Time and Space Evolution of Impurity Charge-State Distributions in Tokamak Plasmas

V. A. Shurygin

Institute of Nuclear Fusion, RRC "Kurchatov Institute",
123182 Moscow, Russia

1. Introduction.

Interpretation of anomalous impurity transport in tokamak plasmas is usually associated with enhanced particle transport. However, in analyzing the experimental spectroscopic data care must be taken to distinguish between the impurity particle (mass) transport and the motion of impurity charge-state distributions (called charge-state transport) across magnetic field due to atomic processes in plasma [1]. The latter transport has diffusive character and its diffusivity is the half sum of atomic rates [2]. More recently, the concept of impurity charge-state transport in tokamak plasmas has been developed and compared with conventional approach to the interpretation of anomalous impurity transport [3]. It has been shown that a set of conventional transport equations can be reduced to a single kinetic equation for the introduced charge-state distribution function where ionic charge k is assumed to be a continuous variable. It was shown that diffusivity of impurity charge states due to atomic processes can be as large as 5-300 m^2/s that strongly exceeds usual empirical magnitudes of anomalous particle diffusivity 0.1-1 m^2/s.

The present paper concentrates on the time-dependent solution of derived kinetic equation (neglecting the particle transport terms, for detail see [3]). This solution is assumed to be dependent on a number of the plasma parameters which determine the values of atomic rate coefficients and their reasonable time and profile variations. These are the electron density n_e and temperature T_e , neutral population density $\xi_n=n_n/n_e$ and relative population of superthermal electrons ε. In particular, large influxes of fast neutral atoms with energies 0.7-1 keV towards the plasma core originating from the regions close to the edge of plasma have been observed experimentally during disruptions in tokamak plasmas [4] and can strongly affect (through charge exchange) the impurity charge state distributions in the plasma core [1]. Recently, it has been also found that a self-consistent consideration of charge exchange processes (including excited states) can result in complete interpretation of high-resolution spectroscopic observations without using anomalous impurity diffusion coefficients [5].

2. Time-dependent impurity charge-state transport.

A kinetic equation for the continuous charge-state distribution function $f(k,\rho,t)$ can be used in the following form [3]

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial k}\left[(R-S)\cdot f + \frac{\partial Df}{\partial k}\right], \qquad (1)$$

where all particle transport terms are neglected, R, S - the recombination and ionization rates (in s^{-1}) respectively and $D=(R+S)/2$. Further, we assume that the value of R is dominated by charge exchange recombination. The steady state case of Eq. (1) was considered in detail in [3]. We shall consider the effect of instantaneous variations of the above-mentioned parameters (including profile changes, which are inherent for disruptive phenomena in tokamak plasmas) at $t_0=0$ on the impurity charge-state transport and charge-state distributions. The transport of impurity charge-states will be considered previously as a motion of charge-state distribution function from the initial $f_i\ (k,m_i,\alpha_i,t)$ to the final one $f_f\ (k,m_f,\alpha_f,t)$, where m_i, m_f are the locations of charge-state distribution maxima in the space of charge states $0 \le k \le Z$ and $\alpha_i=R_i/S_i$, $\alpha_f=R/S$, $R\equiv R_f$, $S\equiv S_f$, $D\equiv D_f$. The solution of Eq. (1) we find in the following form

$$f(k,t)=f_f(k)\cdot\left[b+\varphi(k,t)\right], \tag{2}$$

where b=const. Then, Eq. (1) can be reduced to

$$\frac{g_f}{D}\cdot\frac{\partial\varphi}{\partial t}=\frac{\partial}{\partial k}\left(g_f\cdot\frac{\partial\varphi}{\partial k}\right), \tag{3}$$

with

$$g_f\left(k,m_f,\alpha_f\right)=\exp\left(-2\int_k^{m_f}\frac{\alpha_f'-1}{\alpha_f'+1}dk'\right), \tag{4}$$

where m_f can be found (and similarly m_i) as a solution of equation

$$\alpha_f\left(m_f-\frac{1}{2}\right)=\frac{2m_f+1}{2m_f-1}, \tag{5}$$

which is more correct than that given in [3]. The boundary conditions for the introduced function $\varphi(k,t)$ can be presented as follows

$$\left.\frac{\partial\varphi}{\partial k}\right|_{k=0}=\left.\frac{\partial\varphi}{\partial k}\right|_{Z=0}=0. \tag{6}$$

