26^{th} *EPS Conf. on Contr. Fusion and Plasma Physics, Maastricht, 14 - 18 June 1999 ECA Vol.23J (1999) 925 - 928*

Self-consistent Properties of Tokamak Plasmas in a State of Turbulent Equipartition

Igor A.Ivonin,[1,2] Vladimir P.Pavlenko [1] and Hans Persson[1,3]

[1]*Department of Space and Plasma Physics,*
Uppsala University, S-755 91 Uppsala, Sweden
[2]*RRC Kurchatov Institute, 123182 Moscow, Russia*
[3]*Studsvik Ecosafe AB, S - 611 32 Nyköping, Sweden*

The TEP distribution of the plasma density and temperature can be considered as an attractor of all possible steady distributions of the plasma in tokamak. This is because the unique TEP distribution corresponds to distribution of adiabatically conserved invariants of motion which is constant in tokamak volume . Namely, the turbulent perturbations of the electron motion will sooner or late provide a constant volume distribution of their adiabatic invariants. No details of the turbulence are important, except the requirement of a quick rate of the turbulent mixing in comparison with the rate of the violation of the adiabatic invariants (due to collisions, resonant interaction with high-frequency waves, etc.). In this case the TEP distribution may be steady on the timescale of the energy confinement time. This is the main idea of the *global TEP* approach (Yankov V.V.).

The experimentally observed quick response (within $10 \div 20ms$) of a plasma on external perturbation and the experimental observations of canonical profiles of plasma density (D.R.Baker, 1997-1998), temperature (T.C.Luce, 1992), radial electric field (R.A.Moyer et.al., 1995), plasma rotation and turbulence (J.Kim et.al., 1994) speaks in favor of as the cause of the establishment of a global TEP distribution.

In some basic ideal cases it is possible to obtain the TEP distribution of the electrons from their adiabatic invariants – the perpendicular adiabatic invariant $\mu \equiv m\vec{V}_\perp^2/(2B)$ and the *general* longitudinal invariant $J_0 \equiv \oint \frac{\vec{P}_e\vec{\Omega}_e}{\Omega_e} dl$ with $\vec{P}_e = m\vec{V} - \frac{e}{c}\vec{A}$ and the integration along the frozen-in field $\vec{\Omega}_e \equiv rot\vec{P}_e$ is assumed. According to the TEP concept this gives a uniform volume distribution of the adiabatic invariants for every group of electrons with the same values of adiabatic invariants. Each electron from this group is a passing one in the internal tokamak region and has the possibility to be trapped in the outside region.

For example, the TEP profiles of the electron density can be obtained in the following simple way. Let us consider the closed frozen-in field tube of the trapped (banana) electrons. The knowledge of the length $L_b(r) = 2\pi R_0 q(r)$ of this tube gives the profile of the density of the banana particles $n_b(r) \sim 1/L_b \sim 1/q(r)$ (Yankov V.V., 1994) through the safety factor profile $q(r)$ averaged on the magnetic surface r. This is because the invariant value of the total number of the banana electrons in this tube is a constant in the whole tokamak volume according the TEP idea. (Here we used also the conservation of the frozen-in field flux $dF = \Omega_e dS$). The effective length of the infinite contour for the passing electrons (the relative distance along the *infinite* magnetic tube between two Lagrange particles) can be obtained from the constant distribution of the general longitudinal invariant. In this case the mechanical part of this integral is negligible in comparison with the magnetic one, hence, $J_0 \approx \int(\vec{A}\vec{B})dl$ which gives $n_p(r) \sim 1/L_p(r) \sim (\vec{A}\vec{B})(r)$, i.e. the profile of the density of only passing electrons follows the helicity profile (Ivonin I.A., Pavlenko V.P., Persson H., 1998). On Fig.1 we present the normalized profiles of the values $1/q(r)$, $(\vec{A}\vec{B})(r)$ for the typical plasma parameters of the DIII-D tokamak

$(a_0 = 60cm,\ R_0 = 180cm,\ q_a = 4,\ \beta_\phi(0) = 0.01)$ and numerically calculated combined profile $n(r)$ of the density of both the trapped and the passing electrons. The best power fitting gives $n(r) \sim 1/q(r)^{0.5\div0.6}$ in accordance with the usual experimental scaling.

For *banana* electrons from a particular group, the *general* longitudinal invariant is half the mechanical one. Its value in TEP distribution $J_\parallel = J_\parallel(r, E_0(r)) = const$, does not depend on the magnetic surface r. This gives *both* the energy $E(r)$ profile (and the temperature $T(r)$ after the averaging over all groups of the electrons) of the banana electrons. This profile is rather flat in comparison with the experimentally observed data.

