Reconstruction of the ECRH power deposition profile in T-10 Tokamak

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1. Introduction

The ECRH system of T-10 consist of 4×0.4 MW gyrotrons at 140 GHz (2-nd harmonic, X-mode). The gyrotrons have fixed mirror systems for oblique launching EC waves relative to the magnetic field. This allows us to provide the experiments to control the plasma current profile by EC driven current. Determination of the EC power deposition profile is important for the analysis of the experimental results.

In this paper we determine the ECRH power deposition profile from the time evolution of measured electron temperature by solving the inverse problem for transport equations and reconstructing the transport coefficients and the heat sources. The inverse problem is formulated for transients process with switch-on or switch-off the additional heating power. This process is considered on a small temporal interval, thus the change of integral plasma parameters can be neglected.

2. Statement of the problem

The dynamic process, which happens during the gyrotrons switch-off or switch-on, can be described as follows. The heat conductivity equation for the steady-state electron temperature $T^S(r,t)$, (with index 's') before switch-off can be written as

$$\frac{3}{2}\frac{\partial}{\partial t}\left(n^S T^S\right)=\frac{1}{r}\frac{\partial}{\partial r}\left(rK^S\frac{\partial T^S}{\partial r}\right)-\frac{1}{r}\frac{\partial}{\partial r}\left(rV^S T^S\right)+P_{OH}^S+Q^S+P_{EC},$$

$$\frac{\partial T^S}{\partial r}(r=0,t)=0,\ T^S(r=1,t)=T_0,\quad 0<r<1,\ t=t_S, \tag{1}$$

and heat conductivity equation for the electron temperature $T(r,t)$, corresponding to the transient process after the gyrotrons switch-off (without index) is

$$\frac{3}{2}\frac{\partial}{\partial t}(nT)=\frac{1}{r}\frac{\partial}{\partial r}\left(rK\frac{\partial T}{\partial r}\right)-\frac{1}{r}\frac{\partial}{\partial r}(rVT)+P_{OH}+Q,$$

$$\frac{\partial T}{\partial r}(r=0,t)=0,\ T(r=1,t)=T_0,\ T(r,t=t_S)=T^S(r),\ 0<r<1,\ t>t_S. \tag{2}$$

Here $n^S(r,t)$ and $n(r,t)$ are the electron densities, t_S is the time instant, when the gyrotron is switched-off, T_0 is the boundary temperature, $K^S=n^S\cdot\chi_e^S$ and $K=n\cdot\chi_e$ are the heat conductivities, $V^S=n^S\cdot u_e^S$ and $V=n\cdot u_e$ are the heat pinch velocities, P_{OH}^S, P_{OH} are the ohmic heating powers, Q^S, Q are another heat sinks before and after the gyrotron switch-off, P_{EC} is the additional heating power. During the transient process the electron temperature $T(r,t)$ can be presented as the sum of its steady state value $T^S(r,t)$ and an increment $\Delta T(r,t)$, i.e.

$T(r,t) = T^S(r,t) + \Delta T(r,t).$

We subtract Eq. (1) from Eq. (2) and expand the ohmic heating power P_{OH} and heat loss power Q relatively ΔT, omitting the second order terms O(ΔT^2). Thus we obtain the following equation:

$$\frac{3}{2}\frac{\partial}{\partial t}(n\Delta T) = \frac{1}{r}\frac{\partial}{\partial r}\left(rK\frac{\partial \Delta T}{\partial r}\right) - \frac{1}{r}\frac{\partial}{\partial r}(rV\Delta T) - P_{EC} + P'_{OH}(T^S)\cdot\Delta T + Q'(T^S)\cdot\Delta T$$
$$+\left\{-\frac{3}{2}\frac{\partial}{\partial t}\left[\left(n-n^S\right)T^S\right] + \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(K-K^S\right)\frac{\partial T^S}{\partial r}\right] - \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(V-V^S\right)T^S\right]\right\}. \quad (3)$$

We assume that the steady state temperature gradients are much less than the dynamic temperature gradients, therefore as a first approximation, we do not take into account the terms describing the change of transport coefficients. In the report we study shots where the ohmic heating power P_{OH} and heat loss Q are considerably less than the deposited power P_{EC}, therefore we can also omit the terms describing the change of P_{OH} and Q. As a result, we obtain the approximate linear equation for ΔT :

$$\frac{3}{2}\frac{\partial}{\partial t}(n\Delta T) = \frac{1}{r}\frac{\partial}{\partial r}\left(rK\frac{\partial \Delta T}{\partial r}\right) - \frac{1}{r}\frac{\partial}{\partial r}(rV\Delta T) - P_{EC},$$
$$\frac{\partial \Delta T}{\partial r}(r=0,t) = 0,\ \Delta T(r=1,t) = 0,\ \Delta T(r,t=t_0) = 0. \quad (4)$$

3. Numerical algorithm of the solution

Let we know the experimental values of increments of the electron temperature f_i^k, measured in N radial points: $i = 1,\ldots, N$, and in M temporal points: $k = 1,\ldots, M$, also we know the global parameters: the total current, ohmic heating power, major and minor radii. The discrepancy functional can be written as:

$$J = \frac{1}{2}\sum_{k=1}^{M}\sum_{i=1}^{N}\gamma_k\left[\Delta T(\rho_k,t_i) - f_i^k\right]^2 / \sum_{k=1}^{M}\sum_{i=1}^{N}\gamma_k\left[f_i^k\right]^2. \quad (5)$$

where γ_k are the weight factors, which are selected in accordance with the reliability of measurements in the every channel.

