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On the Scaling Description of Electromagnetic Fluctuations in Strong Magnetized Plasmas

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1.Introduction. Theoretical studies of electromagnetic plasma fluctuations are of great importance for the development of efficient methods of plasma diagnostics in laboratory fusion research devices as well as in the near and outer space. Electromagnetic fluctuations in nonequilibrium plasmas are determined by the spectral distribution of the Langevin current along with the dielectric permittivity tensor. The spectral distributions of current density fluctuations in the plasma disregarding the Coulomb interaction of charged particles is usually taken for the spectral distribution of the Langevin current:

$$\langle j_i j_j \rangle^0_{\vec{k}\omega} = e^2 \int d\vec{v} \int d\vec{v}' v_i v_j W_{\vec{k}\omega}(\vec{v},\vec{v}') f_0(\vec{v}), \tag{1}$$

where $W_{\vec{k}\omega}(\vec{v},\vec{v}')$ is the Fourier transformation of the probability density in the phase space for the particle transition from the point $\vec{v}$ to the point $\vec{v}'$ for the time t. If the charged particle distribution is axially symmetric with respect to the external magnetic field, the spectral distribution of the Langevin current is as follows:

$$\langle j_i j_j \rangle^{0\alpha}_{\vec{k}\omega} = 2\pi \sum_\alpha e_\alpha^2 \int d\vec{v} \sum_n \prod_{ij}^{(n\alpha)} (v_\perp, v_\parallel) \delta(\omega - k_\parallel v_\parallel - n\omega_{B_\alpha}) f_{0\alpha}(v_\perp, v_\parallel), \tag{2}$$

where the tensor $\prod_{ij}^{(n)} (v_\perp, v_\parallel)$ is defined in [1]. The dielectric permittivity tensor for a nonequilibrium plasma may be found on the basis of inverting the fluctuation-dissipation relationship:

$$\varepsilon_{ij}(\omega,\vec{k}) = \delta_{ij} + 4\pi \sum_\alpha \kappa_{ij}^\alpha(\omega,\vec{k}),\ 4\pi\kappa_{ij}^\alpha(\omega,\vec{k}) = -\sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2} \left\{ \delta_{ij} + \frac{2}{\omega_{p\alpha}^2} \int d\omega' \frac{\omega' \frac{\partial}{\partial E_\alpha} \langle j_i j_j \rangle^{0\alpha}_{\vec{k}\omega'}}{\omega' - \omega - i0} \right\}, \tag{3}$$

where $\frac{\partial}{\partial E_\alpha} \langle j_i j_j \rangle^{0\alpha}_{\vec{k}\omega}$ is the correlation function averaged over the derivative of the energy distribution.

2.Low-frequency fluctuations in strong magnetized plasmas. For a plasma with strong external magnetic field $\vec{B}_0$, we introduce a small dimensionless parameter $q^2 = \frac{k^2 s^2}{\omega_{B_i}^2} << 1$, where $s = \sqrt{\frac{3T}{m}}$ is the electron thermal velocity and $\omega_{B_i} = \frac{eB_0}{Mc}$ is the ion cyclotron frequency. We restrict the consideration to the spectral range of frequencies ω lower than the electron cyclotron frequency $\omega_{B_e} = \frac{eB_0}{mc}$ $(\omega << \omega_{B_e})$ and employ approximate expressions for the plasma dielectric permittivity components, obtained under the assumption that $\frac{m}{M} << 1$ and expended in power series of the small parameter q^2. Making use of the fluctuation-dissipation theorem, low-frequency fluctuations of charge density, electric and magnetic fields in equilibrium strong magnetized plasmas are considered [2].

Similarly to the spectral distribution of electric field fluctuations, the spectral distribution of magnetic field fluctuations has two maxima given rise to by Alfven and magnetosonic fluctuation oscillations and a low-frequency maximum associated with incoherent fluctuations. Like in the case of electric field fluctuations, differing polarizations of Alfven and magnetosonic waves are responsible for the fact that relevant magnetic field fluctuation maxima are manifested in different components of the spectral distribution tensor. The magnetosonic perturbation of the electric field is transverse (the polarization vector of the fast magnetosonic wave is perpendicular to the wave vector); the electric field of Alfven perturbations has a longitudinal component as well. The spectral distribution of charge density fluctuations associated with Alfven perturbations in the plasma is determined by the relation

