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DIOCOTRON INSTABILITY OF TRAVELLING ELECTRONS IN MAGNETIC GAPS OF MAGNETO-ELECTROSTATIC TRAPS

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The non-compensated volume charge of electrons in magnetic gaps of magneto-electrostatic traps (MET) leads to the development of the diocotron instability (DI) observed in single-slit [1] and multi-slit toroidal ATOLL [2] traps. The DI theory for the oscillations propagating strictly across the magnetic field was constructed for the application to the development of electronic devices [3]. The non-compensated electron layer in the MET gaps consists of travelling and trapped electrons [4]. At the early stage of plasma accumulation the density of travelling electrons exceeds that of the trapped ones considerably. Travelling electrons are formed by two interpenetrating electron flows. One of the flows builds up when the electrons flow along the magnetic lines of force under the action of the electric field between the negatively charged plasma and the cathode. Another one is due to the reflection of this flow by the electric field near the cathode.

The report considers the stability of inter-penetrating electron beams in magnetic gaps of MET. As the radius of the device R exceeds the width x_a of the magnetic gap considerably ($R>>x_a$) [1, 2, 4], one may regard the electron layer to be planar. Two planar inter-penetrating beams of electrons with the densities $n/2$ each form the electron layer located symmetrically between conducting walls and possessing sharp boundaries.

We consider low frequency ($\omega << \omega_C$, $\omega_C = |e|B/mc$) curl-free oscillations of the planar electron layer with the non-compensated volume charge consisting of travelling electrons corresponding to the case of small density beams in the strong magnetic field when the parameters $q = \omega_p/\omega_C$ is small ($q<<1$, $\omega_p = \left(4\pi e^2 n/m\right)^{1/2}$).

We start from the linearized equations of motion and the continuity equation for electron flows and the Poisson's equation for the electrostatic potential. Here we deal with low frequency oscillations, when all perturbed quantities are proportional to $\propto \exp\left[i\left(k_y y + k_z z - \omega t\right)\right]$, ($\left|\omega - k_y v_D(x) \pm k_z V\right| << \omega_C$, k_y, k_z are the projections of the wave vector on the directions of the drift and the external magnetic field, respectively, $v_D(x) = q^2\omega_C(x - x_0) + cU/x_a B$, $x_0 = (x_2 - x_1)/2$, $x_{1,2}$ are the limits of the electron layer, U is the potential difference between metallic walls with the coordinates x_0 and x_a. We have obtained the following differential equation for the y-projection of the electric field:

$$\frac{d^2 E_y}{dx^2} - \left(k_y^2 + k_z^2\right)E_y = -\frac{4\pi ec}{B} E_y \left[f\left(\langle n^+\rangle, \langle V^+\rangle\right) + f\left(\langle n^-\rangle, \langle V^-\rangle\right)\right] \quad (1),$$

where $$f(\langle n\rangle,\langle V\rangle)=\frac{k_y\dfrac{d\langle n\rangle}{dx}}{\Omega}+\frac{k_z\langle n\rangle\left(k_y\dfrac{d\langle V\rangle}{dx}+k_z\omega_C\right)}{\Omega^2},$$

$$\langle n^{\pm}\rangle=\frac{n}{2},\ \langle V^{\pm}\rangle=\pm V,\ \Omega^{\pm}=\omega-k_y v_D(x)\pm k_z V.$$

We limit ourselves to considering the perturbations stretched along the magnetic field $\kappa=k_z/k_y<<1$. (This is the region of the DI maximum.) In this case one can neglect the perturbation of the space charge of the diocotron wave inside the beam under the condition that

$$\frac{\kappa^2}{2q^2}\left|\frac{1}{(\tilde{\omega}-\tilde{\eta}-k\tilde{x})^2}+\frac{1}{(\tilde{\omega}+\tilde{\eta}-k\tilde{x})^2}\right|<<1.$$

$$\tilde{\omega}=\omega/\omega_p q,\ \tilde{\eta}=k_z V/\omega_p q,\ k=k_y\Delta,\ \Delta=(x_2-x_1)/2,\ \tilde{x}=(x-x_0)/2.$$

