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Turbulent Diffusion Influence on Large-Scale Fluctuations in Plasmas

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1. Introduction

In spite of considerable progress in statistical theory of plasma turbulence, consistent description of electromagnetic fluctuations in turbulent plasmas has not yet been performed. That is why phenomenological approaches to the description of turbulence influence on electromagnetic fluctuations still remain an important tool for the studies of this problem [1–4]. Within such approaches however, it is impossible to take into account explicitly the effects associated with turbulent diffusion in the velocity space. The purpose of this contribution is to describe electromagnetic fluctuations in turbulent plasmas in external magnetic fields with regard for such diffusion.

2. Basic Set of Equations

We consider plasma with saturated turbulence assuming that the characteristic time of turbulent field changes τ_T is much shorter than the correlation times for the fluctuations under consideration. In such a case, averaging the equation for the microscopic phase density over the physically-infinitesimal time $\tau_{\rm ph} \gg \tau_T$ yields the kinetic equation

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial r} + \frac{1}{m}\left(\mathbf{F}^{\rm ext} + \widetilde{\mathbf{F}}\right)\frac{\partial}{\partial \mathbf{v}}\right\}\widetilde{\mathcal{F}}(X,t) = \left(\frac{\partial \widetilde{\mathcal{F}}}{\partial t}\right)_T \tag{1}$$

with the collision term $(\partial\widetilde{\mathcal{F}}/\partial t)_T)$ describing the influence of turbulent fields on the evolution of the smoothened microscopic phase density $\widetilde{\mathcal{F}}(X,t)$. Here, $X \equiv (\mathbf{r},\mathbf{v})$, $\widetilde{\mathbf{F}}^{\rm ext}$ and $\widetilde{\mathbf{F}}$ are the forces associated with external and intrinsic plasma fields (the latter is averaged over turbulent perturbations), the rest of notation is traditional.

The collision term can be represented in the Fokker-Planck form with the kinetic coefficients determined by the turbulent field correlation functions. In the general case such kinetic coefficients should be calculated selfconsistently with regard for their temporary evolution [5]. In the case under consideration the kinetic coefficients are assumed to be known. In what follows we also suggest that the turbulence is isotropic in the directions perpendicular to the external magnetic field and that the velocity-dependence of the diffusion coefficients may be neglected. Within such model Eq. (1) has the form

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + [\mathbf{v}\times\mathbf{\Omega}]\frac{\partial}{\partial \mathbf{v}} + \frac{e}{m}\widetilde{\mathbf{E}}\frac{\partial}{\partial \mathbf{v}}\right\}\widetilde{\mathcal{F}}(X,t) = \tag{2}$$

$$= \frac{\partial}{\partial v_\parallel}\left\{\nu_\parallel v_\parallel \widetilde{\mathcal{F}}(X,t) + D_\parallel \frac{\partial \widetilde{\mathcal{F}}(X,t)}{\partial v_\parallel}\right\} + \frac{\partial}{\partial \mathbf{v}_\perp}\left\{\nu_\perp \mathbf{v}_\perp \widetilde{\mathcal{F}}(X,t) + D_\perp \frac{\partial \widetilde{\mathcal{F}}(X,t)}{\partial \mathbf{v}_\perp}\right\}.$$

Here, $\mathbf{v} \equiv (\mathbf{v}_\perp, v_\parallel)$, $\mathbf{\Omega} = e\mathbf{B}_0/mc \equiv \mathbf{e}_z\Omega$, $\nu_\parallel$ and $\nu_\perp$ are the friction coefficients for motions in the directions parallel or perpendicular to the external magnetic field $\mathbf{B}_0 \equiv \mathbf{e}_z B_0$, $D_\parallel$ and $D_\perp$ are the relevant diffusion coefficients.

