26th EPS Conf. on Contr. Fusion and Plasma Physics, Maastricht, 14 - 18 June 1999 ECA Vol.23J (1999) 729 - 732

Wave Power Flux and Ray-Tracing in Regions of Electron Cyclotron Resonant Absorption

E. Westerhof*, M.D. Tokman†, M. Gavrilova†

* *FOM-Instituut voor Plasmafysica 'Rijnhuizen', Associatie Euratom-FOM, Trilateral Euregio Cluster, PO Box 1207, 3430 BE Nieuwegein, The Netherlands*

† *Institute of Applied Physics of the Russian Academy of Sciences 603600, Nizhny Novgorod, Russia*

1. Introduction

Recently, the problem of wave propagation in media with resonant dissipation [1] has received renewed attention in connection with wave propagation through electron cyclotron (EC) resonant layers (e.g., [2]–[9]). In many cases dissipation remains weak in the zone of resonant absorption, and one might expect most results of wave theory for nondissipative media to apply also near EC resonance. However, near resonance the anti-Hermitian parts of the dielectric tensor elements ϵ_{nm} are of the same order of magnitude as the Hermitian parts and both may be very much larger than unity. That spatial dissipation nevertheless is weak, is due to the specific polarization of the waves [5,10]. As shown by Piliya and Fedorov [5] the anti-Hermitian part of the dielectric tensor leads to an additional term in the averaged Poynting theorem. This invalidates the usual expression for the wave energy flux (see e.g. [11]) near EC resonance.

In the present paper, this additional term is rewritten such that the Poynting theorem again takes the form of the divergence of an energy flux balanced by a single source term (Section 2). This way, a new expression for the wave energy flux is obtained, which is then shown to be consistent with the propagation of wave beams as obtained in Ref. [9]. Ray-tracing is reconsidered in Section 3.

2. Wave Power Flux

The starting point of the discussion of the wave power flux is the Poynting theorem: $\frac{1}{8\pi}\frac{\partial}{\partial t}\left(\tilde{\mathbf{B}}^2+\tilde{\mathbf{E}}^2\right)+\frac{1}{4\pi}\frac{\partial}{\partial \mathbf{r}}\left(\tilde{\mathbf{E}}\times\tilde{\mathbf{B}}\right)+\tilde{\mathbf{j}}\cdot\tilde{\mathbf{E}}=0$, where $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$ are the fluctuating electric and magnetic fields and $\tilde{\mathbf{j}}$ is the fluctuating current density. We are interested in the trajectory of quasi-stationary wave beams, and restrict the discussion to the time independent case. We consider a quasi-monochromatic wave $\tilde{\mathbf{E}}=\mathbf{E}(\mathbf{r})\exp(i\mathbf{k}_0\cdot\mathbf{r}-i\omega t)$, where $\mathbf{E}(\mathbf{r})$ is the weakly varying amplitude, $\mathbf{k}_0$ is the real wave vector, and ω the frequency. Absorption is described by the spatial dependence $E(\mathbf{r})$. The requirement, $k_0 \gg |(\partial\, E/\partial r)/E|$, implies weak absorption. A homogeneous medium is assumed. According to Ref. [5,] the Poynting theorem averaged over a wave period is, $\nabla\cdot\mathbf{S}+Q+q=0$, where (with the convention of summation over repeated indices, and the superscript $(a)H$ denoting the (anti-)Hermitian part of tensors)

$$\mathbf{S}=\frac{c^2}{16\pi\omega}\frac{\partial}{\partial\mathbf{k}}\left(D^H_{nm}\right)\Big|_{\mathbf{k}=\mathbf{k}_0}E^*_nE_m \qquad (1)$$

is the standard dielectric energy flux [11] in a dispersive medium in terms of the dispersion tensor $D_{nm} = \delta_{nm}k^2 - k_m k_n - \omega^2/c^2\varepsilon_{nm}$, and

$$Q = \frac{ic^2}{8\pi\omega}\left(D^{aH}_{nm}\right)\Big|_{\mathbf{k}=\mathbf{k}_0} E^*_n E_m \tag{2}$$

is the standard energy loss of the wave to the medium [11], while the last term

$$q = \frac{c^2}{16\pi\omega}\left(\frac{\partial D^{aH}_{nm}}{\partial \mathbf{k}}\right)\Bigg|_{\mathbf{k}=\mathbf{k}_0}\left(E^*_n\frac{\partial E_m}{\partial \mathbf{r}} - \frac{\partial E^*_n}{\partial \mathbf{r}}E_m\right) \tag{3}$$

arises in dissipative media with non-negligible anti-Hermitian part of the dielectric tensor. This last term shows that it is not allowed to identify the vector $\mathbf{S}$ with the wave energy flux, when the anti-Hermitian elements of the dielectric tensor ϵ^{aH}_{nm} do not vanish.

