FUNDAMENTAL ION CYCLOTRON ABSORPTION OF FAST MAGNETOSONIC WAVE PROPAGATING ACROSS NONUNIFORM MAGNETIC FIELD

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Introduction. Taking into account the nonuniformity of a straight magnetic field in the direction perpendicular to the lines of force at the Larmor orbit of charged particles leads to a possibility of resonant interaction between particles with various transverse velocities and the electromagnetic wave and to the appearance of the finite cyclotron damping of waves propagating strictly across the magnetic field ($k_{\|} = 0$) [1].

In papers [1,2] there was found the coefficient of cyclotron damping of fast magnetosonic waves (FMSW) at $k_{\|} = 0$ in small pressure plasma $(\beta = 4\pi n_0 T_i / B_o^2) << 1$ under the resonance $\omega = n\omega_{Ci}$, $n \geq 2$. The fundamental resonance $\omega = \omega_{Ci}$ for ion minority was studied in paper [3] in which on the ground of the numerical solution of Maxwell's equations for FMSW there were determined the coefficients of reflection, transmission and absorption. In this case the absorption takes into account the FMSW conversion into short-wavelength oscillations in the region of the ion-ion hybrid resonance disguising the "actual" cyclotron absorption.

In this paper we make an attempt of analytical determination of the coefficients of cyclotron FMSW absorption and reflection at $k_{\|} = 0$ under the fundamental resonance $\omega = \omega_{Ci}$ in the plasma with a single ion species. In this case within the resonance region with the width order of the ion Larmor radius ρ_i the FMSW refractive index varies substantially [2], and this nonuniformity leads to the reflection of the wave. The appearance of the small cyclotron damping of the wave is associated, as was the case at $k_{\|} \neq 0$, with a small component of the FMSW electric field rotating in the direction of ion rotation in the constant magnetic field $\vec{B}_0$.

In Section 1 there are obtained the integral equations for the wave electric field strength within the resonance region. In Section 2 they are reduced to the ordinary second-order differential equation in the limit of zero Larmor radii. This equation is solved according to the narrow-layer technique [4] using the small parameter $k_A \rho_i << 1$ where $k_A = \omega / v_A$ is the FMSW wavenumber, v_A is the Alfven velocity. Section 3 takes into account the effects of the first order in the Larmor radius. In this Section the set of integral equations is reduced to the third-order equation that is also solved according to the narrow –layer technique. In Conclusion we discuss the results obtained.

1. Integral equation for the FMSW electric field. Consider the FMSW propagation with the components E_x, E_y, B_z through a uniform plasma layer across the steady magnetic field $B = B_0(1 + x/L_B)$ directed along z-axis. We assume β to be small and $\rho_i << 1/k_A << a << L_B$,

a is the scale length of the density nonuniformity. For simplicity we will take $k_y=0$. Then integrating Vlasov equation for the perturbed ion distribution function along the characteristics, we obtain the following expressions for the plasma dielectric permittivity tensor ε_{ij}:

$$\varepsilon_{11}=\varepsilon_1-\frac{\omega_{pi}^2}{4\omega_{Ci}^2},\ \varepsilon_{22}=\varepsilon_2-\frac{\omega_{pi}^2}{4\omega_{Ci}^2},\ \varepsilon_{12}=i\varepsilon_1+i\frac{5}{4}\frac{\omega_{pi}^2}{\omega_{Ci}^2},\ \varepsilon_{21}=-i\varepsilon_2-i\frac{5}{4}\frac{\omega_{pi}^2}{\omega_{Ci}^2},\quad (1)$$

where

$$\varepsilon_1=i\frac{\sqrt{2\pi}}{4}\frac{\omega_{pi}^2}{\omega}\frac{L_B}{v_{Ti}}W\left(\zeta-i\frac{k_x\rho_i}{\sqrt{2}}\right),$$

$$\varepsilon_2=\frac{\omega_{pi}^2L_B}{2\omega v_{Ti}}\left\{\left[1+i\sqrt{\pi}\zeta W\left(\zeta-i\frac{k_x\rho_i}{\sqrt{2}}\right)\right]\zeta\sqrt{2}+ik_x\rho_i\right\}$$

H $\quad \rho_i=\frac{v_{Ti}}{\omega_{Ci}},\ \omega_{Ci}=\frac{eB_0}{m_ic},\ v_{Ti}=\sqrt{\frac{T_i}{m_i}},\ \omega_{pi}^2=\frac{4\pi e^2n_0}{m_i},$

$$W(\zeta)=\exp\left(-\zeta^2\right)\left[1+\frac{2i}{\sqrt{\pi}}\int_0^{\zeta}\exp\left(t^2\right)dt\right]=u(\zeta)+iv(\zeta),\quad \zeta=\frac{x}{\sqrt{2}\rho_i},$$

