

On the Steady State Distribution of Charged Fusion Products

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Introduction

This report is devoted to the description of phenomena occurring in a magnetically confined fusion plasma by employing a multiple timescale approach [1,2]. A three-component plasma including the slowing-down alpha particles is considered, where the parameter range of a typical fusion plasma is assumed. For the determination of the fast-ion distribution function the two-dimensional in the velocity space kinetic equation with a Fokker-Planck collision term is solved analytically. The present investigation concerns the fast ion distribution in velocity space only, without considering the spatial dependence.

The multiple timescale approach within kinetic theory

The starting point of the investigations is a kinetic description of a multi-species plasma by the Fokker-Planck equation

$$\frac{\partial f_j}{\partial t} + \vec{V} \cdot \vec{\nabla} f_j + q_j \left(\vec{E} + \frac{1}{c} \vec{V} \times \vec{B} \right) \cdot \frac{\partial f_j}{\partial \vec{V}} = C_j + (S_j - L_j), \quad (1)$$

to which the multiple timescale approach [1] is applied. Here C_j is the collision integral and S_j and L_j are the source and collisionless loss terms of species j , respectively. For the electrons and ions stationary conditions are assumed, it means that the phase space averaged source and loss term cancel each other. Concerning the energetic particles, the phase space averaged source term and the total loss term also have to cancel each other, thus expressing the conservation of the total number of particles.

The considered kinetic timescales are the Larmor time $\tau_{j0} = \Omega_j^{-1}$, the transit time $\tau_{j1} = \omega_j^{-1}$, the inverse collision rate $\tau_{j2} = \nu_j^{-1}$, and the transport timescale τ_{j3} , which satisfy for a typical fusion plasma for each species the ordering:

$$\tau_{j0} = \Omega_j^{-1} \ll \tau_{j1} = \omega_j^{-1} \ll \tau_{j2} = \nu_j^{-1} \ll \tau_{j3}.$$

The natural expansion parameter employed is $\delta_j = \omega_j / \Omega_j$. For timescales much shorter than the collisional ones, the interaction between the particles is taken into account in the collisionless Fokker-Planck (Vlasov) equation through the average electric and magnetic fields. The selection of a particular distribution from the infinity of possible solutions of Eq.(1) is outside the scope of the Vlasov equation. This choice would involve the history of the plasma or the known features of the underlying plasma model. For example, if the system has existed for many collision times, the Maxwellian distribution is an appropriate choice. Another example is a system prepared by injecting a monoenergetic beam of particles. In this case the distribution function may be significantly different from a Maxwellian one.

From the zero-order Fokker-Planck equation and the zero-order Maxwell's equations, one concludes that the zeroth order distribution functions are independent of t_{j0} , i.e. $\partial f_{j0}/\partial t_{j0} = 0$, and that in the zeroth-order there is no influence of the energetic particles. The solution for the first order distribution function f_{j1} has to be obtained from the first order Vlasov equation. In order to prevent a secular growth of f_{j1} , it must be required that

$$\left\langle \frac{\partial f_{j0}}{\partial t_{j1}} + \vec{V} \cdot \vec{\nabla} f_{j0} \right\rangle = 0, \quad (2)$$

where $\langle \dots \rangle$ denotes time-averaging over the zeroth order timescale t_{j0} . Thus, on the first timescale the distribution function f_{j0} is governed by the drift-kinetic equation and depends on the constants of motion. Therefore, on the Alfvén timescale the inclusion of the energetic particles has no influence on the single-fluid ideal MHD equations. In [1] it is shown, that the zero-order distribution function for electrons and ions approaches a drifted Maxwellian (provided that the confinement time is much larger than the collision time). The case for the energetic particles is different and their distribution function, f_{h0} , can be obtained from the solution of the second order equation with a Fokker-Planck collision operator [2]. On the t_{h2} -timescale, the only particle flows are due to collisions. As result one obtains for f_{h0} the following equation

$$\frac{\partial f_{h0}}{\partial t_{h2}} = \langle \langle \Lambda_h C_h(f_{h0}, f_{j0}) + S_{h0} - L_{h0} \rangle \rangle, \quad (3)$$

where $\langle \langle \dots \rangle \rangle$ denotes time-averaging over the zeroth and first order timescales t_{h0}, t_{h1} .

