

Features of Surface Electromagnetic Waves in Magnetized Plasma Cylinder

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The surface waves (SW) are applied in plasma sources and plasma electronics. Also they are excited at RF plasma in magnetic traps (so called coaxial modes) and effect plasma periphery. These stimulate theoretical investigations of SW. They often use the model of homogeneous circular plasma cylinder with axial magnetic field, separated from metal wall by vacuum for SW studies. The axially symmetric $m = 0$ SW with $k_{\parallel} \neq 0$ (k_{\parallel} is a wave vector along confining magnetic field) were analyzed in details e.g. in [1]. Since [2] axially nonsymmetric ($m \neq 0$, $k_{\parallel} = 0$, $E_z = 0$) SW are intensively studied. Such waves is named as azimuth surface waves (ASW). As for fast magnetosonic waves the ASW with $m > 0$ and with $m < 0$ propagates in different ways due to plasma gyrotropy.

Both for $m = 0$ case and for $k_{\parallel} = 0$ case one can extract from Maxwell equations two independent differential equations of the second order. One of them is for the B_z component of wave and by analogy to fast magnetosonic waves we shall call these waves as fast surface waves (FSW). And another is for E_z component of wave. We shall call these waves as slow surface waves (SSW). At the boundary plasma - the metal SSW does not exist. Moreover even in systems with vacuum gap it was not considered as a rule. The real plasma devices have final length along confining magnetic field. Therefore it is worth-while to take into account $k_{\parallel} \neq 0$ in SW studies.

The paper presented concerns with analytical studies of the excitation and propagation of SW. We consider a homogeneous plasma cylinder of radius $r = a$ with axial magnetic field $B_0 \parallel Oz$. It is separated from metal wall of $r = b$ ($a < b$) by vacuum layer. The SW are excited by an azimuth surface current of radius $r = d$ ($a < d < b$) $\vec{j} = \vec{e}_{\phi} j_0 \exp(i(m\phi + k_{\parallel}z - \omega t))$.

We take into account presence of a surface charge $\rho = -i \operatorname{div} \vec{j} / \omega$. The RF field takes the form $\vec{B}, \vec{E} \sim \vec{B}, \vec{E}(r) \exp(i(m\phi + k_{\parallel}z - \omega t))$. From Maxwell equations we have

$$\begin{aligned}
 (\epsilon_1 - N_{\parallel}^2) \left[\frac{1}{r} \frac{d}{dr} \left(r \frac{d}{dr} \right) - \frac{m^2}{r^2} \right] B_z + \frac{\omega^2}{c^2} \left[(\epsilon_1 - N_{\parallel}^2)^2 - \epsilon_2^2 \right] B_z - i N_{\parallel} \epsilon_2 \left[\frac{1}{r} \frac{d}{dr} \left(r \frac{d}{dr} \right) - \frac{m^2}{r^2} \right] E_z = 0 \\
 i N_{\parallel} \epsilon_2 \left[\frac{1}{r} \frac{d}{dr} \left(r \frac{d}{dr} \right) - \frac{m^2}{r^2} \right] B_z + \left[\epsilon_1 (\epsilon_1 - N_{\parallel}^2) - \epsilon_2^2 \right] \left[\frac{1}{r} \frac{d}{dr} \left(r \frac{d}{dr} \right) - \frac{m^2}{r^2} \right] E_z + \\
 + \frac{\omega^2}{c^2} \epsilon_3 \left[(\epsilon_1 - N_{\parallel}^2)^2 - \epsilon_2^2 \right] E_z = 0
 \end{aligned} \tag{1}$$

Here ϵ_{α} are components of dielectric permeability tensor of cold two-component (ions and electrons) plasma. The collisional absorption of waves is taken into account. The effective collision frequency $\nu \ll \omega$ is included in ϵ_{α} . We carry out our research for $\omega_{ci} < \omega < \omega_{ce}$

($\omega_{c\alpha}$ is cyclotron frequency). The solutions of Eqs.(1) can be written as $B_z(r) = BI_m(k_{\perp}r)$, $E_z(r) = EI_m(k_{\perp}r)$ (B, E are constants, $I_m(x)$ is modified Bessel function). Then for $N_{\perp} = k_{\perp}c / \omega$ we have

$$N_{\perp F,S}^2 = \frac{I}{2\varepsilon_I} \left[\varepsilon_2^2 - (\varepsilon_I + \varepsilon_3)(\varepsilon_I - N_{\parallel}^2) \pm \left[\left[\varepsilon_2^2 - (\varepsilon_I + \varepsilon_3)(\varepsilon_I - N_{\parallel}^2) \right]^2 - 4\varepsilon_I\varepsilon_3 \left[(\varepsilon_I - N_{\parallel}^2)^2 - \varepsilon_2^2 \right] \right]^{1/2} \right] \quad (2)$$