Eq. (2) generates the following linearized equation for fluctuation evolution

$$\hat{L}_0 \delta f(X,t) = -\frac{e}{m}\delta\mathbf{E}(\mathbf{r},t)\frac{\partial f_0(\mathbf{v})}{\partial \mathbf{v}} \tag{3}$$

where

$$\hat{L}_0 = \frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + [\mathbf{v}\times\mathbf{\Omega}]\frac{\partial}{\partial \mathbf{v}} + \left(\beta_\parallel + \beta_\parallel v_\parallel \frac{\partial}{\partial v_\parallel} + D_\parallel \frac{\partial^2}{\partial v_\parallel^2}\right)$$
$$- \left(2\beta_\perp + \beta_\perp \mathbf{v}_\perp \frac{\partial}{\partial \mathbf{v}_\perp} + D_\perp \frac{\partial^2}{\partial \mathbf{v}_\perp^2}\right);$$
$$\delta f(X,t) = \widetilde{\mathcal{F}}(X,t) - f_0(\mathbf{v});$$

$f_0(\mathbf{v}) = \langle \widetilde{\mathcal{F}}(X,t)\rangle$ is the unperturbed distribution function, $\delta\mathbf{E}$ is the electric field fluctuation satisfying the Poisson equation

$$\mathrm{div}\delta\mathbf{E} = 4\pi \sum en \int d\mathbf{v}\delta f(X,t). \tag{4}$$

3. Transition probability of a particle in a magnetoactive plasma with regard for the dynamical friction and diffusion in the velocity space

The formal solution of Eq. (3) is given by

$$\delta f(X,t) = \delta f^{(0)}(X,t) - \frac{e}{m}\int_{-\infty}^{t} dt' \int dX' W(X,X',t-t')\delta\mathbf{E}(\mathbf{r}',t')\frac{\partial f_0(\mathbf{v}')}{\partial \mathbf{v}'} \tag{5}$$

where $\delta f^{(0)}(X,t)$ is the fluctuation in the system with no selfconsistent electric interaction, $W(X,X',t-t')$ is the transition probability in such system. Obviously, these quantities satisfy the equation

$$\hat{L}_0 \delta f^{(0)}(X,t) = 0 \tag{6}$$

and

$$\hat{L}_0 W(X,X',t-t') = 0. \tag{7}$$

Eq. (7) should be supplemented with the initial condition $W(X,X',0) = \delta(X-X')$.

The solution of the initial-value problem for the transition probability is

$$W(X,X',\tau) = \frac{1}{8\pi^3 \Delta_\perp \Delta_\parallel^{1/2}} \exp\left[-\frac{1}{2\Delta_\parallel}(a_\parallel \rho_\parallel^2 + b_\parallel P_\parallel^2 + 2h_\parallel \rho_\parallel P_\parallel)\right] \times$$
$$\times \exp\left[-\frac{1}{2\Delta_\perp}(a_\perp \boldsymbol{\rho}_\perp^2 + b_\perp \mathbf{P}_\perp^2 + 2h_\perp \boldsymbol{\rho}_\perp \mathbf{P}_\perp - 2q_\perp \mathbf{e}_{\mathbf{\Omega}}[\boldsymbol{\rho}_\perp \mathbf{P}_\perp])\right]. \tag{8}$$

Here $\mathbf{e}_{\mathbf{\Omega}} = \mathbf{B}_0/B_0 = \mathbf{e}_z$,

$$\rho_x = e^{-\nu_\perp \tau}(v_x \cos\Omega\tau - v_y \sin\Omega\tau) - v_x'$$
$$\rho_y = e^{-\nu_\perp \tau}(v_x \sin\Omega\tau + v_y \cos\Omega\tau) - v_y'$$
$$\rho_z = e^{-\nu_\parallel \tau} v_z - v_z'$$