For a sharp boundary dividing vacuum and the electron layer we obtain from equation (1) the following boundary conditions

$$(E)_{ext}=-i\left(\frac{dE_y}{dx}\right)_{\text{int}}-\frac{i}{2}\times \tag{2}$$

$$\times\left[\frac{1}{(\tilde{\omega}-\tilde{\eta}-k\tilde{x})}+\frac{1}{(\tilde{\omega}+\tilde{\eta}-k\tilde{x})}+\eta\left(\frac{1}{(\tilde{\omega}-\tilde{\eta}-k\tilde{x})^2}+\frac{1}{(\tilde{\omega}+\tilde{\eta}-k\tilde{x})^2}\right)\right]E_{y\text{int}}.$$

In what follows we will omit the sign "~" over reduced quantities for brevity. Relating the solutions of equation (1) inside the electron layer with the solutions within the vacuum regions with the help of boundary conditions (2), we obtain the dispersion relation for diocotron oscillations of the electron layer located symmetrically between conducting walls including the effects due to the shear of the longitudinal velocity.

$$C_0\omega^8+C_1\omega^6+C_2\omega^4+C_3\omega^2+C_4=0, \tag{3}$$

where

$$C_0=A_0=sh[2k(\delta+1)],$$
$$C_1=A_2-2(k^2+\eta^2)A_0,$$
$$C_2=A_4-2(k^2+\eta^2)A_2+(k^2-\eta^2)A_0+2\eta^2B_0,$$
$$C_3=-2(k^2+\eta^2)A_4+(k^2-\eta^2)A_2+2\eta^2B_2,$$
$$C_4=(k^2-\eta^2)A_4+2\eta^2B_4,$$
$$A_2=-2sh[2k(\delta+1)](k^2+\eta^2)+2kshk\delta sh[k(\delta+2)]-sh^2k\delta sh2k,$$
$$A_4=sh[2k(\delta+1)](k^2-\eta^2)^2+2k(\eta^2-k^2)shk\delta sh[k(\delta+2)]+k^2sh^2k\delta sh2k,$$

$$B_0=2shk\delta[3ksh[k(\delta+2)]-shk\delta sh2k],$$
$$B_2=-4k(k^2+\eta^2)sh[k(\delta+2)]shk\delta,$$

$$B_4 = -2kshk\delta\left[\left(k^2 - \eta^2\right)^2 sh[k(\delta+2)] - k^3 shk\delta sh2k\right].$$

δ is the ratio of the vacuum gap width to the halfwidth of the electron layer.

For the perturbations propagating strictly across the magnetic field $k_z = 0$ and the parameter η=0. In this case our dispersion equation coincides with the one obtained in [3] and possesses two solutions. The instability for long wavelengths ($k<<1$) appears when the width of the vacuum gap exceeds the halfwidth of the electron layer ($\delta>1$). For perturbations stretched along the magnetic field $\kappa<<1$, $\eta\neq0$ the analysis of the dispersion equation was performed numerically.

In the presence of the shear eight branches of oscillations are found. In contrast to the oscillations propagating strictly across the magnetic field, the DI growth rate maximum increases and the instability region broadens.

In a MET-based fusion reactor (the length of the magnetic gap is $L \geq 10\,cm$, the magnetic field induction is $B \approx 5T$, the electron velocity along the magnetic field within the gap is $V \sim 10^{10}\,cm/s$, the concentration of travelling electrons is $n \approx 5\cdot10^{12}cm^{-3}$) the DI is the most dangerous because the time interval during which the perturbations are taken out of the gap exceeds the period of the DI development ($t >> \tau$, $t \sim L/V \sim 10^{-9}s$, $\tau \sim \gamma^{-1} \sim \omega_c/0.3\omega_p^2 \sim 3\cdot10^{-9}s$). On this ground one can draw a conclusion that in this region where the velocities of the interpenetrating beams are equal, the DI would have time to develop already at the initial injection stage and might affect the process of plasma accumulation in MET.

References

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