The sign “+” meets FSW and sign “-“ - SSW. The electromagnetic field in vacuum consist of two independent modes. One of them is TE_m mode with $E_z = 0$ and another is $E_z = 0$ with $E_z = 0$. In the space between plasma and surface current they look like

$$B_z(r) = AV(k_v r, k_v b) - \frac{\pi}{2} j_0 V(k_v r, k_v d) k_v d, \quad E_z(r) = CW(k_v r, k_v b) + \frac{i\pi}{2} j_0 W(k_v r, k_v d) m N_{\parallel} \quad (3)$$

Here $W(k_v r, k_v b) = Y_m(k_v b) J_m(k_v r) - Y_m(k_v r) J_m(k_v b)$, $V(k_v r, k_v b) = Y'_m(k_v b) J_m(k_v r) - Y'_m(k_v r) J'_m(k_v b)$, $Y_m(x), J_m(x)$ - Bessel functions. Equating tangential components of electric and magnetic fields at the plasma - vacuum boundary we have a system of four linear equations to obtain A, C, B and E . The right side of this system is proportional to j_0 . Putting to zero the real part of determinant of this system we get the eigenfrequencies that is dispersion curves of SW. We investigate SW dispersion using $\Omega - N_A^2$ plane (here $\Omega = \omega / \omega_{ci}$ and $N_A = c / v_A$, v_A is Alfvén velocity).

Previously we want to mark.

While $k_{\parallel} \neq 0$ the region on $\Omega - N_A^2$ plane appears where SW don't exist (Fig.1, region $D < 0$).

The dispersion of SW in principle is well described by the relations

$$\frac{V'(k_v b, k_v a)}{V(k_v b, k_v a)} + \frac{I}{N_{\perp F}} \frac{I'_m(k_{\perp F} a)}{I_m(k_{\perp F} a)} + \frac{m}{k_{\perp F} a} \frac{I}{N_{\perp F}} \frac{\varepsilon_2}{\varepsilon_I - N_{\parallel}^2} = 0 \quad (4)$$

$$\frac{W'(k_v a, k_v b)}{W(k_v a, k_v b)} - N_{\perp S} \frac{I'_m(k_{\perp S} a)}{I_m(k_{\perp S} a)} = 0 .$$

These equations turn out from (1) ignoring the interaction of FSW and SSW. However in the dispersion equation of SSW $N_{\perp S}$ should be taken from (2).

Further to number of parameters which characterize device and remain fixed at dispersion curve we shall refer a, b, d, B_0 and k_{\parallel} . They are included in simulations as $b_0 = b\omega_{ci} / c$,

$a / b, d / b$ and $N_{\parallel 0} = k_{\parallel} c / \omega_{ci}$. Then changing the plasma density we change N_A^2 .

The calculations carried out allow us to make the following conclusions. The parameter b_0 is principal at the studies of SW dispersion.

At small $b_0 < 3 \times 10^{-4}$ only FSW with $m < 0$ can exist.

At larger values of b_0 ($3 \times 10^{-4} < b_0 < 3 \times 10^{-2}$) both the waves with $m < 0$ and $m > 0$ can be excited. Let's name them FSW of the first type (Fig. 2). The eigenfrequency of these waves grows with growth of $|m|$ and $\frac{b-a}{b}$. At such b_0 values there is no SSW. For this range of parameter we point out the remarkable fact. At $a / b \rightarrow 0$ the solution with $m > 0$ tends to solution of a vacuum cylindrical waveguide $J'_m(k_v b) = 0$. But the dispersion relation for $m < 0$ takes a form

$$\Omega = N_A^2 (2 + N_A^2)^{-1} \quad (5)$$

This means that for any b_0 of the range and any $|m|$ there is a "limiting" curve (5) (see Fig.3).

