INFLUENCE OF TRAPPED PARTICLES WITH ANISOTROPIC TEMPERATURE ON THE DIELECTRIC PERMITTIVITY OF A MAGNETOSPHERIC PLASMA

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The energetic particles (both the protons and electrons) with anisotropic temperature can lead to the wide class of proton/electron cyclotron instabilities in the Earth's magnetosphere.[1] These instabilities should be analyzed by solving the Vlasov-Maxwell's equations, taking into account the two-dimensional nonuniformity of the geomagnetic field and the bounce-resonant wave-particle interactions there.[2−4] In this paper, the linearized drift kinetic equation is solved for the trapped particles with the bi-maxwellian steady-state distribution function in an axisymmetric magnetospheric plasma model with circular magnetic field lines: $B(R, \phi) = B_0 R_0^3 / (R^3 \cos \phi)$. Here, R_0 is the radius of the Earth, R is the geocentric distance, ϕ is the geomagnetic latitude, B_0 is the magnetic field in an equatorial plane on the Earth's surface ($R = R_0, \phi = 0$). For simplicity, only electrostatic perturbations are considered. To solve the Vlasov equation we use the standard method of switching to new variables associated with the conservation integrals of energy: $v_\perp^2 + v_\parallel^2 = const$, magnetic moment: $v_\perp^2 / 2B = const$, and the equation of **B**-field line: $R/\cos\phi = const$. Introducing the variables

$$v = \sqrt{v_\parallel^2 + v_\perp^2}, \qquad \mu = \frac{v_\perp^2 B(L, 0)}{v^2 B(L, \phi)}, \quad \text{and} \quad L = \frac{R}{R_0 \cos \phi}$$

(instead of $v_\parallel, v_\perp, R$) we seek the perturbed distribution function as

$$f(t, R, \phi, \theta, v_\parallel, v_\perp, \alpha) = \sum_s^{\pm 1} \sum_l^{\pm \infty} f_l^s(\phi, L, v, \mu) \exp(-i\omega t + im\theta + il\alpha),$$

where α is the gyrophase angle in velocity space. So that the linearized Vlasov equation for harmonics f_0^s and $f_{\pm l}^s$ can be rewritten in the next form:

$$\sqrt{1 - \frac{\mu}{\cos^4 \phi}} \frac{\partial f_l^s}{\partial \phi} - is \frac{L R_0}{v} \left(\omega + \frac{l \, \omega_{co}}{L^3 \cos^4 \phi} \right) f_l^s = Q_l^s, \qquad l = 0, \pm 1, \tag{1}$$

$$Q_0^s = \frac{e}{T_\parallel} R_0 L \sqrt{1 - \frac{\mu}{\cos^4 \phi}} F_0 E_\parallel, \qquad Q_{\pm 1}^s = \frac{se}{2T_\perp} R_0 L \frac{\sqrt{\mu}}{\cos^2 \phi} F_0 E_\pm, \qquad E_\pm = E_n \mp E_b,$$

$$F_0 = \frac{N(L)}{\pi^{1.5} v_{T\parallel} v_{T\perp}^2} \exp \left\{ -\frac{v^2}{v_{T\parallel}^2} \left[1 - \frac{\mu}{\cos^4 \phi} \left(1 - \frac{T_\parallel}{T_\perp} \right) \right] \right\}, \qquad v_{T\parallel}^2 = \frac{2T_\parallel}{M}, \qquad v_{T\perp}^2 = \frac{2T_\perp}{M},$$

where $E_\parallel, E_n, E_b$ are, respectively, the parallel, normal and binormal perturbed electric field components relative to **B**; F_0 is the steady-state distribution function of plasma particles with the density N, parallel and perpendicular temperature $T_\parallel$ and $T_\perp$, respectively, charge e and mass M. By indexes $s = \pm 1$ we differ the particles with positive

