Plasma Depletion and Electron Current Saturation for Positively Biased Flush-Mounted Probe in Magnetic Field

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Abstract

The problem of electron saturation current for flush-mounted probe is treated on the basis of the model including effects of various mechanisms of transverse conductivity and anomalous diffusion. It is shown, that electron current saturation is essentially determined by the plasma density depletion in the channel in front of the probe and by corresponding diffusive fluxes. At the same time, the scales of the depleted zone are determined by the processes of conductivity.

1. Introduction

Theory of a probe was developed by many authors (see for example [1-3]) for the case when plasma density was assumed to be undisturbed. Models based on such an approach describe only transitional part of I-V characteristics, when modest positive voltages are applied to the probe, and ion saturation current collection. The situation is quite different for large positive probe potentials, when electron saturation current is collected. Plasma density in this situation is strongly perturbed, and this perturbation crucially determines all the processes.

Here further development of transverse conductivity model [1] is presented. New model takes into account plasma density depletion and thus is able to describe the structure of electric fields and particle fluxes for the case of large probe voltages and predicts the value of electron saturation current as well as all the I-V characteristic.

2. Model

We consider a flush-mounted probe on the infinite plain conductive wall exposed to magnetized fully ionized plasma, see Fig.1. Magnetic field B in plasma is assumed to be constant and normal to the wall. We restrict ourselves to the 2D problem, so geometry considered is slab or cylindrical. In slab geometry probe is an infinite strap with semi-width a; in cylindrical geometry probe is a disk with radius a. Plasma with constant (but not necessarily equal) electron and ion temperatures T_e and T_i is created at infinity and flows to the wall with sound speed $c_s = [(T_e + \gamma T_i)/m_i]^{1/2}$ according to the Bohm criterion.

Voltage V is applied to the probe with respect to the grounded wall (in the present paper we focus on positive probe voltages). Potential in probe channel increases correspondingly and transverse electric field arises. Processes of effective or real transverse ion conductivity (caused by ion viscosity, inertia or ion-neutral collisions [1,3]) drives ions across B away from the probe channel on their way parallel to B towards the wall. At the same time electrons are extracted along B to the positively biased probe. Thus, plasma density is depleted in the probe channel. Perpendicular density gradient leads to anomalous ambipolar diffusive flux. Situation is stationary: electrons coming from infinity along the magnetic field are dragged across B by diffusion to the probe channel and then are collected by the probe; transverse motion of ions consists of two contributions, perpendicular electric field extracts them out of the probe channel, while diffusive flux has an opposite direction.

For modest positive voltages balance is established for slightly perturbed plasma density, since anomalous diffusion is quite large. However, for sufficiently large V density

depletion becomes significant, and electron saturation current to the probe is limited by the perpendicular diffusion flux to the depleted channel, i.e., by the value of the diffusion coefficient. The larger the diffusion is, the larger voltage should be applied to the probe to reach current saturation. Hence, models that consider density as undisturbed are valid for modest applied voltages. We consider anomalous diffusion which is modelled by constant coefficient D .

Initial equations for the analysis are the particle and momentum balance equations for electrons and ions

$$\nabla \cdot \left(n\vec{u}_{e,i} \right) = 0 \ , \tag{1}$$

$$0 = -\nabla p_e + en\nabla \varphi - en\left[\vec{u}_e \times \vec{B} \right] + \vec{R}_{ei} \ , \tag{2}$$

$$m_i \nabla \cdot \left(n\vec{u}_i \vec{u}_i \right) = -\nabla p_i - en\nabla \varphi + en\left[\vec{u}_i \times \vec{B} \right] - \nabla \cdot \vec{\pi}_i - \vec{R}_{ei} + \vec{R}_{iN} \ , \tag{3}$$

where $\vec{\pi}_i$ is the ion stress tensor, $\vec{R}_{ei}$ is the electron ion friction force

$$R_{ei\|} = 0.51 n m_e \nu_{ei} \left(u_{i\|} - u_{e\|} \right) \quad , \qquad \vec{R}_{ei\perp} = n m_e \nu_{ei} \left(\vec{u}_{i\perp} - \vec{u}_{e\perp} \right) \ , \tag{4}$$

and $\vec{R}_{iN}$ is the ion-neutral friction force

$$\vec{R}_{iN} = n\mu_{iN} \nu_{iN} \left(\vec{u}_N - \vec{u}_i \right) \ , \tag{5}$$

where μ_{iN} is the effective mass for charge exchange collisions, ν is the collision frequency. The velocity of neutrals is assumed to be smaller than that of ions.