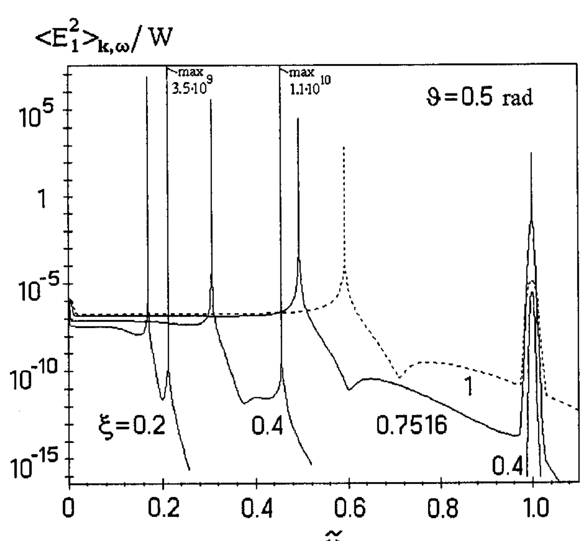
$$\left\langle \rho^2 \right\rangle^{(A)}_{\vec{k}\omega} = \frac{k^2}{16\pi^2}\left\langle E_l^2 \right\rangle^{(A)}_{\vec{k}\omega} = \frac{1}{6}\sqrt{\frac{\pi}{2}}\,\frac{e^2 n_0}{\omega_{p_i}}\,\frac{\omega^2}{\omega^2_{p_e}}\,\frac{s^2}{v_A^2}\,ka\sin\vartheta\, tg^3\vartheta\, e^{-\frac{M}{m}z^2}, \quad a^2 = \frac{T}{4\pi n_0 e^2}. \tag{4}$$

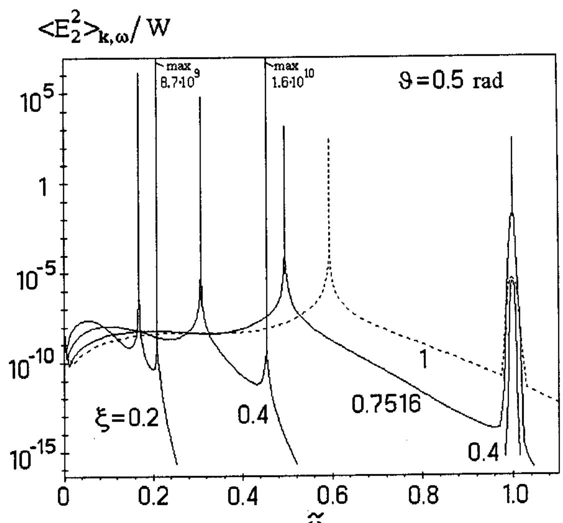
We note that intensities of both charge density and electric field fluctuations tend to zero in the limit $\omega \to 0$. As distinct from the case of electric field fluctuations, the maximum of incoherent fluctuations of the magnetic field corresponds to the zero frequency:

$$\left\langle B_2^2 \right\rangle^{(A)}_{\vec{k}\omega} = 2\pi\sqrt{\frac{\pi}{6}\frac{m}{M}}\frac{T}{ks}q^2\sin\vartheta tg\vartheta e^{-\frac{M}{m}z^2}, \quad \left\langle B_i B_j \right\rangle^{(S)}_{\vec{k}\omega} = b_i^{(S)} b_j^{(S)} 8\pi\sqrt{\frac{2\pi}{3}\frac{m}{M}}\frac{T}{kv_A}\sin\vartheta tg\vartheta e^{-\frac{M}{m}z^2}. \tag{5}$$

The widths of the distributions are determined by the ion thermal velocity. In the low-frequency domain dominant contribution to the incoherent charge density fluctuations is associated with the longitudinal electric field.