Initial condition for $\varphi(k,t)$ can be obtained as

$$\varphi(k,0)=\frac{f_s(k)}{f_f(k)}-b. \tag{7}$$

Second order parabolic Eq. (3) with conditions (6) and (7) can be solved by the standard way (see, for example [6]) and its solution is

$$\varphi(k,t)=\sum_{n=1}^{\infty}C_n\exp(-\lambda_n t)\cdot X_n(k) \tag{8}$$

where C_n are constants, λ_n and $X_n(k)$ are eigenvalues and eigenfunctions respectively for the considered problem for the function $\varphi(k,t)$. To find them corresponding asymptotic formulas presented in [6] can be used as follows

$$\begin{gathered}X_n=\sqrt{\frac{2}{z}}\cdot\frac{\sqrt[4]{D}}{\sqrt{g_f}}\cdot\sin\left(\pi n\cdot\frac{x}{z}\right)\\ x=\int_0^k\frac{dk'}{\sqrt{D}},\ z=\int_0^Z\frac{dk'}{\sqrt{D}}\\ \lambda_n=\left(\frac{\pi n}{z}\right)^2\end{gathered} \tag{9}$$

where n=1, 2, 3,... . C_n coincides with Fourier coefficients for $\varphi(k,0)$

$$C_n=\int_1^Z\left(\frac{f_i(k,m_i,\alpha_i)}{f_f(k,m_f,\alpha_f)}-b\right)\cdot X_n(k)\cdot\frac{g_f(k,m_f)}{D}dk. \tag{10}$$

Characteristic equation for the time-dependent maximum of $f(k,t)$ derived from (1) is

$$\frac{m-m_i}{t-t_0} \cong R(m)-S(m)+2\frac{\partial D(m)}{\partial m} \cong R(m+1)-S(m-1). \quad (11)$$

3. Modelling.

Transport model (1)-(10) allows one to calculate time evolution of charge-state distribution function if the corresponding dependences of atomic rate coefficients are presented in the form of continuous functions. In fact, this is an analytical solution of the conventional set of transport equations for impurities, which takes into account the dominant effect of charge-state diffusion compared with particle one and, consequently, is an equivalent of the time-dependent impurity transport code.

First, we have simulated the time evolution of charge-state distribution function (for iron impurity) due to instantaneous changes of neutral population density from ξ_n=5·10^{-6} up to ξ_n=10^{-4} and of T_e from 1.5 keV up to 1 keV. It means that impurity recombination starts to dominate over ionization and the ionization balance between corresponding atomic processes appears to be shifted. The results are shown in Fig.1.

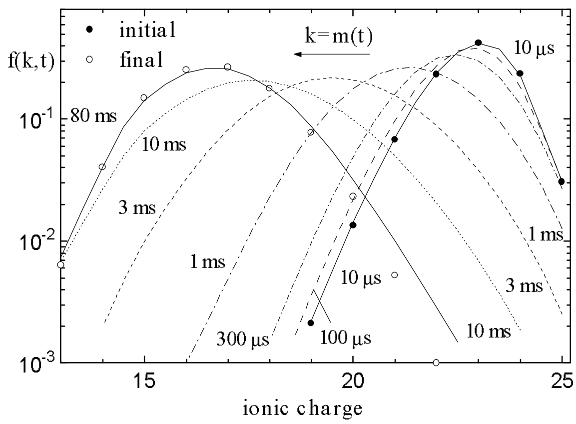

Fig.1. Time evolution of impurity (iron) charge-state distribution function simulated with proposed transport model (1)-(10).

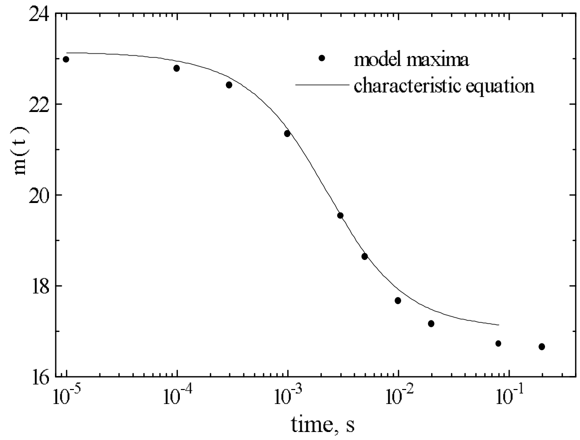

Fig. 2. Time evolution of charge-state distribution maximum obtained from model distributions (see Fig.1) and calculated from Eq. (11).

From these results it follows that *f(k,t)* begins to move shortly after the start of these changes (at t_0=t=0) when charge-state diffusion coefficient *D* changes as assumed instantaneously. 10-20 % relative changes of *f(k,t)* occurs in 100 μs. Further, charge-state distribution shifts rapidly as a whole towards lower *k* and the width of *f(k,t)* on *k* increases also as compared with the widths of initial and final distributions (see, for example, the curve for *t*=3 ms on Fig.1), while the maximum of *f(m(t),t)* firstly decreases and then gradually increases again. Time evolution of *m(t)* obtained from modelling agrees quantitatively with that derived from Eq.(11) (see Fig.2). The final distribution occurs only a few hundreds of milliseconds later.