The basic equation system Eqs.(1-3) must be completed with the set of boundary conditions. At infinity ions and electrons are moving along the magnetic field with sound speed c_s , the density is equal to undisturbed density n_0 and the potential is equal to the undisturbed plasma potential $\varphi_f = (T_e/2e)\ln(T_e/2\pi m_e c_s^2)$ with respect to the grounded wall. At plasma-sheath boundary ($z = 0$) the longitudinal current coming from plasma must be equal to the current passing through the sheath

$$-j_\| \big|_{z=0} = en_s c_s - en_s \sqrt{T_e/2\pi m_e} \exp\left[-e(\varphi_s + \varphi_f - \varphi_p)/T_e \right] \ , \tag{6}$$

where subscript s denotes that quantity is taken at the plasma-sheath boundary, φ_p is equal to the probe potential V inside, or to zero outside the probe channel.

The system of basic equations can be simplified. Particle continuity equations (1) lead to current continuity equation

$$\nabla \cdot \vec{j} = 0 \ . \tag{7}$$

Longitudinal current density can be expressed from longitudinal component of momentum balance equation for electrons Eq.(2) as a function of density n and potential φ profiles

$$j_\| = -\sigma_\| \left[\nabla_\| \varphi - (T_e/e)\nabla_\| \ln(n/n_0) \right] = -\sigma_\| \nabla_\| \psi \quad , \qquad \psi = \varphi - (T_e/e)\ln(n/n_0) - \varphi_f \ . \tag{8}$$

Expressions for transverse current density can be derived from momentum balance equation for ions Eq.(3). These expressions are quite complicated and are the topic of more detailed forthcoming paper, so here we specify only two of them: for viscosity driven transverse conductivity for cases of slab and cylindrical geometry:

$$j_x = \frac{1}{B^2} \frac{\partial}{\partial x}\left[\eta_1 \frac{\partial^2}{\partial x^2}\left(\varphi + \frac{T_i}{e}\ln\frac{n}{n_0} \right) \right] \ , \quad j_r = \frac{1}{B^2} \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^3 \eta_1 \frac{\partial}{\partial r}\left[\frac{\partial}{r\,\partial r}\left(\varphi + \frac{T_i}{e}\ln\frac{n}{n_0} \right) \right] \right) \ , \tag{9}$$

(where η_1 is the viscosity coefficient) and for the case of conductivity caused by ion-neutral friction for arbitrary geometry

$$\vec{j}_\perp = -\frac{n\mu_{iN}\nu_{iN}}{B^2} \nabla_\perp \left(\varphi + \frac{T_i}{e}\ln\frac{n}{n_0} \right) . \tag{10}$$

Substituting expressions (8) and (9) or (10) for current components into current continuity equation (7) one obtains a system of two equations for density and potential. Here this system is written for the case of slab geometry

$$D \frac{\partial^2 n}{\partial x^2} + c_s \frac{\partial n}{\partial z} = \frac{\sigma_\parallel}{e} \frac{\partial^2 \psi}{\partial z^2} \quad \text{and} \tag{11}$$

$$\frac{1}{B^2} \frac{\partial^2}{\partial x^2} \left[\eta_1 \frac{\partial^2}{\partial x^2} \left(\psi + \frac{T_e + T_i}{e} \ln \frac{n}{n_0} \right) \right] = \sigma_\parallel \frac{\partial^2 \psi}{\partial z^2} \quad \text{(the case of viscosity) or} \tag{12}$$

