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Spectral Time Series Analysis of Plasma Turbulence

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Abstract. A combined Doppler/time-of-flight laser anemomeer has been developed at Risø, and is now in operation at the Wendelsten 7-AS stellerator. The apparatus yields time series of the fluctuating plasma density in neighbouring measurement volumes. The subsequent data analysis and interpretation has resulted in the development and implementation of a suite of spectral analysis techniques.

We have solved the Hasegawa-Mima drift wave equation to obtain model data, which serves as a useful verification and testing ground for the analysis techniques. The analysis is applied to experimental data and simulated model data, and it is demonstrated how to extract dispersion relation from coherent wave dynamics and identify non-linear structures.

Background. A hybrid Doppler/time-of-flight anemometer is installed at W7-AS. Coherent scattering from elongated measurements volumes, illuminated by a 25 W CO_2 laser, is heterodynely detected and sampled at 20 ns intervals to yield time series of $\tilde{n}(t)$, the electron density fluctuation in the measurement volume. Total density is $n(t) = \tilde{n}(t) + n_0$, where n_0 is time-averaged density. The use of rotating prisms allows the measurement direction to be continuously changed from poloidal to radial. Scattering is currently maximised at fluctuation wave number $k \sim 32$ cm^{-1}, but can be varied by adjusting laser beam width.

The new configuration detects scattering from *two* measurement volumes, of separation ~ 1 cm, and hence allows for correlation analysis. The use of correlation techniques enhances spatial resolution, and large-scale turbulence structures can be probed. The measurement volumes can be aligned poloidally or radially, so transport in both directions can be investigated.

Figure 1 illustrates power spectra of radial density fluctuations and cross-correlations between toroidally displaced measurement volumes, obtained by laser scattering.

Numerical model. The two-point configuration has prompted several questions regarding correlation techniques and data interpretation. In order to resolve these issues we have recently implemented a numerical turbulence model. The model serves to estimate correlation length and time scales, predict expected signal features and test two-point data analysis procedures by comparison with the full field information available in the numerical simulation.

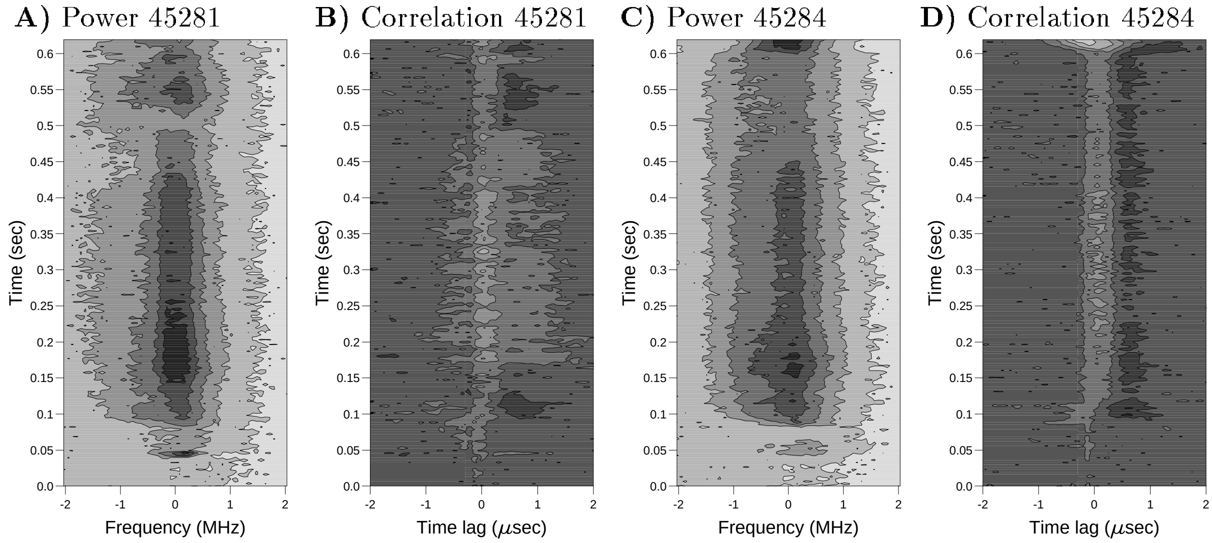

Figure 1. *Time-resolved autopower spectra of radial density fluctuations from shot 45281 (H-mode) and 45284 (L-mode), and their toroidal correlation. Dark shading indicates large values. L-mode exhibits a shift in power spectrum towards negative frequencies, indicative of increased radial transport, and a change in correlation signature.*

