Detailed interpretation of the Mirnov coils data in non-circular low aspect ratio tokamaks

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1. Introduction

Detailed interpretation of the Mirnov coils data in D-shaped and low aspect ratio tokamaks is much more complicated than in the case of cylindrical geometry. This task is important for investigations of the large scale MHD perturbations especially during disruptive instability. Usually it is solved by decomposition of Mirnov probe data into the Fourier series in poloidal angle. A difficulty results from an ambiguity of the poloidal angle. Generally speaking, any function which is periodic when going around the magnetic axis, could be considered as a poloidal angle. Difficulties appear also due to the different distances of Mirnov coils from the plasma boundary and from the phase errors connected with conducting elements of the vacuum chamber placed between some coils and plasma. In Ref. [1,2] some approaches to solving of this problem are developed.

In the present paper we describe a new method of modes decomposition of magnetic probes data, which is rather adequate from the physical point of view. We use a model, in which we suppose that the helical field on the probes is produced by the helical currents, localized on the resonant magnetic surfaces. We suppose that the current density on these magnetic surfaces is directed along of the field lines. In the process of calculations we find the magnetic fields, which would be produced by each of such helical currents with unit amplitude. The magnetic field at the magnetic probes is assumed to be the sum of the fields from helical currents on several resonant surfaces. Then, minimizing the deviation of the magnetic field produced by these currents from the experimentally measured field, we find the amplitudes and phases of the helical currents.

2. Principal equations and description of the method

Current density on each resonant surface m, n obeys the relationship:

$$J_{mn} = J^c_{mn}\cos(m\theta - n\varphi) + J^s_{mn}\sin(m\theta - n\varphi) \ . \qquad (1)$$

Here φ is the toroidal angle, and θ is the poloidal angle in a coordinate system with straight field lines. Contrary to other poloidal angles, this angle which allows the current density to be expressed by the Eq. (1) is unique one for any magnetic surface. The field line equation in this coordinate system has a simple form:

$$\theta = \theta_0 + (n/m)\varphi \ .$$

From the system of equilibrium equations in the coordinate system with nested magnetic surfaces Ref. [3] one can find the equation for θ:

$$\theta = 2\pi \, \frac{\int_0^{\omega}(D/H)d\omega}{\int_0^{2\pi}(D/H)d\omega} \ ; \qquad (2)$$

Here ω is usual poloidal angle, $H = r/R$, $D = r'_a z'_\omega - r'_\omega z'_a$, r is local major radius on a magnetic surface, R is averaged major radius and a is averaged minor radius, or label of a magnetic surface; $r'_a \equiv \partial r/\partial a$, $r'_\omega \equiv \partial r/\partial\omega$, ...

Magnetic field, produced by current element dJ, is:

$$d\mathbf{B} = dJ \, \frac{[d\mathrm{l} \times \boldsymbol{\rho}]}{\rho^3} \ . \qquad (3)$$

Let's consider a magnetic probe, placed at a position $\{x_k, y_k, z_k\}$. Let $\{x, y, z\}$ is some point at a resonant surface.

In this case:

$$\rho = [(x - x_k)^2 + (y - y_k)^2 + (z - z_k)^2]^{1/2} \ .$$

At resonant magnetic surface (a =const) the lines, where the helical current density is constant, are:

$$x = x(\theta_0, \varphi) \ , \quad y = y(\theta_0, \varphi) \ , \quad z = z(\theta_0, \varphi) \ .$$

$$\dot{x} = \frac{\partial x}{\partial \varphi} = \frac{\partial x}{\partial \omega} \, \frac{n/m}{\partial\theta/\partial\omega} \ , \ \ldots$$

Then, the field, produced at this magnetic probe by localized at a resonant surface helical current, is:

$$B_{xk} = J^c_{mn} \int_0^{2\pi} \cos m\theta_0 \, d\theta_0 \int_0^{2\pi} \frac{\dot{y}(z - z_k) - \dot{z}(y - y_k)}{\rho^3} d\varphi$$

$$+ J^s_{mn} \int_0^{2\pi} \sin m\theta_0 \, d\theta_0 \int_0^{2\pi} \frac{\dot{y}(z - z_k) - \dot{z}(y - y_k)}{\rho^3} d\varphi \ ;$$

$$B_{yk} = J^c_{mn} \int_0^{2\pi} \cos m\theta_0 \, d\theta_0 \int_0^{2\pi} \frac{\dot{z}(x - x_k) - \dot{x}(z - z_k)}{\rho^3} d\varphi$$

$$+ J^s_{mn} \int_0^{2\pi} \sin m\theta_0 \, d\theta_0 \int_0^{2\pi} \frac{\dot{z}(x - x_k) - \dot{x}(z - z_k)}{\rho^3} d\varphi \ ;$$

$$B_{zk} = J^c_{mn} \int_0^{2\pi} \cos m\theta_0 \, d\theta_0 \int_0^{2\pi} \frac{\dot{x}(y - y_k) - \dot{y}(x - x_k)}{\rho^3} d\varphi$$

$$+ J^s_{mn} \int_0^{2\pi} \sin m\theta_0 \, d\theta_0 \int_0^{2\pi} \frac{\dot{x}(y - y_k) - \dot{y}(x - x_k)}{\rho^3} d\varphi \ .$$

Produced by each helical current magnetic field at a probe k is:

$$B_k = B_{xk} n_{xk} + B_{yk} n_{yk} + B_{zk} n_{zk} \ , \tag{4}$$

where $\mathbf{n} = \{n_x, n_y, n_z\}$ is a unit vector of probe direction.

Let J_l is the amplitude of helical current number l (numeration is arbitrary), and σ_{lk} is the magnetic field, produced by unit helical current number l at probe number k. The values of σ_{lk} we find using the above relationships, and after it, the values of J_l one can find as a result of minimization of functional:

$$S = \sum_{lk} (J_l \sigma_{lk} - B_k^{exp})^2 \ , \tag{5}$$

where B_k^{exp} represents the measured field at probe k.

The condition of minimization leads to a system of l equations for J_l:

$$\sum_{l'} J_{l'} \sum_k \sigma_{lk} \sigma_{l'k} = \sum_k \sigma_{lk} B_k^{exp} \ . \tag{6}$$

After solution of this system of equations we know all the amplitudes J_l and, hence, the amplitude and phase of each helical mode.

On the basis of this approach we designed a numerical code. This code was tested by comparison of the numerical results with analytical solutions for some model cases. Also the Mirnov coils data, obtained in DIII-D tokamak were used for test of this code. The fact that calculated amplitude and phase of each mode doesn't change dramatically for near time moments is indirect confirmation of correctness of it.

3. Conclusions

A new approach for solving of the problem of mode decomposition of MHD signal, measured by Mirnov coils in D-shaped and low aspect ratio tokamaks was developed. On the basis of the equations derived, a numerical code was designed. This code was tested using the Mirnov coils data, obtained in DIII-D tokamak.

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References

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