$$\frac{\mu_{iN} \nu_{iN}}{B^2} \frac{\partial}{\partial x} \left[n \frac{\partial}{\partial x} \left(\psi + \frac{T_e + T_i}{e} \ln \frac{n}{n_0} \right) \right] = -\sigma_\parallel \frac{\partial^2 \psi}{\partial z^2} \quad \text{(the case of ion-neutral friction)} \tag{13}$$

with the boundary conditions at infinity and boundary condition at plasma boundary

$$\sigma_\parallel \frac{\partial \psi}{\partial z} \Big|_{z=0} = e n_s c_s - e n_0 c_s \exp \left[\frac{e}{T_e} \min \left(\varphi_p - \psi_s, \ln n_s + \varphi_f \right) \right] . \tag{14}$$

For simplicity in Eq.(11) it is assumed that longitudinal ion velocity is equal to the sound speed. Then the system of Eqs.(11-14) is a subject of numerical simulation.

3. Results

The system of equations (11,13) with boundary condition Eq.(14) was solved numerically for the case when the transverse conductivity caused by ion-neutral friction dominates. Typical distributions of the potential and density are shown in Figs.(2,3) for large applied voltages. Longitudinal characteristic scale of perturbation is close to $l_\parallel = \lambda_{mfp} (m_i / m_e)^{1/2}$ as was derived analytically for small probe potentials [1,3]. The transverse characteristic scale is also close to the result obtained from the linearized model [1]

$$R_0^{i-N\,friction} = \sqrt{\mu_{iN} \nu_{iN} n_0 l_\parallel^2 / B^2 \sigma_\parallel} . \tag{15}$$

The I-V characteristics are shown in Fig.(4) for two values of anomalous diffusion coefficient. The slope of the transitional part of the I-V characteristic is close to that derived under the assumption of undisturbed density [1]. The values of the electron saturation current, obtained in the simulation, corresponds to the simple estimate

$$I_e^{sat} \sim 2eD^* (\partial n / \partial x) l_\parallel \sim 2eD^* n_0 l_\parallel / R_0 , \quad \text{or} \quad I_e^{sat} = 2keD^* n_0 l_\parallel / R_0 , \tag{16}$$

where D^* is the effective diffusion coefficient, as it follows from Eq.(11,13), just the sum of anomalous diffusion coefficient D and diffusion coefficient corresponding to ion-neutral collisions

$$D^* = D + \mu_{iN} \nu_{iN} (T_e + T_i) / m_i^2 \omega_{ci}^2 . \tag{17}$$

Expression (16) is obtained from the assumption that the diffusion to the depleted channel of dimensions $l_\parallel$ and R_0 determines the saturation current to the probe. The exact numerical coefficient k in Eq.(16) should be obtained by comparison with the results of numerical simulation and is of the order of unity.

4. Conclusions

Structure of density and potential distribution near the flush-mounted probe is determined by the combination of anomalous diffusion and ion transverse conductivity (caused by viscosity, inertia or collisions with neutrals). For large applied probe voltages plasma density near the probe is significantly depleted. The characteristic longitudinal scale is determined by classical Spitzer conductivity and is equal to $l_\parallel = \lambda_{mfp} (m_i / m_e)^{1/2}$. The characteristic transverse scale is essentially determined by the type of transverse ion

conductivity. The electron saturation current is limited by the diffusion to the depleted channel, therefore the value of saturation current is determined by the ratio of the anomalous diffusion coefficient to the value of the effective conductivity. The calculations presented can explain experimental results for the flush-mounted probes obtained on many tokamaks, in particular the observed small ratio of electron to ion saturation current.