However, with the method as outlined in Ref. [8], it is possible to rewrite the Poynting relation in the form of the divergence of a modified flux and a modified source. The wave field is written as a composition of normal modes by

$$\mathbf{E}(\mathbf{r}) = \int \mathbf{e}(\lambda^{\mathrm{mode}})A_k(\mathbf{k}_0 + \Delta\mathbf{k})\delta(\lambda^{\mathrm{mode}})\exp(i\Delta\mathbf{k}\cdot\mathbf{r})d\Delta\mathbf{k}. \tag{4}$$

Here, λ^{mode} is the eigenvalue of the dispersion tensor $D_{nm}(\mathbf{k}_0 + \Delta\mathbf{k})$ corresponding to the wave mode, which has dispersion relation $\lambda^{\mathrm{mode}} = 0$, and $\mathbf{e}(\lambda^{\mathrm{mode}})$ is the associated unit eigenvector. The spectral density of normal modes is $A_k(\mathbf{k}_0 + \Delta\mathbf{k})\delta(\lambda^{\mathrm{mode}})$, where the Dirac delta function excludes contributions from $\Delta\mathbf{k}$ values that do not satisfy the dispersion relation. Taking into account the narrowness of the spatial spectrum, $k_0 \gg \Delta k$, the slowly varying part of the wave field, $\mathbf{E}(\mathbf{r})$, becomes

$$\mathbf{E}(\mathbf{r}) = \mathbf{e}(\mathbf{k}_0)A(\mathbf{r}) - i\frac{\partial A}{\partial r_n}\left(\frac{\partial \mathbf{e}}{\partial k_n}\right)\Bigg|_{\mathbf{k}=\mathbf{k}_0}, \tag{5}$$

where $\mathbf{e}(\mathbf{k}_0)$ is the unit eigenvector (polarization vector) for the central value in the spectrum, and $A(r)$ is the slowly varying scalar amplitude of the wave field defined by $A(r) = \int A_k(\mathbf{k}_0 + \Delta\mathbf{k})\delta(\lambda^{\mathrm{mode}})\exp(i\Delta\mathbf{k}\cdot\mathbf{r})d\Delta\mathbf{k}$. The second term on the right hand side of Eq. (5) describes a possible effect of absorption on the polarization. Now, expression (5) is substituted into Eqs. (1–3), and the amplitude is represented $A = |A|\exp i\phi$ with slowly varying phase ϕ. After some algebra, using the relations $D^{aH}_{nm}e_m = -D^{H}_{nm}e_m$, $e^*_n D^{aH}_{nm} = e^*_n D^{H}_{nm}$, and neglecting terms with $\partial^2 A/\partial r^2$ or $(\partial A/\partial r)^2$, the Poynting relation is again cast in the form, $\nabla\cdot\tilde{\mathbf{S}} + \tilde{Q} = 0$, with

$$\tilde{\mathbf{S}} = \frac{c^2}{16\pi\omega}|A|^2\frac{\partial}{\partial \mathbf{k}}\left(D^{H}_{nm}e^*_n e_m\right)\Big|_{\mathbf{k}=\mathbf{k}_0}, \tag{6}$$

giving the modified wave energy flux and with the modified source [8]

$$\tilde{Q} = \frac{ic^2}{8\pi\omega}|A|^2\left[\left(1 + \frac{\partial\phi}{\partial r_j}\frac{\partial}{\partial k_j}\right)e^*_n e_m D^{aH}_{nm}\right]_{\mathbf{k}=\mathbf{k}_0} \approx \frac{ic^2}{8\pi\omega}|A|^2\left(e^*_n e_m D^{aH}_{nm}\right)_{\mathbf{k}=\mathbf{k}_0+\nabla\phi}.$$

Although in Eq. (6) only the Hermitian part of the dispersion tensor appears explicitly, the modified wave flux depends on the anti-Hermitian part (D^{aH}_{nm}) through its effect on the polarization. When the effect of (D^{aH}_{nm}) on the polarization is negligible and, consequently, $D^{H}_{mn} e_n = 0$, the wave energy flux (6) reduces to the standard expression (1).