With further magnification increase of b_0 the FSW of the second type appears. As opposite to FSW of the first type its dispersion slightly depends on a sign (and magnitude) m and eigenfrequencies decrease with $\frac{b-a}{b}$ growth. Simultaneously SSW appears. At $b_0 > 5 \times 10^{-2}$ FSW of the first type cannot be excited (Fig. 4). Let us mark that eigenfrequencies of SSW and FSW of the second type approach with density increase. This easy to understand analyzing Eqs.(4). With N_A^2 growth N_{\perp}^2 is increased both for FSW and SSW. Therefore zero of the first equation in (4) are determined by zero of a numerator of the first addend and the zero of the second equation are determined by zero of a denominator of the first addend of this equation. The roots of these functions are very close.

That is why it is interesting to investigate excitation of SW in a range of such "collective" resonance. E_z and B_z at plasma boundary vs. ω are represented in Figs 5,6. At small $v/\omega = 0.009$ two peaks associated to eigenfrequencies of FSW and SSW are clearly visible. With growth of v/ω peaks height is decreased and they broaden. Relative magnitude E_z/B_z grows with N_{\parallel} increase. With magnification of v/ω up to 0.1 one generalized resonance (Fig.6) forms. Let's underline that the SW considered are eigenwaves of a system. Therefore for each separate resonance (either FSW or SSW) the total absorbed power does not depend on value or way of absorption (collisional either kinetic damping or absorption at the local resonance at plasma edge)

The interesting information can be obtained analyzing the total wave power in vacuum region dependencies. a summarized potency of an electromagnetic field in vacuum gap. With magnification of v/ω peak at FSW resonance frequency broadens while remaining constant on a height. (Fig. 7). But peak at SSW resonance frequency violently grows and expands practically suppressing FSW resonance. This effect is stipulated by interaction of FSW and SSW resonances. It is absent with reviewing these waves separately. The increase of N_{\parallel} carries on to relative (on a comparison to SSW resonance frequency) enlarge of power associated to FSW resonance frequency. The further magnification of v/ω results in formation of a generalized resonance (Fig.8).

Finally we studied eigenfrequencies of FSW of elliptic plasma cylinder. Here we supposed $E_z = 0$. The Maxwell equations were written in an elliptic coordinate system. Solution for B_z in plasma and vacuum is expressed through Mathieu's function. The correction to FSW eigenfrequency due to ellipticity is rather bulky. But it becomes simple for a case of $\frac{b-a}{b} \ll 1$ (ϵ - eccentricity)

$$\frac{\Delta\omega}{\omega} = \left[1 - 2 \frac{b-a}{b} k_{\perp}^2 a^2 \left[k_{\perp}^2 a^2 + m^2 \left(1 + \frac{\epsilon_2^2}{(\epsilon_1 - N_{\parallel}^2)^2} \right) \right]^{-1} \right] \frac{\epsilon^2}{4} \quad (6)$$

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References

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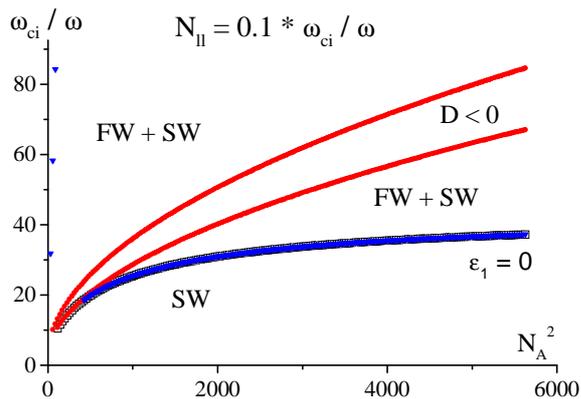