$u(\zeta)$ is the even function of ζ and $v(\zeta)$ is the odd function of ζ. It is assumed that the resonance $\omega=\omega_{Ci}$ is achieved at $x=0$. Relationships (1) determine the components of the electric current density through the Fourier-components of the electric and magnetic fields of the wave $E_y(k_x)$, $E_x(k_x)$ and $B_z(k_x)$ which are proportional to $\exp(ik_x x)$. These components are related through Maxwell's equations yielding that

$$\frac{\partial^2E_y(x)}{\partial x^2}=-\frac{\omega^2}{c^2}\int\left[\varepsilon_{21}(x,k_x)E_x(k_x)+\varepsilon_{22}(x,k_x)E_y(k_x)\right]\exp(ik_xx)dk_x,\quad (2)$$

$$\int\left[\varepsilon_{11}(x,k_x)E_x(k_x)+\varepsilon_{21}(x,k_x)E_y(k_x)\right]\exp(ik_xx)dk_x=0.\quad (3)$$

The set of integral equations (2) and (3) defines a problem on the FMSW propagation through the resonant layer $\omega=\omega_{Ci}$. As the FMSW refractive index $N\sim N_A=c/v_A$ experiences strong changes over the distance $\Delta x\sim\rho_i$, then in equations (2) and (3) the values $k_x\sim 1/\rho_i$ are important. Just such values determine the FMSW reflection from the layer and the absorption of waves in this layer. One can solve the set (2) and (3) only numerically. We will attempt to evaluate the reflection and damping coefficients to the order of magnitude under the assumption that the effects of spatial dispersion taken into account in the coefficients $\varepsilon_{ij}(x,k_x)$ are small. Then we evaluate their contribution expanding $\varepsilon_{ij}(x,k_x)$ in powers of $k_x\rho_i$.

2.<u>Differential equation for E_y neglecting the spatial dispersion.</u> Putting in the expressions for $\varepsilon_{ij}(x,k_x)$ (2) and (3) $k_x=0$ we obtain that $E_x=E_{x0}(x)$ and $E_{y0}(x)$ where

$$E_{x0}(x)=-E_{y0}(x)\frac{\varepsilon_{21}(x,0)}{\varepsilon_{11}(x,0)},\quad (4)$$

$$\frac{d^2E_{y0}}{dx^2}=-k_0^2(x)E_{y0},\quad (5)$$

where $k_0^2(x) = k_A^2[1+f(\zeta)]$, $f(\zeta) = f_R(\zeta) + if_I(\zeta)$,

$$f_R(\zeta) = -\frac{1}{2} + \zeta^2 - \frac{\zeta v(\zeta)}{\sqrt{\pi}|W(\zeta)|^2}, \qquad f_I(\zeta) = -\frac{\zeta \exp(-\zeta^2)}{\sqrt{\pi}|W(\zeta)|^2} \tag{6}$$

The function $f_R(\zeta)$ determining the deviation of the quantity $\mathrm{Re}\, k_0^2(x)$ from the value of the FMSW wavenumber squared far from the layer is less than zero, its minimum value (-1/2) being achieved at $\zeta=0$, for $|\zeta| >> 1$ $f_R \approx -1/2\zeta^2$. The imaginary part of $k_0^2(x)$ is determined by the function $f_I(\zeta)$ that is antisymmetric in ζ. At $\zeta << 1$ this function assumes the form $f_I = -\zeta/\sqrt{\pi}$, at $|\zeta| >> 1$ we have $f_I = -\sqrt{\pi}\zeta^3 \exp(-\zeta^2)$.

Function $k_0^2(x)$ changes substantially at $|\zeta| \le 1$, i.e., within the narrow interval $|\Delta x| \sim \rho_i$ where the field E_y itself experiences weak changes but its derivatives may vary strongly. Therefore one can put a $|x| \le x_0$, where $k_A x_0 << 1$ $E_y = E_{y0} = 1 + E_0^{(1)} + E_0^2 + \ldots$ where $E_0^{(1)}, E_0^{(2)}, \ldots$ are found from the equations

$$\frac{d^2 E_0^{(1)}}{dx^2} = -k_0^2(x), \qquad \frac{d^2 E_0^{(2)}}{dx^2} = -k_0^2(x) E_0^{(1)}. \tag{7}$$

