# Anomalous Diffusion in Magnetoactive Plasma with Hybrid Pump

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Transport processes in a plasma are most fully described by a Boltzmann kinetic equation which treats the charged-particle collisions. In fact, since important macroscopic plasma properties such as the electric conductivity and the viscosity and heat-conduction coefficients are governed by particle collisions, their calculation calls for solution of a kinetic equation with a collision integral.

The determination of the collision integral, and consequently the turbulence spectrum and the associated diffusion of particles are important aspects of plasma nonlinear theory.

It is well known that the collision integral of charged particles in the plasma may be presented in the form:

$$I_\alpha(\vec{p}) = -\frac{e_\alpha}{n_\alpha}\,\mathrm{Re}\int\frac{d\omega}{2\pi}\int\frac{d\vec{k}}{(2\pi)^3}\times\frac{d}{d\vec{p}} < \delta f_\alpha(\omega,\vec{k},\vec{p})\,\delta\vec{E}(\omega,\vec{k}) > \qquad (1)$$

where $\alpha$ means the kind of particles.
In (1) the fluctuation of the distribution function is given by [1]:

$$\delta f_\alpha(\vec{k},\vec{p},t) = -e_\alpha\int_0^\infty d\tau\times e^{i\vec{k}(\vec{R}-\vec{r})}\,\delta\vec{E}(\vec{k},t-\tau)\frac{df_{0\alpha}(\vec{p})}{d\vec{p}} \qquad (2)$$

where $\delta\vec{E}$ is the fluctuating electric field. We shell consider fluctuations in an electron-ion magnetized plasma ( with the constant magnetic field $\vec{B}_0$ directed along the z axis ) under the influence of a radio-frequency pump wave field $\vec{E}_0(t) = E_0\vec{y}\cos\omega_0 t$, which is taken in the dipole approximation. In the linear limit into account that

$$div\,\delta\vec{E} = 4\pi\sum e_\alpha\int d\vec{p}\,\delta f_\alpha$$

and using the method of integration along unperturbed orbits we find:

$$\delta\vec{E}(\omega,\vec{k}) = \delta\vec{E}^s(\omega,\vec{k}) + \sum_\alpha\sum_{m,n} I_m(a_\alpha)I_n(a_\alpha)\chi_\alpha^n\,\delta\vec{E}(\omega+(n-m)\omega_0,\vec{k}) \qquad (3)$$

where $I_n$ - the Bessel function, $\chi_\alpha^n$ is the linear magnetized plasma susceptibiling in the presence of HF electric field and

$$a_\alpha = \mu\gamma_\alpha, \qquad \mu = E_0 k_\perp c/\omega_0 B_0, \qquad \gamma_\alpha = \Omega_\alpha^2/(\omega_0^2-\Omega_\alpha^2) \qquad ,,, [1,2].$$

Using in (1) the expressions (2) and (3) and carrying out the statistical averaging and the averaging over the period $(2\pi/\omega_0)$ we find the Fokker-Plank's type collision integral:

$$I_\alpha(\vec{p}) = \sum_{n=-\infty}^{\infty} d\vec{k}(\hat{L}_{\alpha n} D_{\alpha n} \hat{L}_{\alpha n} + \hat{L}_{\alpha n} A_{\alpha n}) f(p) \tag{4}$$

The quantities $D_{\alpha n}$ and $A_{\alpha n}$ are respectively the diffusion coefficient in velocity space and the dynamic friction coefficient and the notation $\hat{L}_{\alpha n}$ means:

$$\hat{L}_{\alpha n} = k_{II} \frac{d}{dp_{II}} + \frac{n\Omega_\alpha}{\upsilon_T} \frac{d}{dp_\perp}$$

Consider the parametric excitation of short-wave length ($k_i \rho_i > 1$), two-dimensional electrostatic convective cells with $\vec{k} \perp \vec{B}_0$ by lower-hybrid pump wave

$$\omega_0 \sim \omega_{LH} \approx \omega_{pi}(1 + \omega_{pe}^2 / \Omega_e^2)^{-\frac{1}{2}}$$

where $\omega_{p\alpha}$ and $\Omega_\alpha$ are the plasma and cyclotron frequencies of the particles of kind $\alpha$.

The convective cell modes is a zero-frequency, purely damped, electrostatic mode involving only particle motions and pertubations in the direction perpendicular to the external magnetic field. It is a widely-held opinion that these regards cross-field particle diffusion of charged particles in a magnetized uniform plasma.

One of the frequency bands in which effective HF power dissipation mechanisms exist in a thermonuclear plasma is the lower hybrid band (frequencies of the order of several GHz). In particular an r.f. field wich sufficiently large amplitude and frequency near the lower hybrid frequency may excite the parametric instabilities in the plasma.

It has been shown in [2] that parametric coupling of lower hybrid waves with purely-damped convective cells leads to a purely-growing instability.Under parametric instability conditions superthermal fluctuation field intensity is substantially higher than the level of thermal noise. Therefore the main contribution to the collision integral is made the diffusion coefficient.

Taking into account the connection between the turbulent collision frequency, that describes the particle scattering by the turbulent fluctuations with the coefficient of turbulent diffusion $D_\perp$ [3]

$$\nu_{eff} = k_\perp^2 D_\perp \tag{5}$$

and using the equation for the plasma energy balance

$$\frac{1}{2} \frac{\nu_{eff} e^2 n}{m_e \omega_0^2} E_0^2 = \sum_\alpha n_\alpha \int d\vec{p} \frac{p}{2m_\alpha} I_\alpha(\vec{p})$$

as a result we have:

$$D_\perp \approx \frac{1}{8} \frac{\mu^2}{(k_\perp r_{De})^2} \frac{\omega_{LH}}{k_\perp^2} \tag{6}$$

Comparing equation (6) with the corresponding expression in Ref.[3], which is

$$D_\perp = \frac{1}{2\pi} \frac{\nu_i}{n^{1/2}} (\ln \frac{\rho_i}{r_{De}})^{1/2} \tag{7}$$

we have that for typical laboratory plasma parameters the magnitude of the diffusion (6) is essentially greater than (7). Note that expression (7) is also obtained for low-frequency ($\omega < \Omega_i$) short-wavelength convective cell fluctuations but when external pump wave is absent, i.e. for the case of stable plasma.

We investigate the inhomogeneous magnetized plasma with exponential density gradient when the distribution function $f_{o\alpha}$ is proportional to $\exp(\alpha' y)$, $\alpha' = n_0^{-1} \frac{dn_0}{dy}$ is the plasma inhomogeneity parameter. We assume that such plasma subjected upper hybrid pump wave with

$$\omega_0 \sim \omega_{UH} \approx \Omega_e (1 + \omega_{pe}^2 k_z^2 / 2\Omega_e^2 k^2)$$

which decays into the upper hybrid wave and electron drift wave with frequency $\omega_D = -k_\perp \alpha\ T_e / m_e \Omega_e$ [4]. In the region above the instability threshold plasma becomes turbulent. Calculating in similar manner the turbulent diffusion coefficient as a result we have:

$$D_\perp = \frac{1}{8} \frac{\Omega_e^3 \omega_D}{\omega_{pe}^2 \nu_{ei}} \frac{c^2 E_0^2}{\omega_0^2 B_0^2} \qquad (8)$$

It can be seen from (8) that $D_\perp$ grows with increasing density gradient and intensity of pump wave and is independent of the magnetic field.

The present results can be of interest for studying the anomalous diffusion.

References

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