The solution of the kinetic equation for charged fusion products

The confined fast ions slow down and scatter in the pitch-angle as they transfer energy and momentum to the background plasma. The kinetic equation for the highly energetic particles which collide with the Maxwellian electron and ion background, may be written in the following simple form

$$\frac{\partial f_{h0}}{\partial t_{h2}} = \frac{V_0^3}{\tau V^2} \left\{ \frac{\partial}{\partial V} \left(\frac{a(V)}{2V} \frac{\partial f_{h0}}{\partial V} + b(V) f_{h0} \right) + \frac{c(V)}{V_0} \frac{\partial}{\partial \xi} (1 - \xi^2) \frac{\partial f_{h0}}{\partial \xi} \right\} + S_{h0} - L_{h0} = 0, \quad (4)$$

together with the boundary conditions

$$f_{h0}(V=0) \neq 0, \quad f_{h0}(V \rightarrow \infty) = 0, \quad \frac{\partial f_{h0}}{\partial \xi} (\xi \rightarrow \pm 1) = 0. \quad (5)$$

Here $\xi = V_{\parallel}/V = \cos \vartheta$ is the cosine of the velocity-space pitch-angle variable, $a(V)$, $b(V)$ and $c(V)$ describe the slowing down, parallel diffusion and pitch-angle scattering of the energetic particles in the velocity space:

$$\begin{aligned} a(V) &= \frac{m_h}{E_h} \left(\frac{a_e T_e}{m_e} + \frac{a_i T_i}{m_i} \right) & b(V) &= m_h \left(\frac{b_e}{m_e} + \frac{b_i}{m_i} \right) & a_j(V) &= b_j(V) = \frac{4}{\sqrt{\pi}} \int_0^{V/V_{Tj}} s^2 \exp(-s^2) ds, \\ c(V) &= V_0 \left(\frac{c_e}{V_{Te}} + \frac{c_i}{V_{Ti}} \right) & c_j(V) &= \frac{1}{\sqrt{\pi}} \left[\frac{V_{Tj}}{2V} \exp\left(-\frac{V^2}{V_{Tj}^2}\right) + \left(1 + \frac{V_{Tj}^2}{2V^2}\right) \int_0^{V/V_{Tj}} \exp(-s^2) ds \right]. \end{aligned} \quad (6)$$

A reasonable assumption for the source term is that the alpha particles are born isotropically,

$$S_h = S_0 \delta(V - V_0), \quad (7)$$

whereas for the loss term the following assumption is made

$$L_h = (L_0 V^\gamma + L_1 V^\delta \lambda(\xi)) H(V_0 - V), \quad \lambda(\xi) = \sum_n C_n P_n(\xi), \quad (8)$$

where γ and δ are some constants, which allow the investigation of different loss spectra, $P_n(\xi)$ are the Legendre polynomials, and $H(V)$ is the Heaviside step-function. For the case of magnetically confined alphas $L_n(\xi)$ attains its maximum for ξ corresponding to the maximum loss rate determining the appropriate choice of C_n . For instance, in the case of a tokamak reactor the appropriate choice of C_n , corresponding to the spectra of a partially thermalized charged fusion product loss (see, for example, [3]), leads to the $\lambda(\xi)$ dependence shown in Fig.1 ($C_0 = 8/15$, $C_1 = 8/35$, $C_2 = -16/21$, $C_3 = -16/45$, $C_4 = 8/35$, $C_5 = 8/63$).

If one neglects both the velocity diffusion and the pitch-angle scattering, then the steady-state solution for the energetic particle zeroth-order distribution function is given by

$$f_{h0} = \frac{\tau_s S_0}{V_0} \frac{V_c^3}{W^3} \left(\frac{V}{V_0} \right)^{\gamma+3} H(V_0 - V), \quad W^3 = V^3 + V_c^3, \quad (9)$$

which coincides for $\gamma=-3$ with the well known thermonuclear distribution [4]. Here V_c is the critical velocity at which the contribution of electrons and ions to the slowing down becomes equal, $V_c^3 = \frac{3\sqrt{\pi}}{4} \frac{m_e}{m_i} V_{Te}^3$. If one neglects only the pitch-angle scattering, then the steady-state solution for the energetic particle zeroth-order distribution is written [2]

$$f_{h0} = f_0 \exp\left(-\frac{m_h V^2}{2T_j}\right) + \frac{\tau_s S_0}{V_0} \frac{V_c^3}{W^3} \left(\frac{V}{V_0} \right)^{\gamma+3} H(V_0 - V), \quad (10)$$

where f_0 determines the fraction of thermalized alpha-particles.