The inverse problem is formulated as follows. We should find the additional heating power P_{EC}, heat conductivity K and heat pinch V such as the solution $\Delta T(r,t)$ of equation (4), provides the minimum of the functional (5).

We expand the unknown functions K, V and P_{EC} over some given basis [1,2]:

$$K(r) = \sum_{j=1}^{M_K} k_j\cdot\varphi_j^K(r),\ V(r) = \sum_{j=1}^{M_V} v_j\cdot\varphi_j^V(r),\ P_{add}(r) = A\cdot\exp\left[-\left(\frac{r-r_0}{2w}\right)^{\alpha}\right], \quad (6)$$

where $\varphi_j^K, \varphi_j^V = \{1, x, x^2, x^3, \ldots\}$ are the polynomial, r_0, w are the position of the centre and e-fold width of the additional heating profile, A and α are the constants. Thus the solution of the inverse problem is reduced to the finding of the unknown parameters $P = \{k_j,\ j=1,\ldots,M_K,\ v_j,\ j=1,\ldots,M_V,\ A,\ r_0,\ w,\ \alpha\}$ from the condition of the functional (5) is minimum.

We use the method of iterational regularization [3] for the solution of the inverse problem, which is as follows: 1) we set the initial vector of parameters to be found P^S, $s = 1$, and solve the equation (4); 2) we calculate the gradient of the discrepancy functional (5), ∇J^S, and the vector of depth of descent, h^S; 3) we find the new approximation of parameters to be found from the relation $P^{S+1} = P^S + h^S \cdot \nabla J^S$. The minimum of functional (5) gives the solution of the inverse problem (4)-(6).

4. Results of numerical calculations

We use the distribution of soft X-ray intensity, $I_{SXR}(r)$, for determination of $T_e(r,t)$. On T-10 the intensity I_{SXR} was measured along 40 chords with the spatial resolution Δr=1-1.5 cm and temporal resolution Δt = 40 μs. The intensity $I_{SXR}(r)$ and temperature $T_e(r,t)$ are connected by nonlinear Abel-like integral:

$$I_{SXR} \propto \frac{1}{\sqrt{T_e}} \int_0^\infty e^{-\frac{E}{T_e}} \varphi(E) dE. \tag{7}$$

In (7) the Maxwell distribution of electrons is suggested, and $\varphi(E)$ is the energy dependence of the SXR-detector sensitivity. We reconstruct the temperature $T_e(r,t)$ from equation (7) and then use it in the calculations described before.

Figure 1 presents SXR-intensity signals for some various chords for shot #23281. Time t=800.5 ms corresponds to gyrotron switch-off. Figure 2 presents the increments of the electron temperature $\Delta T(r,t)$ at different time instants. The crosses correspond to the experimental values of the temperature, the solid lines show results of solution the inverse problem (4)-(6). The solid line in Figure 3 shows the obtained profile of deposited EC power with following parameters: P_{EC}=0.65 MW, $r_0 = 0.113$ m, $w = 0.048$ m, $\alpha = 2$. The dashed line shows the results of ray tracing calculations [4]. In Figure 4 the calculated heat diffusivity $\chi_e(r)$ and heat pinch velocity $u_e(r)$ are shown.

5. Conclusion

A new method of reconstruction of the ECRH power deposition profile in T-10, based on solution of inverse problem is proposed. It takes into account a broadening of the electron temperature profile with time due to the heat conductivity. This allows us to use the experimental data from not very small time interval and to find the power deposition profile with a high accuracy. This method allows us to find in parallel the heat diffusivity and heat pinch velocity profiles also.

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References

1. Andreev V.F., Dnestrovskij Yu.N., Popov A.M., Nucl. Fusion, v. 33 (1993) p. 499-504.
2. Andreev V.F., Dnestrovskij Yu.N., Razumova K.A, Sushkov A.V., 24-th EPS Conf. Control. Fusion and Plasma Phys. Part II, (1997), p. 937.
3. Alifanov O.M., Artjuhov E.A., Rumjantsev S.V. Extremal Method of the Solution of Ill-posed Problems, Nauka, Moscow, 1988. (in Russian)
4. Cohen R. H. Effect of trapped electrons on current drive. Phys. Fluids, 30(8) 1987, p. 2442-2449.