As a model of drift wave turbulence in the plane perpendicular to the confining magnetic field, we have chosen the well-known Hasegawa-Mima equation[1]

$$\frac{\partial}{\partial t}(\phi - \nabla^2\phi) = \{\phi, \nabla^2\phi\} - \beta\frac{\partial\phi}{\partial x}, \tag{1}$$

where $\{\cdot,\cdot\}$ denotes the Jacobian $\{f,g\} = \partial_x f \partial_y g - \partial_y f \partial_x g$, and ϕ is the electrostatic potential. The model parameter $\beta = -1/L$ is defined in terms of density scale length, assuming an exponential background density profile in y-direction (radial direction) $n_0 \propto \exp(y/L)$. The model assumes adiabatic electron response with density fluctuation level $\tilde{n}/n_0 = \phi$.

Equation (1) is given in dimensionless units, by normalising potential by T_e/e, space by C_s/Ω_i and time by $1/\Omega_i$, where $C_s = (T_e/m_i)$ is sound speed and $\Omega_i = eB/m_i$ is ion cyclotron frequency.

The HM equation (1) was solved on a double-periodic rectangular domain using a highly accurate Fourier-Galerkin spectral method with semi-implicit time forwarding. The solution of the full non-linear system (1), was compared to the solution of the linearised system.

The non-linear dynamics are characterised by wave triad interactions and phase couplings, inverse spectral cascade of energy towards the large scales, and the presence of coherent vortical structures. These features are absent in the linear system, which is dominated by uncorrelated waves.

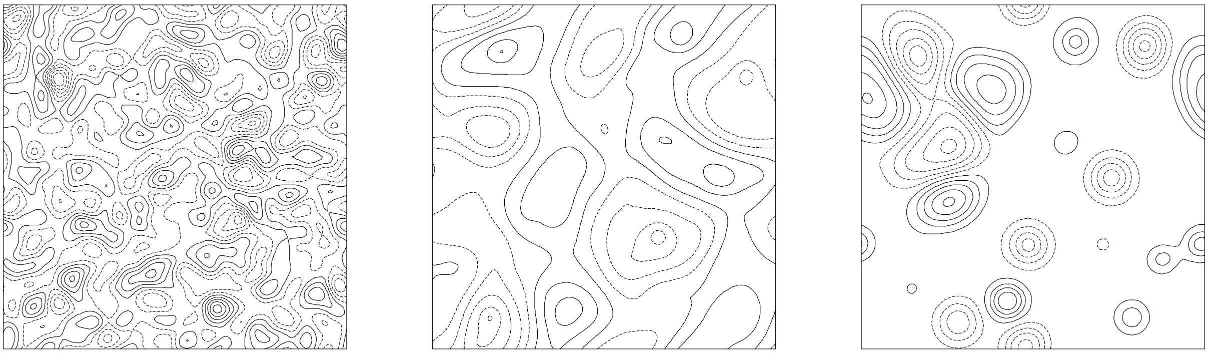

Figure 2. *Contour plots of density fluctuations for three solutions of the Hasegawa-Mima model. Left: linear conditions yields Gaussian field of uncorrelated waves. Middle: weakly non-linear conditions yields dispersive structures. Right: strongly non-linear conditions yields coherent vortices.*

Obviously, non-linear mode-coupling, spectral redistribution, and cascade phenomena profoundly influence the transport and correlation properties of the system. We have adressed these issues by solving the model equation for three scenarios, corresponding to linear, weakly non-linear, and strongly non-linear conditions (figure 2).

Spectral Analysis. The numerical solution was sampled at fixed points to yield time series of density fluctuations. These were subsequently analysed to study the imprint of various dynamics on the analysis.

Figure 3 shows autopower spectra from the simulations. The linear simulation (A) exhibits a wide range of frequencies corresponding to the wide range of uncorrelated wavenumbers excited. The weakly non-linear simulation (B) exhibits a dominant mode interacting with its first and second harmonic. Finally, the strongly non-linear simulation (C) contains only low frequencies associated with the slow drift of vortices.