Hence we obtain that

$$E_0^{(1)}(x) = -\int_{-x_0}^{x} dx_1 \int_{-x_0}^{x_1} dx_2 k_0^2(x_2) + C_1(x + x_0) + C_2, \tag{8}$$

$$E_0^{(2)}(x) = -\int_{-x_0}^{x} dx_1 \int_{-x_0}^{x_1} dx_2 k_0^2(x_2) E_0^{(1)}. \tag{9}$$

Outside the layer $|x| > x_0$ we have

$$E_y = e^{ik_A x} + re^{-ik_A x}, \ (x < -x_0), E_y = Te^{ik_A x}, \ (x > x_0)$$

Coefficients C_1 and C_2 are determined from the continuity conditions for E_y and dE_y/dx at $x = -x_0$, $C_2 = e^{-ik_A x_0} + re^{ik_A x} - 1, C_1 = ik_A\left(e^{-ik_A x_0} - re^{ik_A x_0}\right)$.

.Satisfying the continuity conditions for the fields $E_y = 1 + E_y^{(1)}$ yields the reflection and transmission coefficients

$$r = -i(\alpha + \beta - \gamma) - \alpha^2 - 2\alpha\beta, \quad T = 1 - \alpha^2 - i(\alpha - \beta - \gamma) \tag{10}$$

$$\alpha = -k_A \rho_i \sqrt{2} \int_0^\infty d\zeta f_R(\zeta). \tag{11}$$

$$\beta = -k_A^2 \rho_i^2 \int_{-\infty}^{\infty} d\zeta \int_{-\infty}^{\zeta} d\zeta_1 f_I(\zeta_1) \qquad \gamma = -k_A^2 \rho_i^2 \int_{-\infty}^{\infty} d\zeta \zeta f_I(\zeta) \tag{12}$$

Hence we obtain for the absorbed power

$$Q = 1 - \left|T^2\right| - \left|r\right|^2 = 4\alpha\gamma \sim k_A^3 \rho_i^3 . \tag{13}$$

3. Accounting for the spatial dispersion. Let us assume in equations (2) and (3) the parameter $k_x \rho_i$ to be small and develop the function $\varepsilon_{ij}(x, k_x)$ in the integrands in powers of k_x,.

Then putting $E_y = E_{y0} + E_{y1}, E_x = E_{x0} + E_{x1}$ where $E_{y0} = 1 + E_0^{(1)} + E_0^{(2)}$, we obtain for $E_{y1}(x)$ the equation

$$\frac{d^2 E_{y1}}{dx^2} = -k_1^2(x) E_{y0}(x), \tag{15}$$

where $k_1^2(x) = k_A^2 [\varphi_R(\zeta) + i\varphi_I(\zeta)], \varphi_R = \operatorname{Re}\varphi,\ \varphi_I = \operatorname{Im}\varphi$

$$\varphi(\zeta) = \frac{3}{\pi} \frac{1 + i\sqrt{\pi}\zeta W(\zeta) + \left(2\zeta^2 - 1\right)\left(1 + i\sqrt{\pi}\zeta W\right)^2}{W^2(\zeta)}, \tag{16}$$

the function $\varphi_R(\zeta)$ is even in ζ, the function $\varphi_I(\zeta)$ is odd in ζ. From (18) we obtain that

$$E_{y1}(x) = -\int_{-x_0}^{x} dx_1 \int_{-x_0}^{x_1} dx_2 k_1^2(x_2) E_{y0}(x_2). \tag{17}$$

Satisfying the continuity conditions at $x = \pm x_0$, we obtain the coefficients r, T and Q the expressions (10) and (13) in which the following substitution should be made $\alpha \to \alpha + \alpha_1$, $\beta \to \beta + \beta_1$, $\gamma \to \gamma + \gamma_1$ where

$$\alpha_1 = -k_A \rho_i \sqrt{2} \int_0^{\infty} d\zeta \varphi_R(\zeta)$$

$$\beta_1 = -k_A^2 \rho_i^2 \int_{-\infty}^{\infty} d\zeta \int_{-\infty}^{\zeta} d\zeta_1 \varphi_I(\zeta_1), \qquad \gamma_1 = -k_A^2 \rho_i^2 \int_{-\infty}^{\infty} d\zeta \zeta \varphi_I(\zeta) \tag{18}$$

Conclusion. The consideration performed has shown that at $k_{\|} = 0$ the FMSW reflection and absorption coefficients are $|r|^2 \sim k_A^2 \rho_i^2$ and $Q \sim k_A^3 \rho_i^3$. The obtained evaluations for $|r|$ and Q are valid only to the order of magnitude. In order to obtain their exact values, a numerical solution of equations (2) and (3) is required.

References

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