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Outgoing and Global Drift Waves in Rotating Toroidal Plasma Configuration

I.Sandberg[1], V.P. Pavlenko[1], S.M. Revenchuk[2]

[1]*Department of Space and Plasma Physics, Uppsala University, S-755 91 Uppsala, Sweden EURATOM/NFR Fusion Association*

[2]*Institute for Nuclear research, National Academy of Sciences of Ukraine, Kiev, Ukraine*

Introduction

The drift waves in rotating toroidal plasmas are studied for an axisymmetric, large-aspect-ratio tokamak with concentric and circular magnetic surfaces. Plasma rotation is driven by the radial electrostatic field typical for the high (H) mode confinement regime. Low-frequency electrostatic oscillations of low-β plasma are considered in assumptions of adiabatic electrons and plasma quasineutrallity. In order to describe drift oscillations in the plasma edge region, where radial electric field and plasma rotation velocity are high, a weak coupling approximation that takes into account the toroidal coupling of normal modes centred on the neighbouring rational surfaces, is considered. The derived eigenmode equation has two classes of solutions giving either marginally stable global drift modes or propagating drift waves which experience the shear damping. The corresponding analytical dispersion relations are derived and simplified for some limiting cases.

Basic Equations

We consider low-β toroidal plasma configuration which rotates due to the presence of inhomogeneous electrostatic potential $\Phi(r)$. The plasma is confined by inhomogeneous magnetic field $\mathbf{B}$ with vanishing component along the equilibrium density $n_0(r)$ gradient. The frequency of drift modes is $\omega \ll \omega_{ci}$, and we suppose that electron inertia compared with their thermal motion is small $\omega \ll k_{||} v_{Te}$, so the electrons behave adiabatically. Then, we may use continuity and momentum equations for the ion fluid and Boltzmann distribution for the electrons assuming that quasineutrallity condition is satisfied. For electrostatic perturbations, we decompose electrostatic potential, ion velocity and density into equilibrium and fluctuating parts, assuming that the perturbed terms are of order of ϵ ($\epsilon = r/R \sim \omega/\omega_{ci}$) with respect to equilibrium quantities. In the first order, the considered equations are reduced to

$$\left(\frac{\partial}{\partial t} + \mathbf{v_0}\cdot\nabla\right)\frac{e\phi}{T_e} + \nabla\cdot\mathbf{v} + \mathbf{v}\cdot\nabla\ln n_0 = 0 \tag{1}$$

$$\frac{\partial v_{||}}{\partial t} + \hat{\mathbf{b}}\cdot(\mathbf{v_0}\cdot\nabla)\mathbf{v} + \hat{\mathbf{b}}\cdot(\mathbf{v}\cdot\nabla)\mathbf{v_0} = -\frac{e}{m_i}\hat{\mathbf{b}}\cdot\nabla\phi \tag{2}$$

and

$$\mathbf{v}_\perp = \mathbf{v_E} + \frac{\hat{\mathbf{b}}}{\omega_{ci}}\times\left[\frac{\partial \mathbf{v_E}}{\partial t} + (\mathbf{v_0}\cdot\nabla)\mathbf{v_E} + (\mathbf{v_E}\cdot\nabla)\mathbf{v_0}\right] \tag{3}$$

where $\mathbf{v_0} = (c/B)\hat{\mathbf{b}}\times\nabla\Phi_0$ is the equilibrium plasma rotation velocity, $\mathbf{v_E} = (c/B)\hat{\mathbf{b}}\times\nabla\phi$ is the $\mathbf{E}\times\mathbf{B}$ drift velocity while the usual drift wave ordering is used for the derivation of

Eq.(3). Inserting Eq.(3) into Eqs.(1), (2) we obtain a set of equations for $v_{||}$ and ϕ. Choosing the simple toroidal coordinate system r, φ, θ, we can express the magnetic field as $\mathbf{B} = B_p(r)\hat{\mathbf{e}}_\theta + B_t(1+\epsilon\cos\theta)^{-1}\hat{\mathbf{e}}_\varphi$. Here, $B_p(r)$ and B_t = const are the poloidal and toroidal components respectively. Drift modes are coupled in the poloidal direction due to toroidicity. Consequently, the mode localised on the rational surface $r = r_0$, which is defined by $m_0 - nq(r_0) = 0$ (m, n the poloidal and toroidal mode number and $q(r)$ the safety factor), is coupled poloidally with the modes localised on the neighbouring surfaces $r \pm \Delta r$ such that $m_0 \pm 1 = nq(r \pm \Delta r)$. So, we seek for solutions which in terms of the azimuthial mode spectrum are centred around the poloidal mode m_0 of the reference surface r_0. Since, the symmetry is not broken in the toroidal direction, we express the angular dependence of the perturbed quantities $\phi(r,\theta)$ and $v_{||}(r,\theta)$ in the following Fourier expansion