These calculations can be extended to obtain the radial profile variations of impurity charge-state distributions. Therefore, radial diffusion of impurity charge-state distribution in tokamak plasmas can be simulated similarly assuming instantaneous model radial profile variations of the above-mentioned parameters that is inherent, for example, for disruptive phenomena. In particular, the sharp rise (~10 μs) of a large influx of fast (and more likely excited) neutrals occurs during the energy quench phase of a major disruption [1, 4] originating from the regions close to the edge of plasma. In fact, this acceleration of a few percent of neutral population at the edge regions of plasma results in the large and rapid influx of fast neutrals towards the core. The time evolution of thermal and superthermal electron components has been also taken into account in the modelling as the step changes of

$T_e(\rho)$ and $\varepsilon(\rho)$ in time. So, we assume at t=0 the drop of the central part of the profile of electron temperature and, simultaneously, that superthermal electrons are generated in the outer regions of plasma. The corresponding model profiles are given in Fig.3.

The results of that modelling are shown in Fig. 4. It is seen from the figure that the motion of the central charge-state profiles (Fe22+-Fe20+) towards the core occurs in a few hundreds of microseconds shortly after the start of the model profile changes, while final distributions are expected much more later. The peripheral changes due to generation of superthermal electrons move the corresponding charge-state profiles (Fe14+) towards the edge. Note that the profile Fe17+ in the intermediate radial region becomes wider.

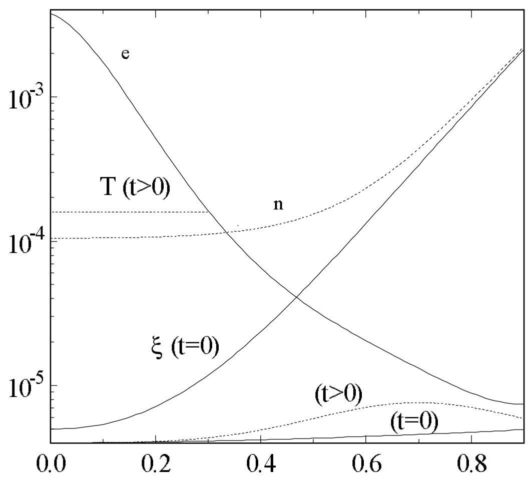

Fig.3. Time evolutions of model radial profiles of ξ_n, T_e and ε.

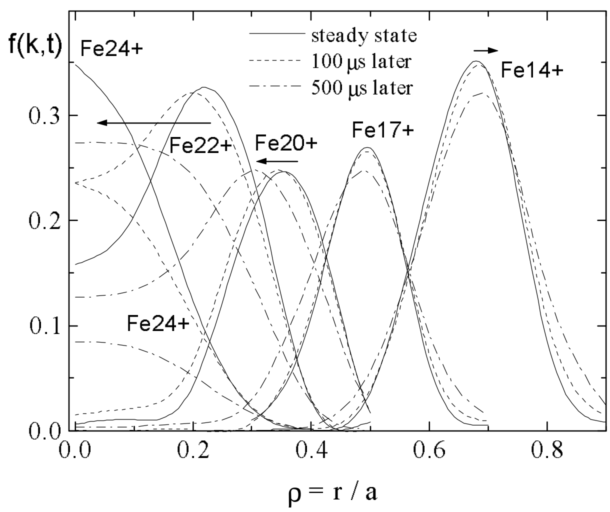

Fig. 4. Time evolution of radial charge-state distributions of iron impurity due to the step changes of model profiles presented in Fig.3.

4. Conclusions.

The developed 1-D transport code allows one to simulate the diffusion of impurity charge-state distributions in time and space. It was found that time-dependent impurity charge-state distributions (rather than impurity particles) can move (with their maxima) on a very rapid time scale by 0.1 - 1 ms due to changes in the rates of atomic processes. The reasonable time and profile evolution of the model parameters, especially, of the profiles of neutrals and superthermal electrons, results in significant effects, which look like 'anomalous' impurity transport. In particular, the significant broadening (both in k and in ρ) of time-dependent charge-state distributions as compared with their initial and final ones are observed during nonequilibrium phase of the above-considered transients. Meanwhile, the diffusion of impurity charge-states due to atomic processes in the plasma core and in the periphery appears to be driven by essentially different processes that qualitatively agrees with available experimental data.

References

[1] Shurygin V.A., *Plasma Phys. Rep.* **22** 975 (1996) and
Rantsev-Kartinov V.A., Shurygin V.A., *Plasma Phys. Rep.* **23** 3 (1997)
[2] Ivanov V.V., Kukushkin A.B., Lisitsa V.S., *Sov. J. Plasma Phys.* **13** 774 (1987)
[3] Shurygin V.A., *Plasma Phys. Control. Fusion* **41** 355 (1999)
[4] Bortnikov A.V. et al., *Plasma Phys. Rep.* **21** 672 (1995)
[5] Rosmej F.B. et al., *Plasma Phys. Control. Fusion* **41** 191 (1999)
[6] Polojij G.N., Equations of mathematical physics, Izdat-vo "Vysshaya shkola", 1964