Tomographic Inversion in Toroidal Geometry Using Flux Coordinates

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At Textor we are developing a new 2D x-ray imaging technique with markedly improved spatial and temporal resolution, thereby we hope to provide more experimental evidence to understand such penomena as the sawtooth crash, the details of disruptions and marfes. The dynamic ergodic divertor projected for TEXTOR–94 will ask for a diagnostic that gives both, high space and time resolution. In order to be able to derive quasi 3D images form the 2D pictures, we make symmetry assumtions, these will be discussed. None of the present day diagnostics does fulfill the requirements concerning the resolution, e.g. the largest number of diodes that has been used for soft x–ray tomography is 400…500.

A tangentially viewing soft x–ray camera [1] system has been build and installed on TEXTOR–94. The device consists of a pinhole camera with a 0.5 μm beryllium window, interchangeable holes ranging from 1…3 mm diameter and a fluorescent screen with a fast phosphor (P47 10 cm dia.). An optical fiber bundle (2 m long) with 15000 plastic fibers guides the light away from the strong magnetic field region into a Hamamatsu light amplifier. Finally the 2.5 cm dia image at the end of the light amplifier is recorded with a CCD camera (Hamamatsu) and the output from this camera is read into a LeCroy 6810 ADC. The storage depth of the Camac module allows us to take 8 pictures of 256×256 pixels each and 12 bit resolution. The positioning on TEXTOR–94 and the layout of the camera system is shown in fig.1 and fig.2.

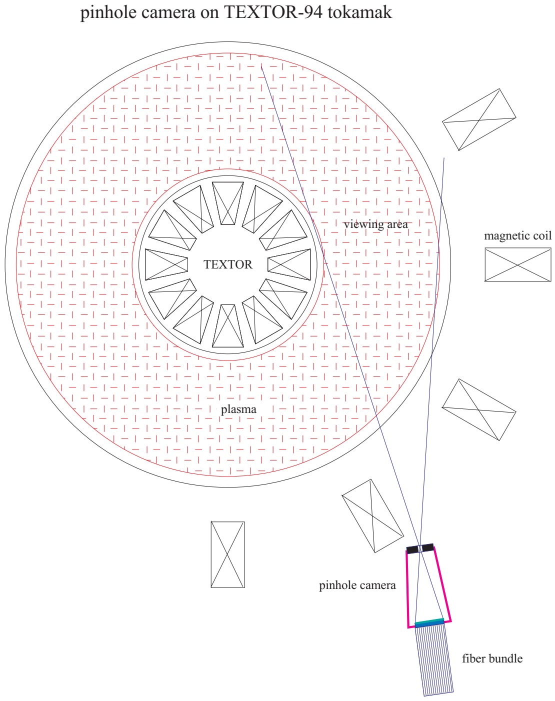

Figure 1: The positioning and viewing angle of the camera on TEXTOR–94.

The CCD camera we have used so far has a framing rate of 30 Hz. We have been able to take pictures with this camera using exposure times of 30 μs for ohmic discharges and 3 μs for such ones with additional heating. In order to be able to take the pictures near a sawtooth crash, we have build an event trigger which may take it's input from signals from e.g. the ECE or the soft x–ray pin diodes. Because we have to predict from the history of the sawtooth crashes the next

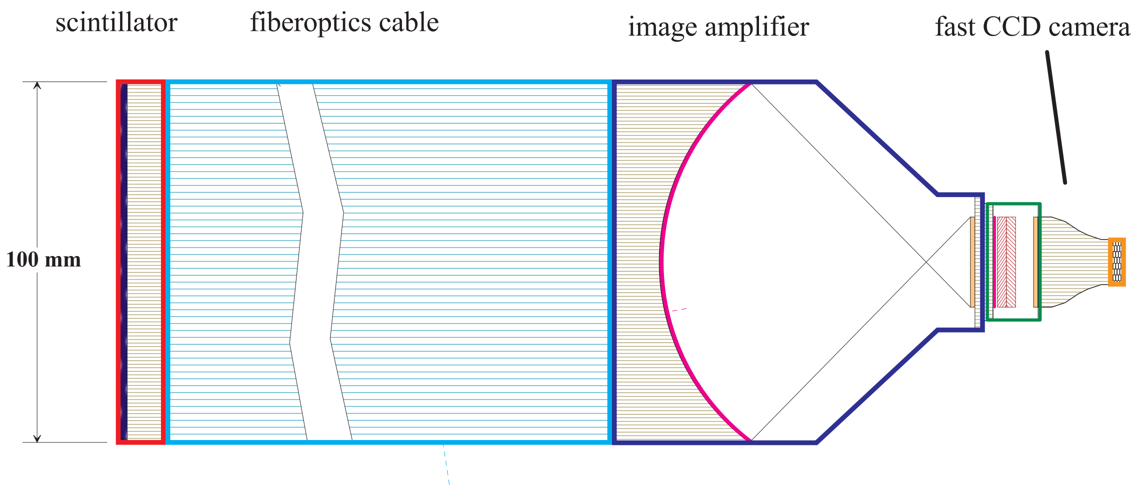

Figure 2: The layout of the camera from the fluorescent plate to the CCD camera.

