included for numerical reasons:

$$\mathcal{D}^{2p} \equiv (-1)^{p+1}\nabla_\perp^{2p}$$

The kinematic viscosity term from the ion vorticity equation has been replaced by the hyperviscosity term.
A self-consistent closed set of equations describing drift waves, driven by a density and an electron temperature gradient, has now been obtained.
The linear growth rate γ, of the system may be obtained by linear stability analysis. Assuming the small amplitude perturbations to be plane waves of the form $e^{i(\mathbf{k}\cdot\mathbf{r}-\omega_t t)}$ we obtain a cubic equation in the frequency ω_t. Only one solution has a positive imaginary part corresponding to a positive growth rate γ, which is presented in Figure 1.

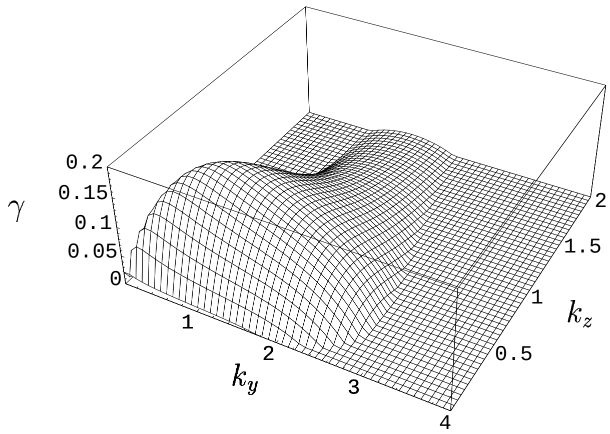

Figure 1: The growth rate as a function of k_y and k_z, for $k_x = 0$, $\mathcal{C} = 1$, $\frac{L_n}{L_T} = 0.5$, $\zeta = 0.5$, $\nu = 2 \cdot 10^{-4}$, and order $2p = 6$. Only positive values are plotted.

The total energy of the system may be expressed as

$$\mathcal{E} = \mathcal{E}_{kin} + \mathcal{E}_{pot} + \mathcal{E}_{thermal} = \frac{1}{2}\int_{\mathcal{V}}\left[(\nabla_\perp\phi)^2 + n^2 + \frac{3}{2}T^2\right] dx\,dy\,dz \tag{4}$$

The sources and the sinks of the system may be obtained by differentiation of (4) with respect to time. We assume that the solutions and their spatial derivatives are localised, i.e. vanish at infinity. Hence by partial integration one obtains:

$$\begin{aligned}\frac{d\mathcal{E}}{dt} &= \int_{\mathcal{V}}\left[-n\frac{\partial\phi}{\partial y} - \frac{3}{2}\frac{L_n}{L_T}T\frac{\partial\phi}{\partial y} + \mathcal{C}(n + \frac{3}{2}\chi T - \phi)\frac{\partial^2}{\partial z^2}(n + T - \phi)\right.\\ &- \left.\frac{3}{2}\zeta\left(\frac{\partial T}{\partial z}\right)^2 - \nu((\nabla_\perp^p n)^2 + \frac{3}{2}(\nabla_\perp^p T)^2 + (\nabla_\perp^{p+1}\phi)^2)\right] dx\,dy\,dz \end{aligned} \tag{5}$$

The first two terms are the turbulent density and heat flux respectively:

$$\Gamma_n = \int_{\mathcal{V}} -n\frac{\partial \phi}{\partial y}\, dx\, dy\, dz = \int_{\mathcal{V}} n v_{E,x}\, dx\, dy\, dz \qquad (6)$$

$$\Gamma_T = \int_{\mathcal{V}} -T\frac{\partial \phi}{\partial y}\, dx\, dy\, dz = \int_{\mathcal{V}} T v_{E,x}\, dx\, dy\, dz \qquad (7)$$

The third term in (5) arises from Ohmic losses and the two last terms are losses due to heat conductivity and hyperviscosity.
The model was implemented numerically using Fourier spectral methods [9] in a triply periodic geometry. The third order Stiffly Stable scheme [10] was used as the temporal integration scheme. The parameters used in the simulation presented below were: domain size $L_x = L_y = L_z = 12\pi$, spatial resolution $M = N = O = 96$ Fourier modes, $\frac{L_n}{L_T} = 0.5$, $\zeta = 0.5$, and a hyperviscosity of the order $2p = 6$ with a viscosity parameter $\nu = 10^{-4}$.
In Figure 2 the total energy $\mathcal{E}$, and the energy of the drift waves $\mathcal{E}(k_\| \neq 0)$ are presented. It may be seen that the energy grows exponentially until nonlinear effects dominate and the system evolves into a turbulent state. In this turbulent state energy is nonlinearly

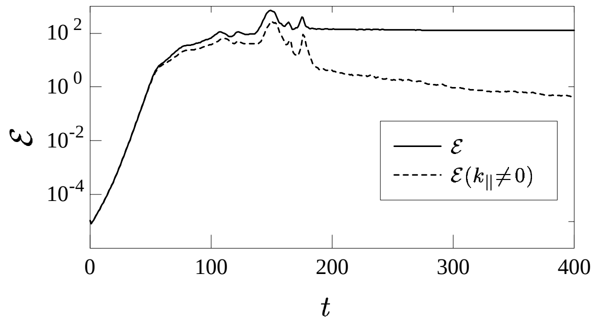

Figure 2: The evolution of the total energy $\mathcal{E}$ and the energy of the drift waves $\mathcal{E}(k_\| \neq 0)$.