and negative values of $v_\parallel = sv\sqrt{1 - \mu/\cos^4\phi}$ relatively to **B**. Note, the longitudinal permittivity elements in magnetospheric plasmas with an equilibrium distribution function (when $T_\parallel = T_\perp$) have been evaluated in Ref. [5] for two plasma models with dipole and circular magnetic field lines. In Eq. (1) we neglect the drift corrections assuming the wave frequency ω is much larger than the drift frequency, that is valid when $mv_{T\perp}^2 L^2/(v_{T\parallel}R_0\omega_{co}) << 1$, where $\omega_{co} = eB_0/Mc$, and m is the azimuthal wave number over θ (east-west) direction. Depending on μ, the domain of perturbed distribution functions is defined by the inequalities $L^{-4} \leq \mu \leq 1$ and $-\phi_t(\mu) \leq \phi \leq \phi_t(\mu)$, where $\pm\phi_t(\mu)$ are the local mirror points for the trapped particles at a given (by L) magnetic field line, which are defined by the zeros of parallel velocity: $\cos^4\phi_t - \mu = 0$. As a result, $\phi_t = \arccos \mu^{0.25}$. Due to the Earth's atmosphere, the trapped particles will be thermalized by the collisions with atmospheric molecules and atoms before they reach the Earth's surface. Any particle with $\mu < L^{-4}$ will not survive more than one half of the bounce time and will be precipitated into the atmosphere.

By solving Eq. (1) the two-dimensional longitudinal, $j_\parallel(\phi, L)$ parallel to **B**, and transverse, $j_{\pm 1}(\phi, L)$, current density components can be expressed as

$$j_\parallel(\phi, L) = \frac{\pi e}{\cos^4\phi} \sum_s^{\pm 1} s \int_0^\infty v^3 \int_{L^{-4}}^{\cos^4\phi} f_0^s(\phi, L, v, \mu)\, d\mu\, dv, \tag{2}$$

$$j_l(\phi, L) = \frac{\pi e}{2\cos^4\phi} \sum_s^{\pm 1} s \int_0^\infty v^3 \int_{L^{-4}}^{\cos^4\phi} \frac{\sqrt{\mu} f_l^s(\phi, L, v, \mu)}{\sqrt{\cos^4\phi - \mu}} d\mu\, dv, \quad l = \pm 1. \tag{3}$$

Note, the normal and binormal to **B** current density components in our notation are equal to $j_n = 0.5(j_1 + j_{-1})$ and $j_b = 0.5i(j_1 - j_{-1})$, respectively. In this paper, we evaluate the longitudinal dielectric permittivity. The transverse permittivity elements can be derived by analogy. Accounting that the trapped particles, with the given μ, execute the periodic motion with the bounce period proportional to

$$\tau_b = \tau_b(\mu) = 4 \int_0^{\phi_t} \frac{\cos^2\phi}{\sqrt{\cos^4\phi - \mu}} d\phi$$

the solution of Eq. (1) is

$$f_0^s(\phi, L, v, \mu) = \sum_{p=-\infty}^{+\infty} f_{0,p}^s(L, v, \mu) \exp\left(ip\frac{2\pi}{\tau_b}\tau(\phi)\right), \tag{4}$$

where

$$f_{0,p}^s(L, v, \mu) = -i\frac{eR_0 Lv G_p(\mu) N \exp(-v^2/v_{T\parallel}^2)}{T_\parallel (2\pi pv - s\omega R_0 L\tau_b)\, \pi^{1.5} v_{T\parallel} v_{T\perp}^2}, \quad \tau = \tau(\phi) = \int_0^\phi \frac{\cos^2\eta}{\sqrt{\cos^4\eta - \mu}} d\eta,$$

$$G_p(\mu) = \int_{-\tau_b/2}^{\tau_b/2} E_\parallel(\tau)\sqrt{1 - \frac{\mu}{\cos^4\phi(\tau)}} \exp\left[-ip\frac{2\pi}{\tau_b}\tau + \frac{\mu v^2(1 - T_\parallel/T_\perp)}{v_{T\parallel}^2 \cos^4\phi(\tau)}\right] d\tau.$$

The perturbed distribution functions defined by Eq. (4) satisfy automatically the corresponding boundary conditions for the trapped particles, namely, the continuity of the distribution functions ($f_0^1 = f_0^{-1}$) at the points $\pm\phi_t$.