one to come and because there is noise on the signals and jitter in the sawtoothing frequency, there is a uncertainty whether or not the crash time is correctly predicted. The probability to expose in the interesting time window is improved by at least a factor of 3 as compared to a fixed framing rate.

We do see profile changes accompanied with the sawtooth crash. However, the pictures we can compare do belong to subsequent crashes and therefore the interpretation is uncertain. Nevertheless, the measurements tell us, that can we use exposure times, which are short enough to resolve profile changes during MHD activity, therefore they do give a proof of principle.

For the future we plan to upgrade the camera system with a fast framing ICCD camera. This camera uses on chip storage, it is capable of a framing rate of 1 MHz together with a resolution of 100×100 pixels and it can store 32 frames at a time. It's pre– and post–trigger capability eases the event triggering, because we do not any more have to predict the next one to come; then we expect to get a 50% probability to expose correctly. The pictures we obtain are sinograms, i.e. they are projections along the lines of sight. These sinograms we get are 2–dimensional; this is, because we do have only 1 camera. If the plasma were truly 3–dimensional, we would need a set of cameras along the toroidal coordinate φ. To make up for this lack of information we assume, that the emission were constant along the magnetic field lines. To invert the projection (Radon transform) i.e. to get the tomogram from the sinogram we use a matrix inversion method. There we employ 2–dim Cartesian coordinates on the sinogram, 3–dim Cartesian coordinates along the lines of sight and flux coordinates when it comes to describe the magnetic configuration. These flux coordinates are action angle variables for the Hamiltonean system making up the magnetic configuration. The action being the toroidal flux Φ enclosed in a flux surface $\Psi = $ const, ϑ is the angle and φ is the toroidal angle and it is equivalent to the time in mechanical systems. Notice, that

$$\iota = \frac{d\Psi}{d\Phi} \tag{1}$$

the rotational transform is the equivalent of the rotation frequency; we assume that we know it e.g. from Faraday rotation measurements as they are done on TEXTOR–94. The coordinate ϑ is introduced in such a way as to make the two–form of the magnetic flux stay constant along flux lines[1].

$$\iota\, \mathrm{d}\Phi \wedge \mathrm{d}\varphi + B_t\, \mathrm{d}\Phi \wedge \mathrm{d}\vartheta = \mathrm{const} \tag{2}$$

In order to be able to set up this coordinate system we need to know both, ι and the shape of the $\Psi = \mathrm{const}$ surfaces. The case is somewhat discussed in [2], in fact it may involve an iteration where ι is obtained from Faraday rotation based itself on the shape of the flux surfaces being derived from the x–ray camera. The magnetic flux lines are straight in these coordinates.
To set up the inversion matrix we proceed as follows: The lines of sight give straight lines in a 3–dimensional coordinate system. The set of points along each of these line is projected along the magnetic field line passing the point into a poloidal plane, the reference plane. Each line of sight gives rise to a curve in this reference plane. The Radon transform in the reference plane is written:

$$\hat{f}(u,v) = \int f(\Phi,\vartheta)\; \delta(\{\Phi,\vartheta\} \in C(u,v))\; \Delta l(u,v,\Phi,\vartheta)\, \mathrm{d}A \tag{3}$$

The delta function sets the condition that the coordinates Φ, ϑ are on the curve C which belongs to the point u, v in sinogram space and $\Delta l(u,v,\Phi,\vartheta)$ is the line element along the line of sight. Equ.3 defines a mapping operator $\mathcal{A}$ and a mapping

$$\hat{f} = \mathcal{A} f \tag{4}$$

The restoring procedure with iteration follows the scheme

$$f^{i+1} = f^i + \lambda^i \mathcal{A}^T(\hat{f} - \mathcal{A} f^i), \tag{5}$$

where λ are parameters, which are to be chosen such as to give both, stable and fast convergence. A sinogram made during a TEXTOR–94 discharge is shown in fig.3. Because of the wire we have put in front for alignment reasons, the inversions are uncertain and will be done later when the wire has been removed.