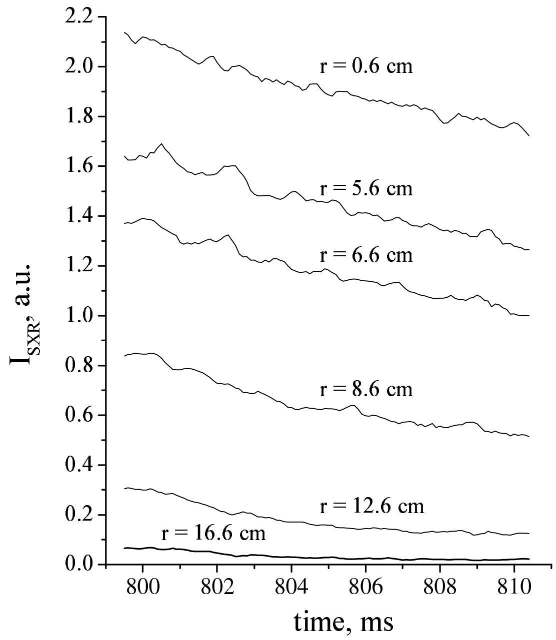

Fig.1. Time dependence of SXR intensity signals for various hords. Time t=800.5 ms correspods to gyrotron switch-off.

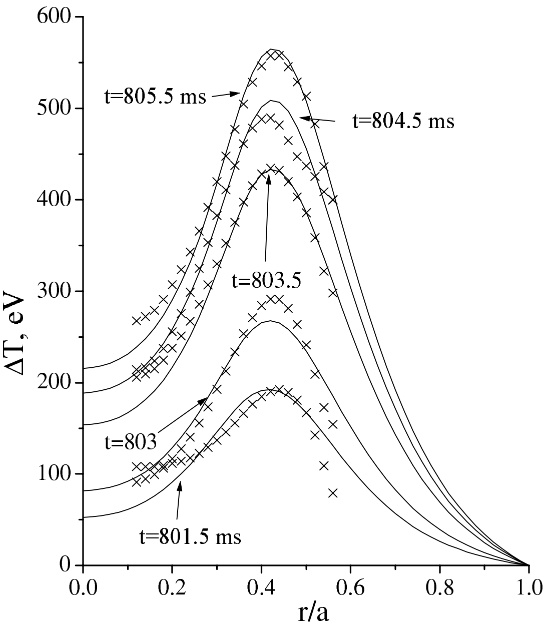

Fig.2. Increment of the temperature for some time instants. Solid lines are the solution of the inverse problem, crosses are experimental temperature.

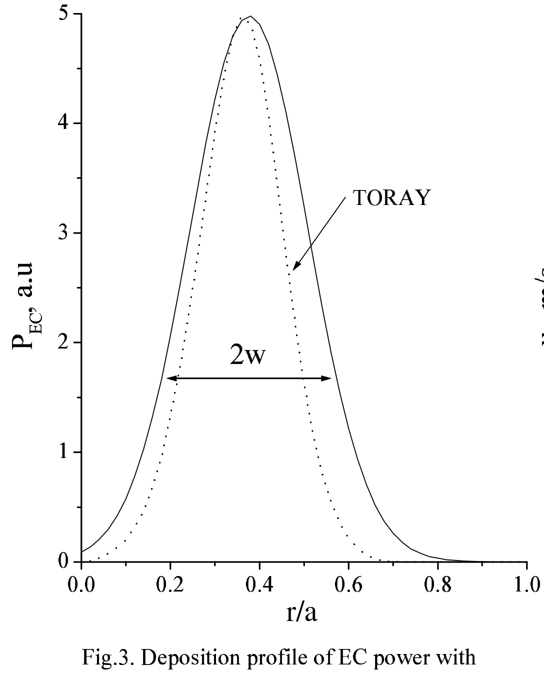

Fig.3. Deposition profile of EC power with parameters r_0=0.113 m, w=0.048 m, α=2. Dashed line shows ray tracing calculation.

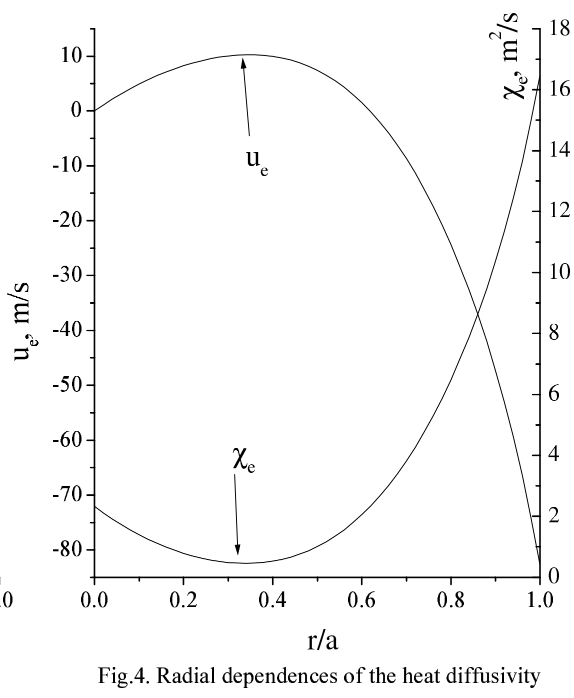

Fig.4. Radial dependences of the heat diffusivity $\chi_e(\rho)$ and heat pinch velocity $u_e(\rho)$.