$$f(r,\theta) = \exp(im_0\theta - in\varphi)\sum_l f_l(r)\exp(il\theta)$$

Further, we expand the coefficients that are functions of r into Taylor series in the vicinity of the reference rational surface $r = r_0$, and we reduce the equations neglecting the high order corrections, into the following form

$$\frac{\partial^2 U_l}{\partial x^2} - DU_l + \sigma^2\left(x - \frac{l}{k_\theta\rho_s s}\right)^2 U_l = \tag{4}$$

$$= \frac{c_s k_\theta \rho_s}{\omega' R}\left(1 - \frac{r_0}{2r_n} - \frac{r_0 V_0}{2c_s\rho_s}\right)(U_{l+1} + U_{l-1}) + \frac{c_s}{\omega' R}\frac{\partial}{\partial x}(U_{l+1} - U_{l-1})$$

This set of equations describes the mode structure and the dispersion properties of a drift wave in a rotating tokamak plasma. The dimensionless variable x and a new potential function $U_l(r)$ have been introduced, through $x = (r - r_0)/\rho_s$ and $\phi_l(r) = U_l(r)\exp\left[(r_0 - r)(1/r_0 - 1/r_n)/2\right]$ respectively. The coefficients in Eq.(4) are

$$D = 1 + k_\theta^2\rho_s^2 - \frac{c_s k_\theta\rho_s}{\omega' r_n}\left[1 - \frac{\rho_s}{r_0}\frac{V_0}{c_s}(1+\xi)\right] + \frac{\rho_s^2}{r_0^2}\frac{k_\theta V_0}{\omega'}(1 - \xi - \xi\xi') \quad \text{and} \quad \sigma = \left|\frac{c_s k_\theta \rho_s s}{\omega' q_0 R}\right|$$

while the following parameters have been also introduced : $\xi = r_0 V_0'/V_0$ is the velocity shear, $\xi' = r_0 V_0''/V_0'$, $c_s{}^2 = T_e/m_i$ is the ion sound velocity, $V_0 = v_0(r_0)$ is the local rotation velocity, $V_0' = (dV_0/dr)_{r=r_0}$, $V_0'' = (d^2V_0/d^2r)_{r=r_0}$, $\omega' = \omega - k_\theta V_0$ is the Doppler-shifted eigenfrequency, $r_n^{-1} = -(d\ln n_0/dr)_{r=r_0}$ is the inverse scale of density inhomogeneity, $s = r_0 q_0'/q_0$ is the magnetic shear parameter with $q_0' = (dq_0/dr)_{r=r_0}$ and $k_\theta = m_0/r_0$ is the poloidal wavenumber.

Weak Coupling Approximation

In the weak coupling approximation, we consider only the coupling of the reference mode $l = 0$ with the two nearest modes $l = \pm 1$. Such approximation is justified, when the Doppler - shifted eigenfrequency ω' is of the order of the local diamagnetic drift frequency ω_*'. The system of equations (4) is reduced now into a set of three coupled differential equations. The equations for $l = \pm 1$ can be simplified by the successive approximation method to a standard form of the Weber equation for the functions of $U_{\pm 1}^{(0)}$. From the asymptotic solutions, we approximate the second derivatives of $U_{\pm 1}^{(0)}$ by keeping only the most rapidly changing term. Substituting this approximation into Eq.(4) for $l = 0$, we finally get

$$\frac{\partial^2 U_0}{\partial x^2} + \left(\Lambda - \zeta x^2\right) U_0 = 0 \tag{5}$$

where

$$\zeta = \sigma^2 DA, \;\; \Lambda = A\left[D^2 - 2\left(\frac{c_s k_\theta \rho_s}{\omega' R}\right)^2 \left(1 - \frac{r_0}{2r_n} - \frac{r_0 V_0}{2c_s\rho_s}\right)^2\right] \text{ with } A^{-1} = 2\left(\frac{c_s}{\omega' R}\right)^2 - D$$

The eigenfunctions of Eq.(5) describe the radial structure of the drift eigenmode, localised at the reference rational surface, and the corresponding eigenvalues define the dispersion properties of the excited drift modes.

• If $\zeta > 0$, Eq.(5) is transformed to a form of the Hermite equation

$$\frac{\partial^2 U_0}{\partial y^2} + \left(\frac{\Lambda}{\sqrt{\zeta}} - y^2\right) U_0 = 0 \tag{6}$$

where $y = \zeta^{1/4} x$. The eigenfunctions of Eq.(6) are in the form of $\exp(-y^2)H_N(y)$ (H_N is the Hermite polynomial of N^{th} order) and the corresponding eigenvalues are defined by the dispersion equation

$$\Lambda/\sqrt{\zeta} = 2N + 1, \qquad\qquad N = 0, 1, 2.. \tag{7}$$

This relation has positive eigenvalues only if $\Lambda > 0$. The corresponding solutions of Eq.(6) describe non-propagating modes localised inside a "potential well", i.e. the global modes. Substituting the expressions of Λ and ζ into Eq.(7), we can derive analytically the dispersion equation for these modes. The necessary conditions ($\zeta > 0$, $\Lambda > 0$) for the formation of global drift modes are satisfied for positive shifted Doppler eigenfrequencies which belong to the interval

