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RFP Confinement - Scalings from Numerical Simulations

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Introduction

Confinement properties of reversed-field pinch (RFP) plasmas are studied by carrying out a series of high resolution, three-dimensional, nonlinear, resistive MHD numerical simulations. The dependence of poloidal beta β_θ, on-axis temperature $T(0)$ and energy confinement time τ_E on experimental parameters are determined. These calculations include the effects of finite pressure, ohmic heating, convection and anisotropic heat conduction. By using plasma parameters relevant for present day RFP experiments, scaling laws that may be useful for predicting performance at higher temperatures and plasma currents are obtained.

Throughout we have attempted to obtain configurations with optimal confinement properties, so that the results of this study should be considered optimistic. No impurity radiation is assumed, and we use a stabilizing conducting wall and classical values for transport coefficients. The simulations were carried out for plasma currents I in the range 18-252 kA and densities n_0 in the range $(0.5\text{-}7.0)\cdot 10^{19}$ m^{-3}, resulting in on-axis temperatures $T(0)$: 26-105 eV and on-axis Lundquist numbers $S(0)$: $2\cdot 10^4$-$7\cdot 10^5$.

The scaling results obtained indicate that the conventional RFP may not extrapolate to thermonuclear conditions, primarily because of persistent magnetic fluctuations, being large enough to make the core magnetic field stochastic. Techniques for enhancing confinement, such as current profile and sheared flow control may thus be required to establish the reactor potential of the RFP.

This is the first numerical study of RFP transport scaling. The absence of earlier attempts can perhaps be ascribed to the apparently large number of independent parameters appearing in the problem. In this study, however, we have found that no more than two dimensionless parameters may be needed to determine optimised confinement within the resistive MHD model [1]. We have also found that confinement properties, and magnetic fluctuations, are largely independent of the exact choice of viscosity. Thus, for reasons of numerical stability, we choose local viscosity to be the largest of classical viscosity or a value sufficient to produce an effective Reynolds' number (usually $\approx$ 30, globally) at the scale length of a grid that is small enough to avoid transition to a turbulent state. In all cases, we have chosen the plasma current to line density ratio to be $I/N = 2.8\cdot 10^{-14}$ A-m, and used the aspect ratio $R/a = 1.25$, $\Theta \equiv B_\theta(a)/\langle B_z \rangle = 1.8$ and $p(a)/p(0) = 0.1$. The mass density is maintained constant throughout. The grid resolution is 300 radial mesh points, 42 axial and 6 poloidal modes. All calculations with the code DEBSP are run until a quasi-steady state is reached, and time-averaged values of poloidal beta and energy confinement time are determined.

Results

Values of poloidal beta for various values of β_0 and S_0 are displayed in Fig.1. These data are obtained by averaging over a sawtooth period, which usually lasts a few percent of the resistive time. Using linear regression analysis, the data points in Fig.1 can be well represented as a power law (indicated as solid curves)

$$\beta_\theta = 3.65\beta_0^{\;0.407\pm0.011} S_0^{\;-0.165\pm0.007} . \qquad (1)$$

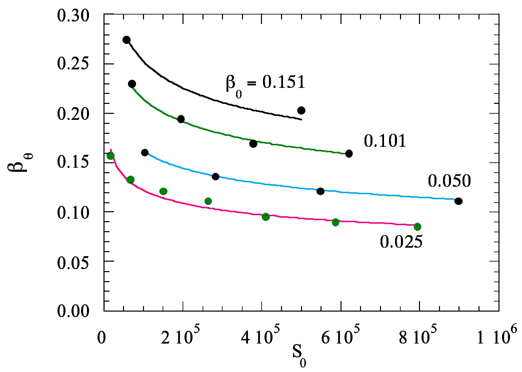

Fig. 1. Poloidal beta β_θ as function of β_0 and S_0. Solid lines represent power law fit.

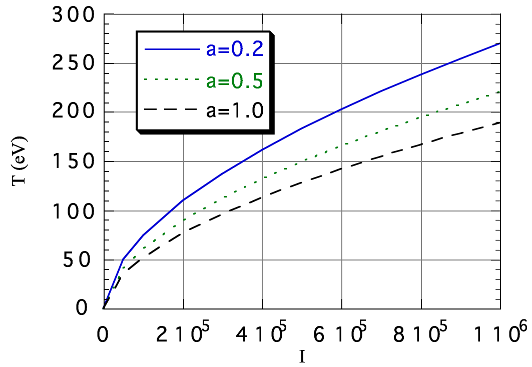

***Fig. 2.** On-axis temperature T(0) as function of current for different minor radii (Z_{eff} = 2, μ = 1).*

In a similar manner, the on-axis temperature $T(0)$ can also be found as a function of β_0 and S_0. The result is (μ is the ion to proton mass ratio)

$$T(0) = 0.221\, a^{-0.5} \mu^{0.25} Z_{eff}^{0.5} \beta_0^{\;-0.320\pm0.018} S_0^{\;0.311\pm0.011} . \qquad (2)$$

We can eliminate β_0 and S_0 from Eq. (1) in favour of $T(0)$ and I, obtaining

$$\beta_\theta = 156\, a^{0.04} \mu^{-0.02} Z_{eff}^{-0.04} T(0)^{1.09} I^{-1.01} . \qquad (3)$$

It is not surprising that the form of Eq. (3) closely resembles the definition of poloidal beta for constant I/N; ie., $\beta_\theta \propto \langle T\rangle/I$, where $\langle T\rangle$ is the volume averaged plasma temperature. The weak dependence on the pinch radius (a) and the ion mass (μ) is a measure of the accuracy of our results. The deviation of the exponent of $T(0)$ from unity is a result of profile effects; the temperature profile is observed to flatten for higher $T(0)$, see Fig. 3, in agreement with experimental results [2]. In the general case β_θ is a function of *both I and I/N*. Any attempt to obtain experimental scalings of β_θ with I or I/N *only* is thus misleading unless the other independent variable remains constant.