If one takes into account the slowing down, parallel diffusion and pitch-angle scattering, then the explicit derivation of the distribution function becomes rather complex [5]. Therefore one considers the low and high energy ranges separately. In the low energy case, $V < V_*$, one can neglect the pitch-angle scattering, whereas in the high energy case, $V > V_*$, the parallel diffusion process can be omitted (with V_* being in the order of a few ion thermal velocities, $V_{Ti} < V_* \ll V_c$). By expanding the distribution function in Legendre polynomials, which constitute the eigenfunctions of the pitch angle scattering operator, one arrives under the requirement of particle and energy conservation at the solution

$$f_{h0} = f_0 \exp\left(-\frac{m_h V^2}{2T_j}\right) + \frac{\tau_s S_0}{V_0} \frac{V_c^3}{W^3} \left[1 - \frac{\beta}{\gamma+3} \left[1 - \left(\frac{V}{V_0} \right)^{\gamma+3} \right] - \frac{\kappa\beta}{\delta+3} \sum_{n=0+5} C_n P_n(\xi) \left(\frac{V^3}{W^3} \right)^{n(n+1)/6} \left\{ F\left(a, b, c; \frac{V_c^3}{V_0^3}\right) - \left(\frac{W^3}{V_0^3} \right)^{(\delta+3)/3} F\left(a, b, c; \frac{V_c^3}{W^3}\right) \right\} \right] H(V_0 - V) \quad (11)$$

where $F(a, b, c; x)$ is the hypergeometric function, and

$$f_0 = \frac{3\tau_s S_0}{V_0}, \beta = \left(\frac{1}{\gamma+3} + \frac{8}{15} \frac{\kappa}{\delta+3} \right)^{-1}, \kappa = \frac{L_1}{L_0} V_0^{\gamma-\delta}, a = \frac{n(n+1) - 2\delta}{6}, b = -\frac{\delta+3}{3}, c = \frac{\delta}{3}.$$

For example, Fig.2 shows the velocity dependence of the charged fusion product distribution for different loss spectra for the case where pitch-angle scattering is neglected.

Conclusions

The Fokker-Planck equations and the corresponding transport equations have been obtained for all three species. The solution of the first-order equations leads under the assumption of a weakly collisional plasma to the ideal MHD equations, where no influence of the high-energy particles is found. For the case of a magnetically confined plasma the steady state distribution function for charged fusion products is obtained for arbitrary loss spectra, which also take into account the anisotropy in velocity space.

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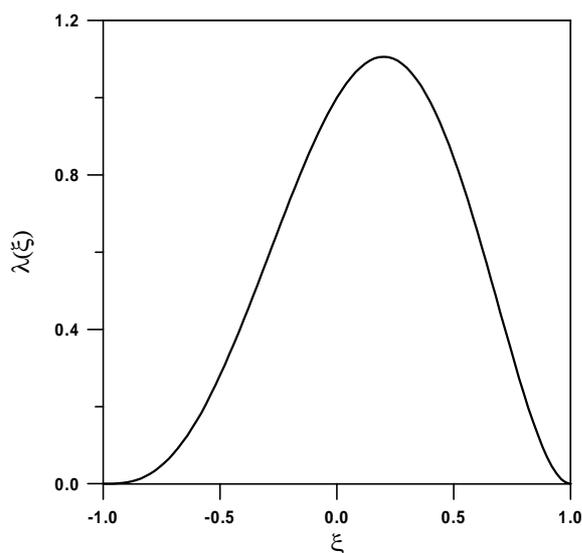


Fig.1. Typical pitch-angle dependence of the loss spectra of charged fusion products.

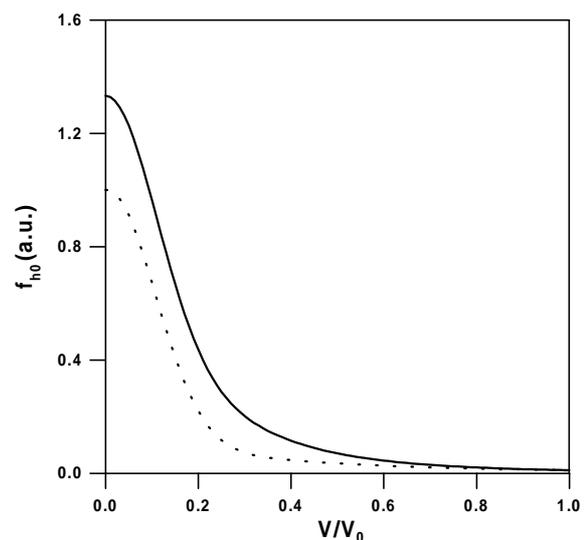


Fig.2. The charged fusion product distribution for different loss spectra (solid line - $\gamma=-3$, dashed line - $\gamma=-2$; $\kappa=0$).