As the wave number increases, the maxima of electric field fluctuation spectra are shifted to the ion cyclotron frequency range and separation of Alfven and magnetosonic waves disappears. The components $\left\langle E_1^2 \right\rangle_{\vec{k}\omega}$ and $\left\langle E_2^2 \right\rangle_{\vec{k}\omega}$ of the spectral distribution of electric field fluctuations in frequency range $\omega \lesssim \omega_{B_i}$ are shown in Figs. as functions of the dimensionless frequency $\tilde{\omega} \equiv \frac{\omega}{\omega_{B_i}}$ for different values of the parameter $\xi \equiv \frac{kc}{\omega_{p_i}}$ ($W = 8\pi\frac{T}{\omega_{B_i}}$).Numerical calculations correspond to the actual conditions of real plasma with the concentration $n_0 = 1.2\cdot 10^{13}\,cm^{-3}$, temperature $T = 10^2$ eV, and external magnetic field $B_0 = 3.4\cdot 10^4$ Gs.





The magnitude of the maximum in the range of ion cyclotron frequency which associated with incoherent fluctuations is increasing when parameter ξ is increasing.

3.Collective fluctuations in the nonequilibrium plasmas. The spectral distributions of electric and magnetic fields fluctuations for a nonequilibrium plasma in the transmittance range reduce to the forms with the effective temperature:

$$\tilde{T}(\omega,\vec{k})=\frac{2\pi^2}{\omega}\frac{\lambda_{kl}}{\operatorname{Im}\Lambda}\langle j_k j_l\rangle^0_{\vec{k}\omega}, \tag{6}$$

where Λ is the dispersion determinant and λ_{ij} is the algebraic complement of the dispersion tensor. We note that the effective temperature $\tilde{T}(\omega,\vec{k})$ depends on the frequency and the wave vector. It is obvious that the value of the effective temperature for the frequency equal to the eigenoscillation frequency should be regarded as the temperature of the relevant eigenoscillations, $T_A \equiv \tilde{T}(kv_A\cos\vartheta,k), T_S \equiv \tilde{T}(kv_A,k)$. The condition of plasma stability with respect to collective excitations reduces to the requirement that the damping rate $\gamma_{A,S}$ of the relevant collective excitation must be greater than zero. The effective temperature of relevant excitations grows infinite at the stability boundary.

The spectral distribution of incoherent fluctuations of the magnetic field is maximum for zero frequency. The ratios of the intensities of these spectral distributions for zero frequencies to the quantities for an equilibrium plasma, may be regarded as effective temperatures of the relevant incoherent fluctuations, i.e.,

$$T_A^N=8\sqrt{6\pi}\sqrt{\frac{M}{m}}\frac{1}{q^2}\frac{\omega_{B_i}^2}{\omega_{p_i}^2}\frac{s}{v_A}\frac{\langle j_1^2\rangle^{(0)}_{\vec{k}\omega}}{kv_A}\frac{1}{\sin^2\vartheta\cos\vartheta},\quad T_S^N=\sqrt{6\pi}\sqrt{\frac{M}{m}}\frac{\omega_{B_i}^2}{\omega_{p_i}^2}\frac{\langle j_2^2\rangle^{(0)}_{\vec{k}\omega}}{ks}\frac{\cos\vartheta}{\sin^2\vartheta}, \tag{7}$$

We note that effective temperatures of incoherent fluctuations are always finite, as distinct from the temperatures of collective Alfven and magnetosonic fluctuations. Thus, it is convenient to describe the state of a nonequilibrium plasma in terms of the sets of temperatures T_A, T_S and T_A^N, T_S^N ; the diagnostics of states of a nonequilibrium plasma with strong external magnetic field may be reduced to the calculation of these temperatures.

4.Fluctuations in turbulent plasmas. We consider plasmas with developed turbulence and assume that there occur large-scale turbulent pulsations. This means that microscopic motion of noninteracting particles reduces to the motion of particles under the influence of the field averaged over a small macroscopic volume, and the stochastic motion of the latter volume [3]. We assume thermal motion of individual particles and chaotic turbulent large-scale motions to occur independently. Therefore, the transition probability density for a turbulent system is given by

$$W^T_{\vec{k}\omega}(\vec{v},\vec{v}')=\int\frac{d\omega'}{2\pi}\int d\vec{v}^T P_{\vec{k}\omega-\omega'}(\vec{v}^T)W_{\vec{k}\omega'}(\vec{v},\vec{v}'+\vec{v}^T), \tag{8}$$

where $P_{\vec{k}\omega}(\vec{v}^T)$ is the factor determined by the stochastic Brownian motion of small macroscopic volumes. If the elementary volume is involved in the diffusion-drift motion, then