coupled between drift waves, having $k_\| \neq 0$, and flute-like convective cells, having $k_\| = 0$. These dynamics resemble those of the Hasegawa-Wakatani system very much, see e.g. [6] and [7]. The turbulent state is only transient since a poloidal flow builds up and most of the energy is transferred to the convective cells.
The norms of the terms in the electron temperature equation are shown in Figure 3. It may be seen that the dominant term initially indeed is the temperature gradient term, which drives energy into the system. From $t \approx 60$ the nonlinear term is dominant.
The evolution of the turbulent fluxes Γ_n and Γ_T is of particular interest in transport studies. It may be seen in Figure 4 that Γ_n and Γ_T are strongly correlated. This is due to n and T having a correlation of ~ 0.99. The difference in the amplitudes of Γ_n and Γ_T reflects to some extent the difference in steepness of the gradients given by $\frac{L_n}{L_T}$.
The effect of varying the relative steepness of the density and temperature gradients $\frac{L_n}{L_T}$ has been investigated. More simulations are needed to obtain sufficient statistics, but the trend is that the heat flux increases with increasing $\frac{L_n}{L_T}$, i.e. when the temperature gradient becomes steeper compared to the density gradient. The changes in the density flux are less pronounced.

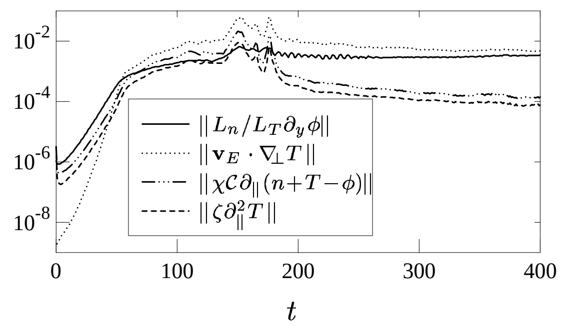

Figure 3: The norms of the terms in the electron temperature equation.

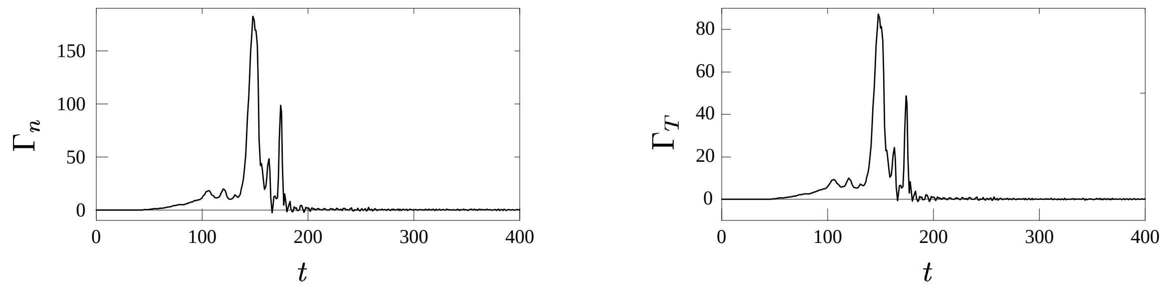

Figure 4: The evolution of the fluxes Γ_n and Γ_T

One may conclude that the behaviour of the model resembles that of the Hasegawa-Wakatani model to a certain extent. However, the strength of the current model is that effects of a temperature gradient may be investigated as well as the correlation between density, temperature, and potential perturbations.

References

[1] M. Endler, H. Niedermeyer, L. Giannone, E. Holzauer, A. Rudyj, G. Theimer, N. Tsois, and ASDEX Team, Nucl. Fusion **35** (1995) 1307–1339.

[2] A. Hasegawa and K. Mima, Phys. Fluids **21** (1978) 87–92.

[3] A. Hasegawa and M. Wakatani, Phys. Rev. Lett. **50** (1983) 682–686.

[4] S. J. Camargo, D. Biskamp, and B. D. Scott, Phys. Plasmas **2** (1995) 48–62.

[5] T. S. Pedersen, P. K. Michelsen, and J. J. Rasmussen, Plasma Phys. Control. Fusion **38** (1996) 2143–2154.

[6] D. Biskamp and A. Zeiler, Phys. Rev. Lett. **74** (1995) 706–709.

[7] S. B. Korsholm, P. K. Michelsen, and V. Naulin, Phys. Plasmas **6** (1999) 2401–2408.

[8] S. I. Braginskii, Rev. Plasma Phys. **1** (1965) 205–311.

[9] E. A. Coutsias, F. R. Hansen, T. Huld, G. Knorr, and J. P. Lynov, Physica Scripta **40** (1989) 270–279.

[10] G. E. Karniadakis, M. Israeli, and S. A. Orszag, J. Comp. Phys. **97** (1991) 414–443.