Fig. 1

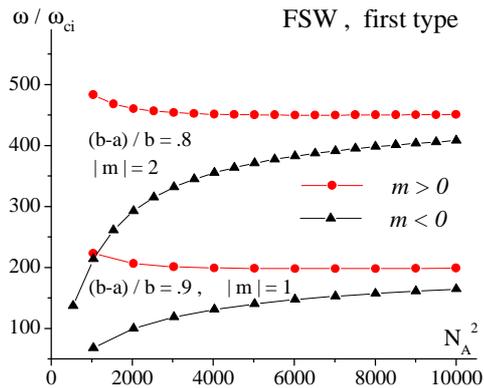


Fig. 2

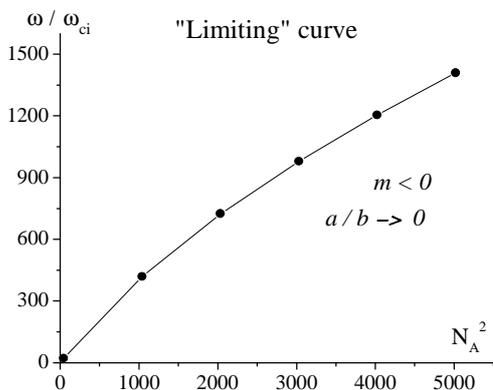


Fig. 3

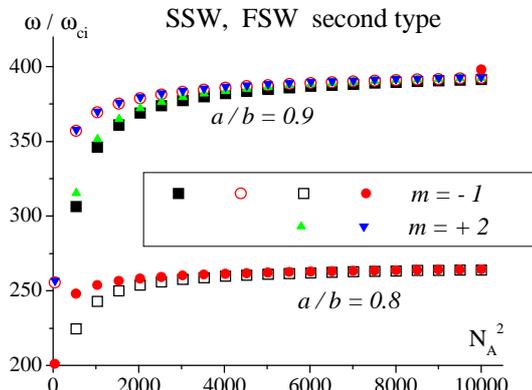


Fig. 4

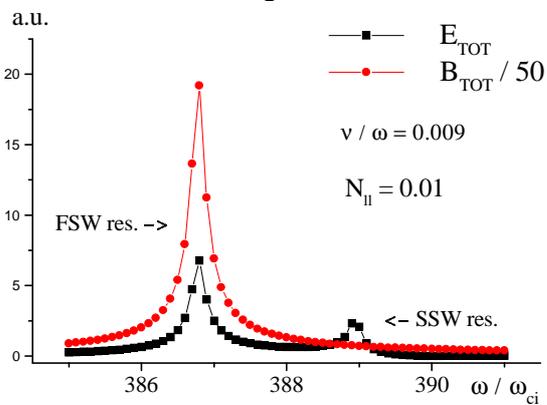


Fig. 5

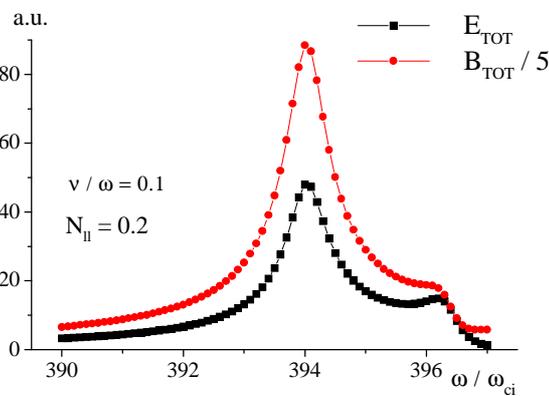


Fig. 6

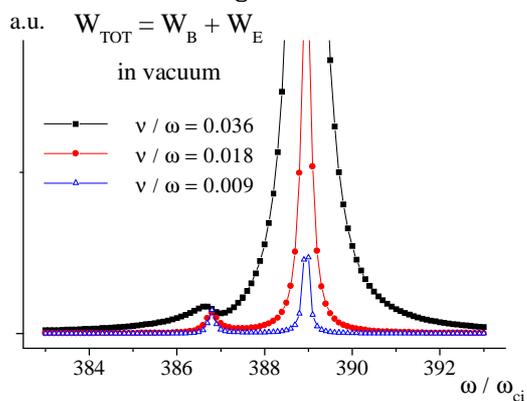


Fig. 7

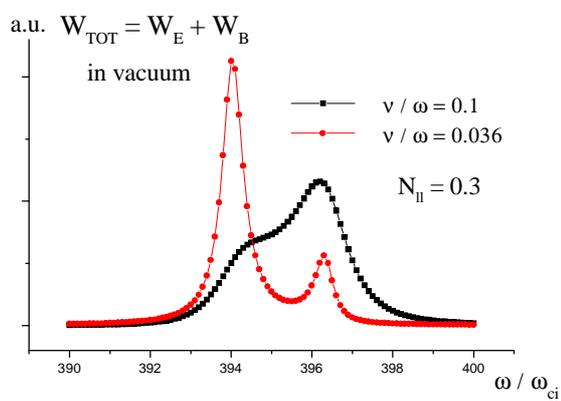


Fig. 8