$$P_{\vec{k}\omega}\equiv\int d\vec{v}^T P_{\vec{k}\omega}(\vec{v}^T)=\frac{2k^2D}{(\omega-\vec{k}\vec{u}_D)^2+k^4D^2}, \tag{9}$$

where $\vec{u}_D$ is the drift velocity and D is the diffusion coefficient. In the more pragmatic model [4] one imagines that small macroscopic plasma volumes move chaotically across the magnetic field and the characteristic function is a Gaussian

$$P_{\vec{k}\omega} = \frac{\sqrt{2\pi}}{\gamma_F} \exp\left\{ -\frac{(\omega - \omega_F)^2}{2\gamma_F^2} \right\} \tag{10}$$

where the mean (drift) velocity $\vec{u}_D$ and the root mean square velocity u, associated with fluid-like motion, determine the Doppler frequency $\omega_F = \vec{k}\vec{u}_D$ (at which the spectrum has its maximum) and the spectral width $\gamma_F = k_\perp u$, respectively. The spectral distribution of the Langevin current for a turbulent system is defined by

$$\langle j_i j_j \rangle^T_{\vec{k}\omega} = \int \frac{d\omega'}{2\pi} P_{\vec{k}\omega-\omega'} \langle j_i j_j \rangle^0_{\vec{k}\omega'} \,. \tag{11}$$

The dielectric permittivity tensor for a turbulent plasma is determined by the formula (3) in which the correlation function $\langle j_i j_j \rangle^0_{\vec{k}\omega}$ has to be changed to $\langle j_i j_j \rangle^T_{\vec{k}\omega}$. The plasma susceptibility tensor takes the form

$$\kappa_{ij}(\omega, \vec{k})^T = \int \frac{d\omega'}{2\pi} P_{\vec{k}\omega-\omega'} \frac{\omega'^2}{\omega^2} \kappa_{ij}(\omega', \vec{k}) \,. \tag{12}$$

Making use of the characteristic function (10), we find the spectral distribution for spontaneous fluctuations in the case of potential field [4]:

$$\left\langle \delta n^2 \right\rangle^T_{\vec{k}\omega} = \sqrt{\frac{2\pi}{C}} \frac{\tilde{n}_0}{\gamma_T} e^{-\frac{(\omega-\omega_F)^2}{2C\gamma_T^2}}, \quad \gamma_T = k_\parallel s_\parallel, \quad \tilde{n}_0 = n_0 e^{-\beta} I_0(\beta), \; \beta = \frac{k_\perp^2 s_\perp^2}{\omega_{B_e}^2}, \quad C = 1 + \left(\frac{k_\perp}{k} \frac{u}{s_\parallel} \right)^2 \tag{13}$$

and the dielectric permittivity for an electron plasma

$$\varepsilon(\omega, \vec{k}) = \varepsilon(k) \left\{ 1 + \frac{1}{C} \frac{1}{\tilde{a}^2 k^2} \left[1 - \varphi\left(\frac{\omega - \omega_F}{\sqrt{2C}\gamma_T} \right) + i\sqrt{\pi} \frac{\omega - \omega_F}{\sqrt{2C}\gamma_T} e^{-\frac{(\omega-\omega_F)^2}{2C\gamma_T^2}} \right] \right\}. \tag{14}$$

The shape parameter C changes in the range from unity to infinity, as it depends on the ratio between the magnitudes of the wavevector perpendicular and parallel components and the ratio between the root mean square turbulent velocity and the particle thermal velocity. The spectral distribution of electron density fluctuations for a turbulent plasma is related to result for a nonmagnetized plasma by the scaling transformations:

$$T \to CT \geq T, \; \omega \to \frac{\omega - \omega_F}{\sqrt{C}}, \; a^2 \to C\,\tilde{a}^2 . \tag{15}$$

The characteristic length scale, which separates incoherent and collective fluctuations according to $C\tilde{a}^2 k^2 \varepsilon(\vec{k}) \sim 1$ depends on the effects of fluid motion $(C > 1)$, particle polarization drift $(\varepsilon(\vec{k}) > 1)$ and finite Larmor radius. The characteristic scale length is large when these effects are important. The spectra are broader for larger C, with collective features being less pronounced. The frequency scale $\sqrt{2C}\,\gamma_T$ depends on particle thermal motion along the magnetic field and on fluid, and it decreases with $k_\parallel$.

References

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