In Fig. 1 the direction of propagation of the maximum of a wave beam through EC resonance as obtained in Ref. [9] is compared with both the standard expression for the energy flux and the modified wave energy flux (6). Whereas, around resonance, the direction of the former deviates strongly from that of the beam maximum, the curve representing the latter is very close to or practically overlaps with the results from Ref. [9]. The small remaining difference in Fig. 1a is due to the assumption of weak absorption Im$k \ll$ Rek, which is only marginally satisfied in this case.

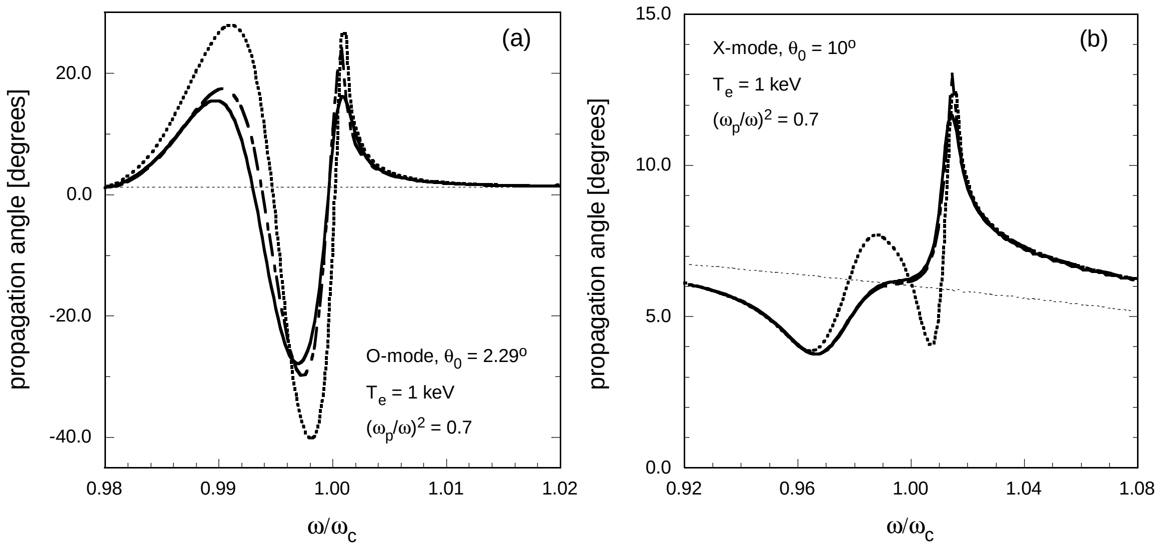

Figure 1. The angle of wave propagation with respect to the normal of the magnetic field: (a) O-mode injected at an angle of 2.29°; (b) X-mode injected at 10°. Full curves show results of Ref. [9], dotted ones the direction of the standard wave flux Eq. (1), and dashed curves the direction of the modified wave flux Eq. (6). Thin dotted lines give the cold plasma approximation.

3. Ray-Tracing

The term between the brackets in the modified wave energy flux (6) is identical to the real part of the eigenvalue for that mode, $\lambda'^{\mathrm{mode}} = D^{H}_{nm} e^{*}_{n} e_m$. A prime (double prime) is used to indicate the real (imaginary) part. This suggests to use the real part of the eigenvalue as Hamiltonian in ray-tracing:

$$\frac{d\mathbf{r}}{d\tau} = \frac{\partial \lambda'^{\mathrm{mode}}}{\partial \mathbf{k}}, \qquad \frac{d\mathbf{k}}{d\tau} = -\frac{\partial \lambda'^{\mathrm{mode}}}{\partial \mathbf{r}}. \tag{7}$$

This agrees with the usual assumption that ray-tracing in weakly dissipative media can be based on just the real part of the dispersion relation. However, this is not identical to neglecting the anti-Hermitian part of the dielectric tensor: first the dispersion tensor is brought in diagonal form using the complete dielectric tensor, only then can the imaginary part of the relevant dielectric tensor element be neglected.

In Fig. 2 the trajectories are shown as obtained from Eqs. (7) for the same parameters as in Fig. 1. For comparison, the trajectories of the maximum of a corresponding wave beam from Ref. [9] is shown. A discrepancy is again found for the O-mode case, which is somewhat larger here as the difference in the direction of wave propagation is integrated along the ray. Almost perfect agreement with the results of Ref. [9] would be obtained if one would correct the eigenvalue for the presence of a finite imaginary part of the wave vector, i.e. use $\lambda'^{\rm mode}(\mathbf{r},\mathbf{k}';k''_\perp)$ with $k''_\perp$ determined by some auxiliary relation. The rays then also overlap with those obtained with the method of Ref. [12].