$$1 + \frac{1}{\sqrt{2}}\frac{r_0}{R}\left(1 + \frac{r_n V_0}{\rho_s c_s}\right) < \frac{\omega'}{\omega_*} < \frac{1}{2} + \frac{1}{2}\left[1 + 8\frac{1 + k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\left(\frac{r_n}{R}\right)^2\right]^{1/2}$$

as well and for the negative Doppler eigenfrequencies ($\omega' < 0$) which belong to

$$0 < \frac{|\omega'|}{\omega_*} < -\frac{1}{2} + \frac{1}{2}\left[1 + 8\frac{1 + k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\left(\frac{r_n}{R}\right)^2\right]^{1/2}$$

The simplified expression for the dispersion equation (7), can be obtained in some limited cases. For example, when

$$\frac{r_n V_0}{\rho_s c_s} \gg 1 \quad \text{and} \quad 8\frac{1 + k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\left(\frac{r_n}{R}\right)^2 >> 1$$

the dispersion relation for the positive shifted eigenfrequencies can be written as

$$\frac{\omega'}{\omega_*} \simeq 1 + \frac{r_0}{\sqrt{2}R}\left[1 + (2N+1)\frac{s\sqrt{2}}{q_0}\left(\frac{r_n}{r_0}\right)^2\left(\frac{1}{\sqrt{2}k_\theta^2\rho_s^2}\frac{r_0}{R} - \frac{1}{4}\frac{r_0^2}{r_n^2}\right)^{1/2}\right]$$

• If $\zeta < 0$, we transform Eq.(5) to a form of the Weber equation

$$\frac{\partial^2 U_0}{\partial y^2} + \left(\frac{\Lambda}{i\sqrt{|\zeta|}} - y^2\right) U_0 = 0 \tag{8}$$

with $y = \sqrt{i|\zeta|^{1/2}}x$. The solutions of Eq.(8) are given by the Hermite functions H_ν and are bounded in the whole interval of y, only when $\nu = N = 0, 1, 2, ...$ The dispersion equation in this case is given by

$$\Lambda/i\sqrt{|\zeta|} = 2N + 1 \tag{9}$$

and the corresponding solutions of (8) describe radial propagating drift waves. The variable ζ takes negative values for positive Doppler eigenfrequencies which belong to

$$0 < \frac{\omega'}{\omega_*} < 1 \qquad \text{or} \qquad \frac{\omega'}{\omega_*} > \frac{1}{2} + \frac{1}{2}\left[1 + 8\frac{1 + k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\left(\frac{r_n}{R}\right)^2\right]^{1/2}$$

and for negative Doppler eigenfrequencies which belong to

$$\frac{|\omega'|}{\omega_*} > -\frac{1}{2} + \frac{1}{2}\left[1 + 8\frac{1 + k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\left(\frac{r_n}{R}\right)^2\right]^{1/2}$$

The dispersion equation for the propagating modes can be expressed in a more simple form, for some limited cases. So, for the case of negative Doppler-shifted eigenfrequencies, we find that if

$$2\frac{1 + k_\theta^2\rho_s^2}{k_\theta^2\rho_s^2}\left(\frac{r_n}{R}\right)^2 \ll \frac{|\omega'|}{\omega_*} < 1$$

the dispersion equation (9) is simplified to

$$\frac{|\omega'|}{\omega_*} \simeq \frac{r_n}{R}\left[\frac{1}{2}\left(\frac{r_0}{r_n} + \frac{r_0 V_0}{c_s\rho_s}\right)^2 - \left(N + \frac{1}{2}\right)^2\frac{s^2}{q_0^2}\right]^{1/2} - 1 - i\left(N + \frac{1}{2}\right)\frac{s}{q_0}\frac{r_n}{R}$$

We underline here, that the obtained dispersion relations for the propagating modes shows that these modes experience damping $\mathrm{Im}\omega' < 0$ due to the magnetic field shear, which found to be the same independently of the eigenfrequency value and sign and the various assumptions we considered in each case. In addition to this, it seems that the linear damping of the propagating drift modes depends on the magnetic shear and inhomogeneity scale, but not on the velocity shear.

Conclusions

The eigenmode equation describing weakly coupled drift waves in a rotating toroidal plasma has two classes of solutions expressing modes with different properties for each class. The global drift mode has a structure of a quasimode, localised in radial direction with a small wavenumber along the confining magnetic field. It includes a number of rational magnetic surfaces due to toroidal coupling of the modes localised on the neighbouring magnetic surfaces. This mode corresponds to the bounded state in a potential well, which is marginally stable. The propagating drift waves correspond to unbounded states and leave the magnetic surface on which they are excited. These waves are characterised by a damping which depends on the magnetic field shear and plasma density inhomogeneity scale, but not on the poloidal rotation velocity shear.