For *passing* electrons the mechanical part of the *general* longitudinal invariant is negligible. Moreover, it cannot be applied separately because of the possibility of a toroidal (longitudinal) deformation of the frozen in field tube. Thus, no information about the energy profile of these passing electrons can be obtained from its second adiabatic invariant. To define the profile of the kinetic energy $E(r)$ for passing electrons we may use the radial component of the kinetic equation of the electron *force equilibrium* which should be treated in an *averaged* sense as the balance of the impulse or “pressure” of the electrons from the particular group on the magnetic surface. Namely

$$\langle\frac{1}{V_\parallel{}^2}\rangle_\theta\{e\phi(r) - E(r)\}'_r = \frac{e}{c}\langle[\frac{\vec{V}_c}{V_\parallel{}^2} \times \vec{B}]_r\rangle_\theta, \tag{1}$$

where brackets denote the averaging over poloidal angle θ, the potential $\phi(r)$ represents the radial electric field, $E(r)$ is the kinetic energy of electron and $n(r,\theta)$ is the density of the electrons of particular group on the magnetic surface. From (1) one can see that the *force equilibrium* equation indeed can be applied only for the *passing* electrons because the value $\langle 1/V_\parallel{}^2\rangle_\theta$ diverges for the banana particles.

Similar to (1) the force equilibrium equation was used, for example, to obtain the TEP profile of the density in the convective zone of the Solar atmosphere,(Yankov V.V., 1997) which is in the force equilibrium with the gravity. The physical sense of (1) is the possibility to transform the energy of the poloidal magnetic field into the kinetic and the potential energy of electrons (Kadomtsev B.B., 1992) and into the energy of turbulence.

The current velocity of the particular group of passing electrons can be obtained from the kinetic momentum equation which for each group of passing electrons has the steady solutions in form:

$$V_0{}^2\langle\frac{V_{c_\phi}}{V_\parallel{}^2}\rangle_\theta = \alpha V_0{}^3 + \delta_1 + \delta_2 V_0\langle\frac{1}{V_\parallel}\rangle_\theta, \qquad V_0{}^2\langle\frac{V_{c_\theta}}{V_\parallel{}^2}\rangle_\theta = \gamma_1 + \gamma_2 V_0\langle\frac{1}{V_\parallel}\rangle_\theta, \tag{2}$$

where the values $\alpha(r),\ \delta_{1,2}(r),\ \gamma_{1,2}(r)$ are approximately the same for all passing electrons. The values $\delta_{1,2}$ and $\gamma_{1,2}$ are determined by the rotation $\vec{u}$ of the ions and by the toroidal drift V_{de} of the banana electrons

$$\{E(r) - e\phi(r)\}'_r = \frac{e}{c}\frac{\langle V_{de}B_\theta/V_\parallel\rangle_\theta}{\langle 1/V_\parallel\rangle_\theta} \tag{3}$$

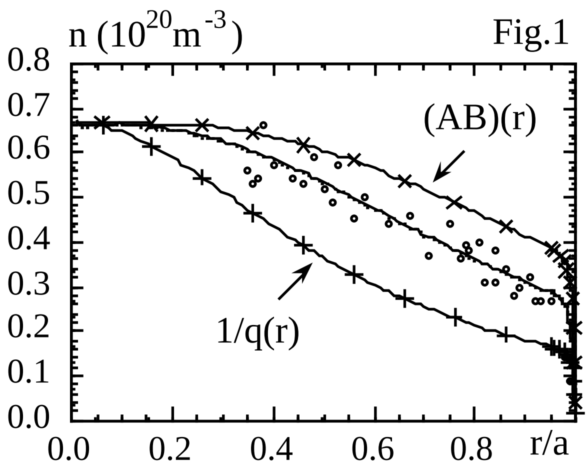
The value α is mainly determined by the loop voltage $V_{loop} \equiv 2\pi R_0 E_\phi$ and by the profile of total density. The above solution for $\langle\vec{V}_c/V_\parallel{}^2\rangle_\theta$ describes the relative possibility to carry the current. Namely, the only deeply passing electrons can carry large current, while the nearly trapped passing electrons carry only small current because the value $\langle 1/V_\parallel{}^2\rangle_\theta$ diverges for the banana electrons. toroidal To determine the radial profiles of plasma rotation we used usual drift equations for the radial equilibrium of the trapped and passing ions in the hydrodynamic approximation together with the force balance of the passing ions along the magnetic field direction

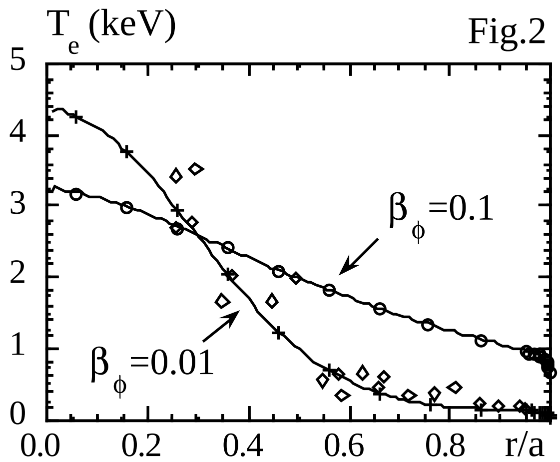
$$\frac{e}{M}E_\phi - \nu_{ii_b}(u_\| - u_{di_b}) - \sum_{\mathrm{p}} \nu_{ei}\frac{m}{M}(u_\| - V_{c\|}) - \sum_{\mathrm{b}} \nu_{ei}\frac{m}{M}(u_\| - V_{de}) = 0, \tag{4}$$