$$
\begin{aligned}
P_x &= x - x' + \frac{\nu_\perp (v_x - v'_x) + \Omega (v_y - v'_y)}{\Omega^2 + \nu_\perp^2} \\
P_y &= y - y' + \frac{\nu_\perp (v_y - v'_y) - \Omega (v_x - v'_x)}{\Omega^2 + \nu_\perp^2} \\
P_z &= z - z' + \frac{v_z - v'_z}{\nu_\parallel} \\
\Delta_\perp &= a_\perp b_\perp - h_\perp^2 - q_\perp^2; \quad \Delta_\parallel = a_\parallel b_\parallel - h_\parallel^2 \\
a_\perp &= \frac{2D_\perp}{\Omega^2 + \nu_\perp^2}\tau; \quad a_\parallel = \frac{2D_\parallel}{\nu_\parallel^2}\tau; \quad b_\perp = \frac{2D_\perp}{\Omega^2 + \nu_\perp^2}(e^{2\nu_\perp \tau} - 1) \\
b_\parallel &= \frac{2D_\parallel}{\nu_\parallel^2}(e^{2\nu_\parallel \tau} - 1); \quad h_\perp = \frac{2D_\perp}{\Omega^2 + \nu_\perp^2}(1 - e^{\nu_\perp \tau}\cos\Omega\tau); \\
h_\parallel &= \frac{2D_\parallel}{\nu_\parallel^2}(1 - e^{\nu_\parallel \tau}); \quad q_\perp = -\frac{2D}{\Omega^2 + \nu_\perp^2} e^{\nu_\perp \tau}\sin\Omega\tau.
\end{aligned}
\tag{9}
$$

For $\Omega = 0$, Eqs. (8), (9) recover the fundamental solution of the generalized Liouville equation obtained by Chandrasekhar [6].

4. Dielectric response functions and correlation functions of the Langevin sources

As follows from Eqs. (4), (5), the Fourier component of the electric field fluctuation potential is given by

$$
\phi\Phi_{\mathbf{k}\omega} = \frac{4\pi \sum e\delta n^{(0)}_{\mathbf{k}\omega}}{k^2 \varepsilon(\mathbf{k},\omega)} \tag{10}
$$

where

$$
\begin{aligned}
\varepsilon(\mathbf{k},\omega) &= 1 + \sum \chi(\mathbf{k},\omega), \qquad \chi(\mathbf{k},\omega) = -i\frac{4\pi e^2 n}{mk^2}\int d\mathbf{v}\int d\mathbf{v}' W_{\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')\mathbf{k}\frac{\partial f_0(\mathbf{v}')}{\partial \mathbf{v}'} \\
W_{\mathbf{k}\omega}(\mathbf{v},\mathbf{v}') &= \int_0^\infty d\tau e^{i\omega\tau}\int d\mathbf{R} e^{-i\mathbf{k}\mathbf{R}} W(X,X',\tau), \qquad \mathbf{R} = \mathbf{r} - \mathbf{r}'; \\
\delta n^{(0)}_{\mathbf{k}\omega} &= n\int d\mathbf{v}\,\delta f^{(0)}_{\mathbf{k}\omega}(\mathbf{v}).
\end{aligned}
\tag{11}
$$

The correlation function of the Langevin sources is also expressed in terms of the transition probability, namely,

$$
\langle \delta n^2 \rangle^{(0)}_{\mathbf{k}\omega} = n\int d\mathbf{v}\int d\mathbf{v}' W_{\mathbf{k}\omega}(\mathbf{v},\mathbf{v}') f_0(\mathbf{v}') + \text{c.c} \tag{12}
$$

Using the explicit form of the transition probability (8) and Eq. (11) one has

$$
\begin{aligned}
&\chi(\mathbf{k},\omega) = \\
&- i\frac{4\pi e^2 n}{mk^2}\int_0^\infty d\tau\int d\mathbf{v}\sum_{h=-\infty}^{\infty} J_n\left(\frac{k_\perp v_\perp}{\tilde{\Omega}}\right) J_n\left(\frac{k_\perp v_\perp}{\tilde{\Omega}} e^{-\nu_\perp \tau}\right)\left\{\frac{n\Omega}{v_\perp}\frac{\partial f_0(\mathbf{v})}{\partial v_\perp} + k_\parallel \frac{\partial f_0(v)}{\partial v_\parallel}\right\} \times \\
&\times \exp\left\{-\left(\frac{k_\perp^2 D_\perp}{\tilde{\Omega}^2}\Phi_\perp(\tau) + \frac{k_\parallel^2 D_\parallel}{\nu_\parallel^2}\Phi_\parallel\right) + i\left[(\omega - n\Omega)\tau - \frac{k_\parallel v_\parallel}{\nu_\parallel}(1 - e^{-\nu_\parallel \tau})\right]\right\}.
\end{aligned}
\tag{13}
$$