References

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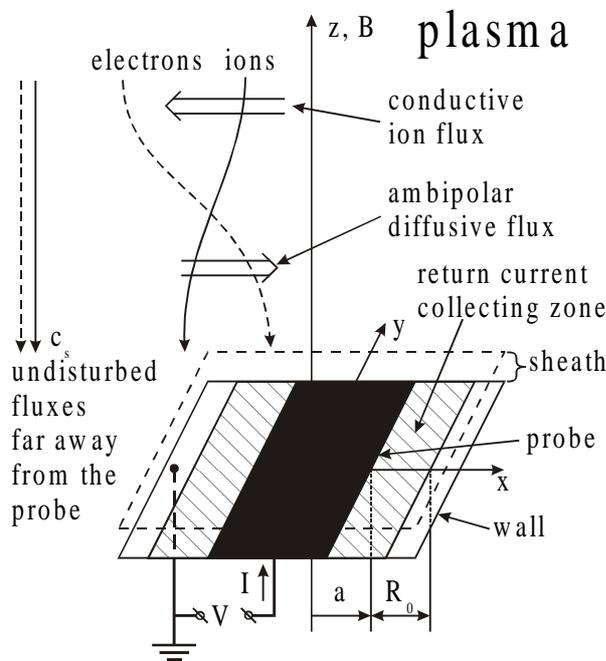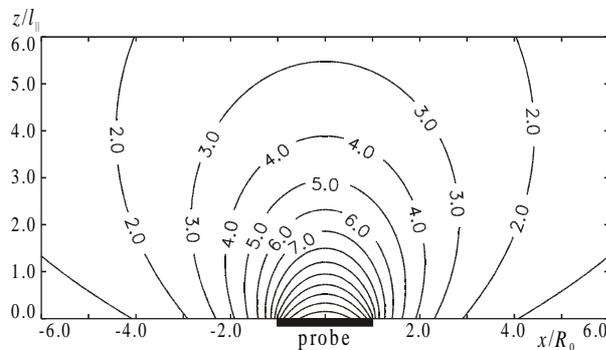
Fig.1. Geometry considered.

Fig.2. Example of the potential perturbation distribution $e\psi/T_e$ in the vicinity of a probe for $D = 5\,R_0^2 c_s/l_\parallel$, $a = R_0$, $V = 20\,T_e/e$; $T_e = T_i$.
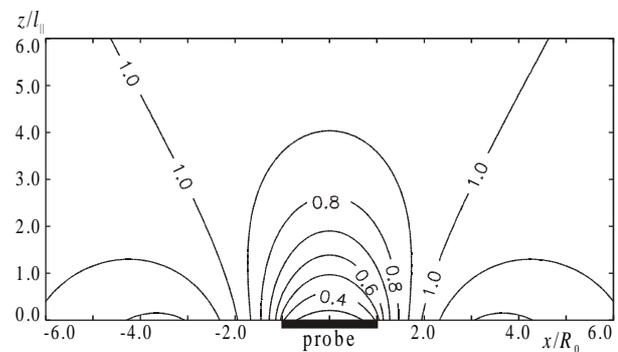

Fig.3. Example of the density distribution n/n_0 in the vicinity of a probe for $D = 5\,R_0^2 c_s/l_\parallel$, $a = R_0$, $V = 20\,T_e/e$; $T_e = T_i$.
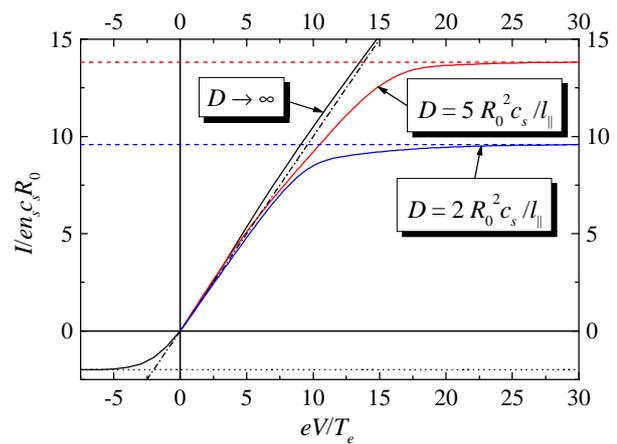

Fig.4. Calculated I-V characteristics (solid lines) for different diffusion coefficients. Broken lines – electron saturation current, dotted line – ion saturation current, dash-dotted line represents the slope derived analytically for strong diffusion and small applied voltages. Probe size $a = R_0$.