where $u_\|$ is the longitudinal rotation velocity of passing ions, u_{di_b} is the toroidal drift of trapped ions and ν_{ii_b} is the collisional frequency of passing and banana ions. The averaged in time toroidal component of the Maxwell equation

$$\frac{1}{r}\{\frac{r^2}{qR/R_0}\}'_r = \frac{R_0}{\omega_{be}(c/\omega_{pe}(r))^2}\{\mu u_\| + (1-\mu)u_{di_{b_\phi}} - \frac{1}{n(r)}\sum_{\mathrm{b}} \frac{\langle n_b\rangle}{\langle 1/V_\|\rangle_\theta}\langle\frac{V_{de}}{V_\|}\rangle_\theta - \frac{1}{n(r)}\sum_{\mathrm{p}} \frac{\langle n_p\rangle}{\langle 1/V_\|\rangle_\theta{}^2}\langle\frac{V_{c\phi}}{V_\|{}^2}\rangle_\theta\} \tag{5}$$

where $\mu(r)$ is the fraction of the passing ions, gives the last equation for the self-consistent determination of the TEP distribution and the profiles $T(r)$, $\phi(r)$, $\vec{u}(r)$ from the Eqs.[(1) - (5)] for given magnetic field geometry with a separatrix. After averaging over all groups of the electrons and elimination of the electric field, this system gives the usual equation of the hydrodynamic plasma equilibrium (Grad-Shafranov Equation). Two types of the pressure profile are possible in this model which is determined by the critical value $\beta_\phi^\star \approx (a/R)^2/(q_a - 1) \approx 4\%$ for the DIII-D parameters. On the Fig.2 we plot the profiles of the effective (longitudinal) electron temperature for the values $\beta_\phi = 1\%$ (typical experimental value) and for $\beta_\phi = 10\%$. One can see that larger value of β_ϕ gives small temperature pinching, but the temperature profile has a sharp boundary similar to that in the H-mode.





The numerical simulations of TEP distribution function were made by the Monte-Carlo method, by the iteration of the TEP distribution. The initial distribution function was the Maxwell function with constant profiles of density and temperature. Then, for each particle from this distribution, the individual TEP distribution of the density and the energies (the total and in the longitudinal degree of freedom) were calculated according to the description above. Thus, after this first iteration, the new distribution function consisting of the sum of the individual electron TEP was determined. Then we calculated the new distribution function, inhomogeneous in the tokamak volume, and used it for the next group of electrons instead of the initial uniform Maxwellian distribution. The convergence to the global TEP was found to be fast (1000 particles are sufficient).

It is important that the TEP distribution function found differs from a flux-surface-local Maxwellian distribution, because it has different longitudinal and perpendicular temperatures. Thus, TEP may relax due to kinetic instabilities. Some energy may be transferred to the turbulence in this relaxation process, which is needed for the TEP establishment. This gives us the possibility to estimate both the turbulence level $U(r)$ and the rate of the turbulent mixing. On Fig.3 we plot the steady level of the ion-sound turbulence which can be generated easily in the DIII-D magnetized plasma (with $r_l < r_{d_e}$). Then, the rate of the turbulent mixing can be estimated through the value of the turbulent diffusion coefficient $D_{turb} \sim U$ (Isichenko M.B. et.al., 1995), which gives the time $\tau_{mix} \leq 20ms$ for the central part $r/a < 0.7$. This may suggest the validity of the global TEP approach at least in the central part of the tokamak where the collision time $\tau_c \approx 5ms$ is not too small in comparison with the mixing time.

The found level of the turbulence is large in the central part of the tokamak because of large electron temperature and large current velocity (which can exceed the marginally stable velocity) of some groups of deeply passing electrons. The level of the turbulence is large also near the separatrix. This is because of the large (due to $n_p|_{r=a} \to 0$) current velocity of passing electrons near the separatrix at the position $r = a$.

Finally, it is interesting to note that the negative radial electric field can be generated near the separatrix by the inhomogeneous level of the turbulence similar to Miller force generation. Indeed, only the passing electrons may produce the turbulence during turbulent movement toward the separatrix. This can be considered as the radial turbulent force that pushes passing electrons from the separatrix. This force generates a charge separation, and the resulting negative radial electric field should compensate the turbulent force. The potential $\Phi(r)$ of this force can be introduces in the kinetic force-equilibrium Eq.(1). On Fig.3 we also plot the preliminary results of numerical calculation of this negative radial electric field. It is important that the position of the zero of this field corresponds to the position the turbulence suppression in agreement with the experimental results in DIII-D tokamak (R.A.Moyer et al., 1995). Finally, in Fig.4 we plot the preliminary results of the profiles of the plasma rotation in $L-$mode.