Here

$$\Phi_\perp(\tau) = \tau + \frac{1}{2\nu_\perp}(1 - e^{-2\nu_\perp\tau}) - \frac{2}{\widetilde{\Omega}^2}\left[\nu_\perp - e^{-\nu_\perp\tau}(\nu_\perp \cos\Omega\tau - \Omega\sin\Omega\tau)\right];$$

$$\Phi_\parallel(\tau) = \tau + \frac{1}{2\nu_\parallel}\left(1 - e^{-2\nu_\parallel\tau}\right) - \frac{2}{\nu_\parallel}\left(1 - e^{-\nu_\parallel\tau}\right); \quad \widetilde{\Omega} = (\Omega^2 + \nu_\perp^2)^{1/2}. \tag{14}$$

Eq. (13) makes it possible to recover various particular cases. For instance, for $D = 0$, $\nu = 0$ one has the well-known result for the Vlasov plasma. For $\nu_\perp \sim \nu_\parallel \sim 0$ (minor friction) and $D_\parallel = 0$ (no diffusion along the magnetic field)

$$\chi(\mathbf{k},\omega) = \frac{4\pi e^2 n}{mk^2}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}(i)^m\int d\mathbf{v}J_n^2\left(\frac{k_\perp v_\perp}{\Omega}\right)\times \tag{15}$$

$$\times\ I_m\left(\frac{\tilde{\nu}}{\Omega}\right)\frac{\frac{n\Omega}{v_\perp}\frac{\partial f_0}{\partial \mathbf{v}_\perp} + k_\parallel\frac{\partial f_0}{\partial v_\parallel}}{\omega - k_\parallel v_\parallel - (n+m)\Omega + i\widetilde{\nu}_0}, \widetilde{\nu}_0 = \frac{k_\perp^2 D_\perp}{\Omega^2}$$

In the case $\nu_\parallel = 0$, $D_\parallel = 0$ and $\beta_\perp \gg 1$

$$\chi(\mathbf{k},\omega) = \frac{4\pi e^2 n}{mk^2}\int d\mathbf{v}\frac{J_0\left(\frac{k_\perp v_\perp}{\widetilde{\Omega}}\right)k_\parallel\frac{\partial f_0}{\partial v_\parallel}}{\omega - k_\parallel v_\parallel + i\widetilde{\nu}}, \widetilde{\nu} = \frac{k_\perp^2 D_\perp}{\widetilde{\Omega}^2}. \tag{16}$$

Similar relations can be derived also for the source correlation functions. In the general case we have

$$\langle\delta n^2\rangle_{\mathbf{k}\omega}^{(0)} = n\int_0^\infty d\tau\int d\mathbf{v}\sum_n J_n\left(\frac{k_\perp v_\perp}{\widetilde{\Omega}}\right)J_n\left(\frac{k_\perp v_\perp}{\widetilde{\Omega}}e^{-\nu_\perp\tau}\right)\times \tag{17}$$

$$\times\ \exp\left\{-\left(\frac{k_\perp^2 D_\perp}{\widetilde{\Omega^2}}\Phi_\perp(\tau) + \frac{k_\parallel^2 D_\parallel}{\nu_\parallel^2}\Phi_\parallel(\tau)\right) + i\left[(\omega - n\Omega)\tau - \frac{k_\parallel v_\parallel}{\nu_\parallel}\left(1 - e^{-\nu_\parallel\tau}\right)\right]\right\}.$$

5. Fluctuation spectra

Relations obtained above lead to the results for the fluctuation spectra very similar to that for ordinary plasmas. For example,

$$\langle\delta n_e^2\rangle_{\mathbf{k}\omega} = |\frac{1 + \chi_i(\mathbf{k},\omega)}{\varepsilon(\mathbf{k},\omega)}|^2\langle\delta n_e^2\rangle_{\mathbf{k}\omega}^{(0)} + Z_i^2|\frac{\chi_e(\mathbf{k},\omega)}{\varepsilon(\mathbf{k},\omega)}|^2\langle\delta n_i^2\rangle_{\mathbf{k}\omega}^{(0)}. \tag{18}$$

However, in the case under consideration the dielectric response functions (13) and the correlation function of the Langevin sources (17) should be substituted into Eq. (18). These relations generalize the result of phenomenological approaches [1–4] which can be reproduced in the relevant particular cases.

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