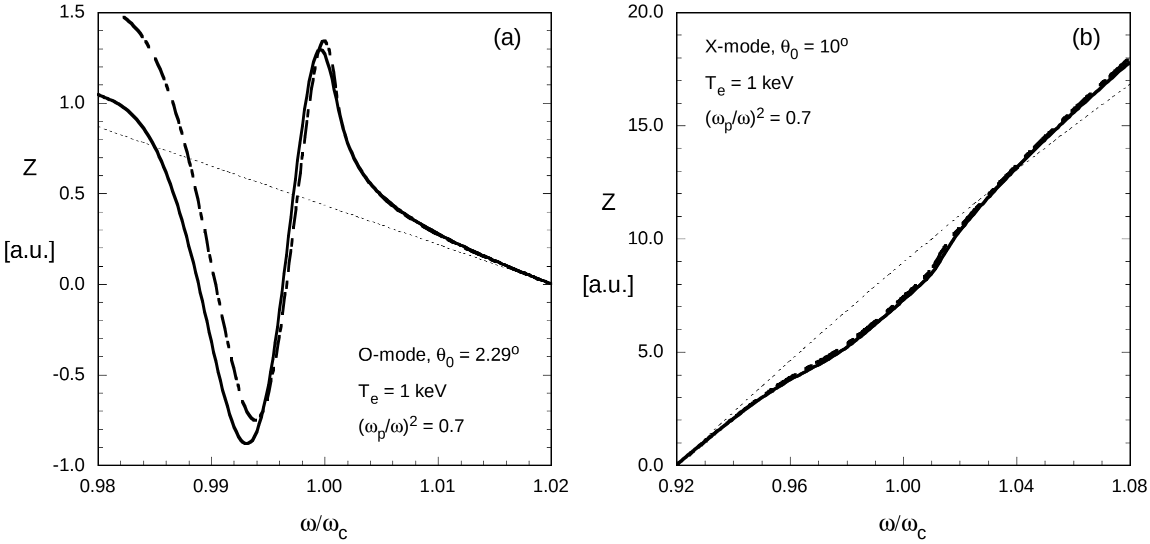

Figure 2. The trajectory of a wave beam: (a) O-mode injected from the low field side at an angle of 2.29°; (b) X-mode injected from the high field side with 10°. Full curves show the results of Ref. [9], and dashed curves the results of ray-tracing using Eqs. (7). Thin dotted lines give the cold plasma approximation.

Acknowledgements This work was performed under the Euratom-FOM association agreement with financial support from NWO, Euratom and INCAS (grant no. 98-2-01).

References

[1] L. Brillouin, *Wave Propagation and Group Velocity*, Academic Press, New York (1960).
[2] C. Maroli, V. Petrillo, Plasma Phys. **23** (1981) 671.
[3] M. Tanaka, M. Fujiwara, H. Ikegami, J. Phys. Soc. Japan **8** (1981) 1358.
[4] M. Bornatici, F. Engelmann, Comments Plasma Phys. Controlled Fusion **8** (1983) 57.
[5] A.D. Piliya, V.A. Fedorov, in *Reviews of Plasma Physics*, Vol. 13, ed. B.B. Kadomtsev, Consultants Bureau, New York (1987) p. 335.
[6] A. Pesić, S. Stojić, J. Phys. Soc. Japan **62** (1993) 2359.
[7] A. Pesić, Europhys. Lett. **35** (1996) 349.
[8] A.I. Smirnov, M.D. Tokman, Sov. Phys.-JETP **83** (1996) 294.
[9] E. Westerhof, Plasma Phys. Controll. Fusion **39** (1997) 1015.
[10] V.V. Alikaev, A.G. Litvak, E.V. Suvorov, A.A. Fraiman, in *High Frequency Plasma Heating*, ed. A.G. Litvak, American Institute of Physics, New York (1992) pp. 1–64.
[11] G. Bekefi, *Radiation Processes in Plasmas*, John Wiley, New York (1966).
[12] E. Westerhof, $10^{\rm th}$ Joint Workshop on ECE and ECH, 6–11 April 1997, Ameland, The Netherlands, World Scientific, p. 39.