[1] S. von Goeler et.al.; Rev.of Sci. Instruments **70** p.599 (1999)

[2] G. Fuchs and V. Pickalov; Plasma Phys. Control. Fusion **40** (1998)

[1] we use the language of differential geometry, here it means the Liouville theorem is being obeyed

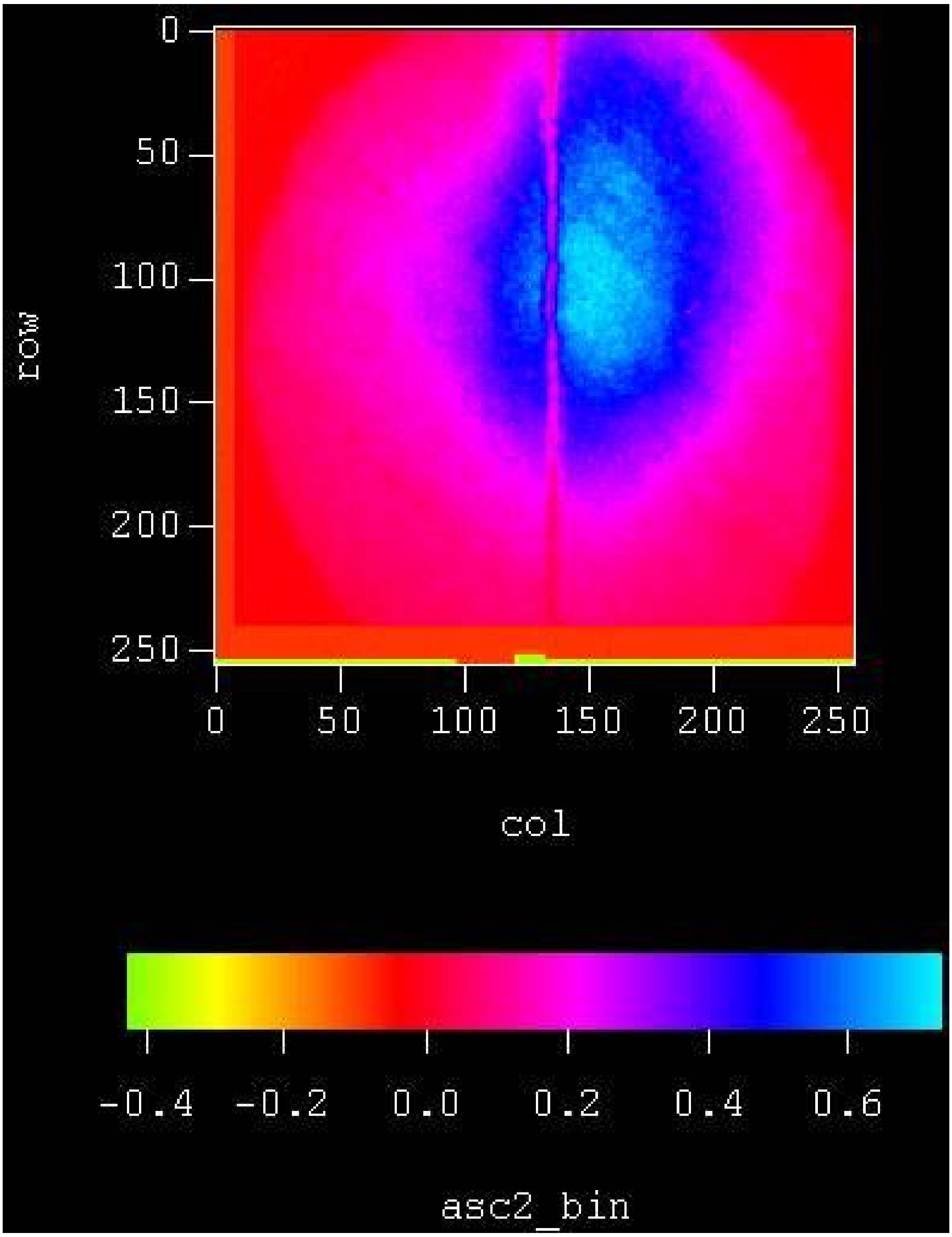

Figure 3: A picture taken with the camera from a TEXTOR–94 discharge. The symmetry axis of the torus is to the right and the picture is upside down. The line in the middle is a wire we put there in order to make sure the magnetic field does not distort the imaging of the light amplifier. This wire will be removed during the next servicing campaign; presently it hampers the inversion for which reason we do not give it.