After the s-summation, the longitudinal current density component can be expressed as

$$\frac{4\pi i}{\omega} j_\parallel(L, \phi) = \frac{4\omega_{po}^2 R_0 Lv_{T\parallel}}{\omega\pi^{1.5} v_{T\perp}^2 \cos^4\phi} \sum_{p=1}^\infty \frac{1}{p} \int_{0.5\sin^2\phi}^{(L^2-1)/2L^2} (1 - 2\kappa) d\kappa \times$$

$$\int_{-\infty}^{+\infty} \frac{u^4 \exp(-u^2)}{u - u_p} \left\{ G_p(\kappa, u) \exp\left[ip\frac{2\pi}{\tau_b}\tau(\phi)\right] + G_{-p}(\kappa, u) \exp\left[-ip\frac{2\pi}{\tau_b}\tau(\phi)\right]\right\} du, \qquad (5)$$

where

$$\omega_{po}^2 = \frac{4\pi N e^2}{M}, \qquad \omega_b = \frac{2\pi v_{T\parallel}}{R_0 L \tau_b}, \qquad u_p(\kappa) = \frac{\omega}{p\omega_b(\kappa)}.$$

In Eq. (5), the new variable $\kappa = 0.5(1 - \sqrt{\mu})$ is introduced instead of μ-variable. So that

$$\phi_t = \arcsin\sqrt{2\kappa}, \quad \tau_b(\kappa) = 2\sqrt{2(1-2\kappa)}\Pi(\kappa), \quad \Pi(\kappa) = \int_0^{\pi/2} \frac{d\phi}{(1 - 2\kappa\sin^2\phi)\sqrt{1 - \kappa\sin^2\phi}},$$

where $\Pi(\kappa)$ is the complete elliptic integral of the third kind. As one can see, by Eq. (5), the longitudinal current density component in an axisymmetric magnetosphere is derived by the p-summation of the bounce-resonant terms. It should be noted that the bounce-resonance conditions for the trapped particles in magnetospheric plasmas are

$$\omega\sqrt{(1-2\kappa)}\Pi(\kappa) + l\frac{\omega_{co}[2E(\kappa) - K(\kappa)]}{L^3(1-2\kappa)^{1.5}} = \frac{p\pi v}{\sqrt{2}R_0 L}, \qquad l = 0, \pm 1, \qquad (6)$$

where $K(\kappa)$ and $E(\kappa)$ are the complete elliptic integrals of the first and second kind, respectively. However, there is no possibility to carry out the Landau integration over the particle energy $u = v/v_{T\parallel}$ since the phase coefficients $G_p(\kappa, u)$ depend on u.

To solve the two-dimensional wave equations, we should expand preliminary the perturbed values in a Fourier series over ϕ. In particular, for the longitudinal components of the current density, $j_\parallel$, and electric field, $E_\parallel$, we have:

$$\cos^4\phi\, j_\parallel(L, \phi) = \sum_n^{\pm\infty} j_\parallel^{(n)}(L)\, \exp\left[\frac{i\pi n\phi}{\phi_0(L)}\right], \qquad E_\parallel(L, \phi) = \sum_{n'}^{\pm\infty} E_\parallel^{(n')}(L)\, \exp\left[\frac{i\pi n'\phi}{\phi_0(L)}\right],$$

where the points $\pm\phi_0(L) = \pm\arccos(1/L)$ are the beginning and the end of a given (by L) magnetic field line on the Earth's surface. This procedure converts the operator, representing the dielectric tensor, into a matrix whose elements are calculated independently on the solutions of Maxwell's equations. As a result,

$$\frac{4\pi i}{\omega} j_\parallel^{(n)} = \sum_{n'}^{\pm\infty} \epsilon_\parallel^{n,n'} E_\parallel^{(n')},$$

and the contribution of a given kind of plasma particles to the longitudinal permittivity elements, $\epsilon_\parallel^{n,n'}(L)$, is