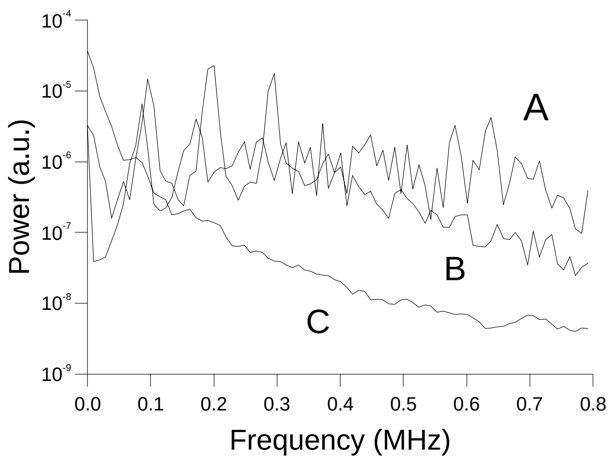

Figure 3. *Autopower spectra of simulated time series.*

Frequency resolved correlation properties are revealed by the cross-coherence of time series, sampled at poloidally displaced positions. Figure 4 demonstrates the enhanced coherence of uncorrelated linear waves.

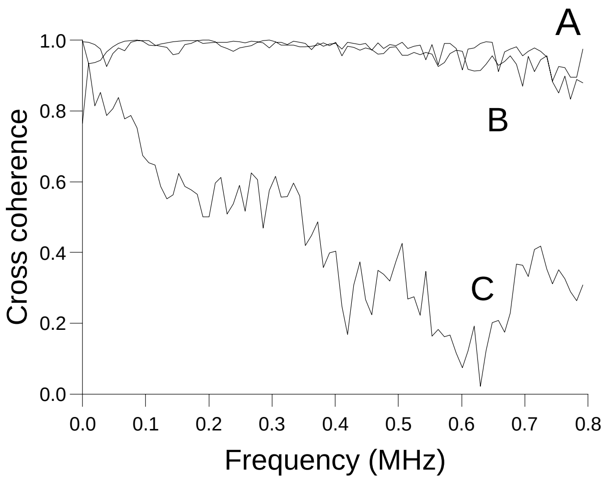

Figure 4. *Cross-coherence spectra of simulated time series under linear conditions (A), weakly non-linear conditions (B), and strongly non-linear conditions (C).*

The use of model data demonstrates that bispectral analysis[2] is is an efficient method of detecting non-linear wave couplings. Figure 5 reveals how strong non-linear interactions are revealed in the bicoherence spectrum, as compared to linear conditions.

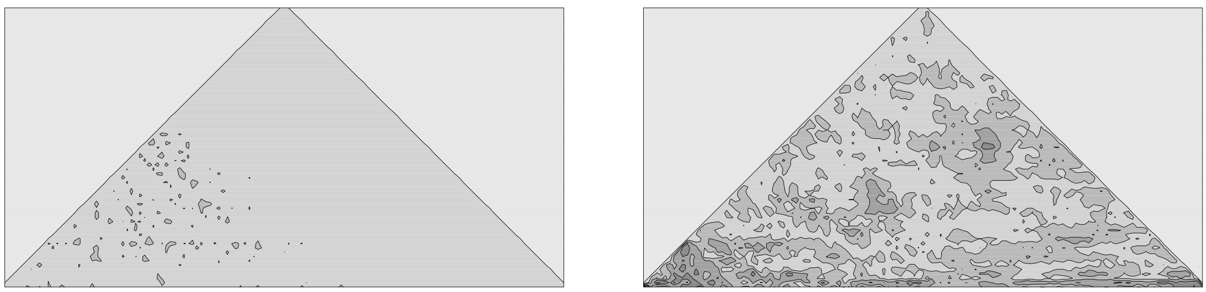

Figure 5. *Bicoherence spectra for linear conditions (left) and strongly non-linear conditions (right). Spectra shown for frequency range* $0 < f < 500$ *kHz.*

Summary. A collective scattering diagnostics based on two-point correlations has prompted numerical investigations of drift wave turbulence and spectral time series analysis. The Hasegawa-Mima model is used to generate test data under versatile dynamical conditions. Linear uncorrelated wave dynamics results in significant cross-coherence levels. Non-linear wave interactions are conveniently studied by bispectal analysis.

References.

1. A. Hasegawa and K. Mima. *Phys. Fluids* **21**, 87 (1978).

2. Y. C. Kim and E. J. Powers. *Phys. Fluids* **21**, 1452 (1978). *IEEE Trans. Plasma Science* **7**, 120 (1979).