For an achieved steady state, we can replace T_0 with $T(0)$. The definitions of β_0 and S_0 then allow us to use Eq. (2) to relate the on-axis temperature $T(0)$ (in eV) to the total current I. The result is (see also Fig. 2)

$$T(0) = 0.071 a^{-0.22} \mu^{0.11} Z_{eff}^{0.22} I^{0.56} . \quad (4)$$

Substituting Eq.(4) into Eq.(3) yields

$$\beta_\theta = 8.8 a^{-0.20} \mu^{0.10} Z_{eff}^{0.20} I^{-0.40} . \quad (5)$$

Eqs. (4) and (5) are scaling laws for steady state on-axis temperature $T(0)$ and poloidal beta β_θ at a constant value of I/N (= $2.8 \cdot 10^{-14}$ Am). For comparison, the experimental scalings $T \propto I/n^{0.5}$ (or $\propto I^{0.5}$ for constant I/N) and $\beta_\theta \propto I^{-0.93} n^{0.46} \propto I^{-0.47}$ were recently found on the Extrap T2 RFP [2]. Earlier confinement experiments on the smaller Extrap T1 RFP resulted in the dependence $\beta_\theta \propto I^{-0.67} n^{0.16} \propto I^{-0.51}$ [3]. The unfavourable β_θ scaling with I is due to ohmic heating becoming less effective as compared to parallel heat conduction, causing an increasingly steep edge temperature gradient, as viewed in Fig. 3.

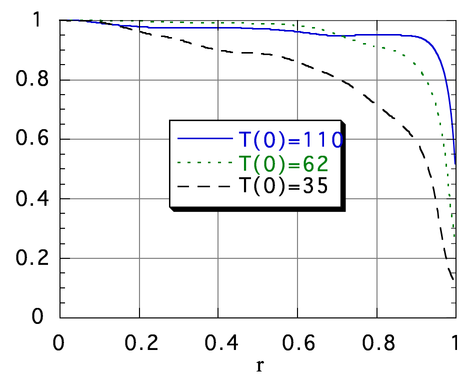

Fig. 3. *Temperature profiles for different on-axis temperatures.*

We now consider the scaling of the energy confinement time $\tau_E = 1.5 \langle p \rangle V_p / (U_{loop} I)$, with $V_p = 2\pi^2 R a^2$ being the plasma volume. Using regression analysis, the fitted energy confinement time can be expressed in experimental variables ($\ln\Lambda = 15$);

$$\tau_E = 2.8 \cdot 10^{-5} a^{1.4} \mu^{0.29} Z_{eff}^{-0.42} I^{0.34} \quad . \quad (6)$$

Eqs. (4), (5) and (6) are our main results. In a coming publication, where we will also include the dependence on I/N and Θ, specific comparisons with individual experiments will be made. Here we briefly remark on the implications of these results for the fusion potential of the conventional RFP. Using parameter values relevant for a compact RFP reactor (a = 0.6 m, I = 18 MA), Eq. (6) predicts $\tau_E \approx$ 0.004 s. This value for the confinement time is well below the design value of 0.2 s assumed in the TITAN reactor study [4] for the same parameters. Further, Eq.(4) implies an on-axis temperature of only about 1.1 keV as compared to the TITAN design value of 10 keV, contradicting the assumption of ohmic heating to ignition. However, the TITAN study used Θ = 1.5, as compared to our value of 1.8. This may yield somewhat better confinement due to a less stochastic plasma core. Care must also be taken when extrapolating to TITAN Lundquist numbers of 10^9, by far exceeding the range employed in our study.

For comparison with analytically and experimentally obtained scaling laws, we rewrite Eq. (6) using Eq. (5). We find (with $I/N = 2.8\cdot10^{-14}$ A-m)

$$\tau_E = 1.5\cdot10^{-7}a^{1.9}\mu^{0.05}Z_{eff}^{-0.90}\beta_\theta^{\ 2.4}I^{1.3} \ . \qquad (7)$$

Connor-Taylor scaling [5], which is based on theory of resistive *g*-mode fluctuations, predicts the dependence $\tau_E \propto I^{1.5}(I/N)^{1.5}a^2$. A fit to an international RFP database [6], using the form $\tau_E = c[I^{1.5}(I/N)^{1.5}a^2]^p$, in units of s, MA, 10^{20} m^{-1} and m, results in the parameter values $c \approx 6\cdot10^{-3}$ and $p \approx 0.87$. Adjusting the current dependence to this form, temporarily assuming constant β_θ, we obtain $c = 2.8\beta_\theta^{\ 2.4}a^{0.15}\mu^{\ 0.05}Z_{eff}^{\ -0.90}$ and p = 0.87, being very similar results. Since β_θ certainly is not constant, the forms for τ_E, suggested in [6], are of questionable utility.

Several deteriorating mechanisms were neglected in these simulations. On the other hand it should be noted, however, that kinetic effects may reduce e.g. edge pressure driven resistive modes, and thus contribute to enhanced confinement.

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