$$\epsilon_\parallel^{n,n'} = \frac{\omega_{po}^2 L^2 R_0^2}{\pi^2 v_{T\perp}^2 \arccos(1/L)} \sum_{p=1}^{\infty} \frac{1}{p^2} \int_0^{\frac{L^2-1}{2L^2}} (1 - 2\kappa)\tau_b C_p^n(\kappa) D_p^{n'}(\kappa)d\kappa, \qquad (7)$$

where

$$C_p^n(\kappa) = \int_0^{\arcsin\sqrt{2\kappa}} \cos\left[\frac{n\pi\phi}{\arccos(1/L)} - 2\pi p\frac{\tau(\phi)}{\tau_b}\right] d\phi$$

$$+ (-1)^{p-1} \int_0^{\arcsin\sqrt{2\kappa}} \cos\left[\frac{n\pi\phi}{\arccos(1/L)} + 2\pi p\frac{\tau(\phi)}{\tau_b}\right] d\phi,$$

$$D_p^n(\kappa) = \int_{-\arcsin\sqrt{2\kappa}}^{\arcsin\sqrt{2\kappa}} \exp\left[\frac{n\pi\phi}{\arccos(1/L)} - 2\pi p\frac{\tau(\phi)}{\tau_b}\right] \frac{1 + 2v_p^2 + 2i\sqrt{\pi}\,v_p^3 W(v_p)}{\left[1 - \frac{1-T_\parallel/T_\perp}{\cos^4\phi}(1-2\kappa)^2\right]^{1.5}}d\phi$$

$$+(-1)^{p-1}\int_{-\arcsin\sqrt{2\kappa}}^{\arcsin\sqrt{2\kappa}} \exp\left[\frac{n\pi\phi}{\arccos(1/L)} + 2\pi p\frac{\tau(\phi)}{\tau_b}\right] \frac{1 + 2v_p^2 + 2i\sqrt{\pi}\,v_p^3 W(v_p)}{\left[1 - \frac{1-T_\parallel/T_\perp}{\cos^4\phi}(1-2\kappa)^2\right]^{1.5}}d\phi,$$

$$W(z) = e^{-z^2}\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2}dt\right), \qquad v_p(\kappa) = \frac{\omega R_0 L \tau_b(\kappa)}{2\pi p v_{T_\parallel}}\sqrt{1 - \frac{1-T_\parallel/T_\perp}{\cos^4\phi}(1-2\kappa)^2}.$$

Thus, due to a geomagnetic field inhomogeneity, the whole spectrum of the electric field (by $\sum_{n'}^{\pm\infty}$) is present in a given (by n) current density harmonic. For the low frequency waves with $\omega/\omega_b << 1$, the first ($p = 1$) bounce-resonant term is main in $\sum_p$. The expression (7) is written by the summation of bounce-resonant terms including the plasma dispersion function $W(z)$; that simplifies substantially the estimations of both the real and imaginary parts of the longitudinal permittivity elements. As was noted above, Eq. (7) describes the contribution of any kind of the trapped particles to the longitudinal permittivity. The corresponding expressions for plasma electrons and ions can be obtained from (7) by replacing T (temperature), N (density), M (mass) by the electron T_e, N_e, m_e and ion T_i, N_i, M_i parameters, respectively. To obtain the total expressions for the longitudinal permittivity, as usual, it is necessary to carry out the summation over all kinds of plasma particles. The same comments should be addressed to derive the total two-dimensional longitudinal current, by Eq. (5). The contribution of trapped particles with anisotropic temperature to the perturbed longitudinal current density component (as well as to $\epsilon_\parallel^{n,n'}$) is derived assuming that the bounce frequency, $\omega_b \sim 2v_{T_\parallel}/LR_0$, is much larger than the drift frequency, $\omega_d \sim 2mv_{T_\perp}^2 L/(R_0^2\omega_{co})$ of ions and/or electrons. It is shown that the bounce-resonance conditions of the trapped particles in magnetospheric plasmas, Eq. (6), are entirely different from the corresponding expressions in the straight magnetic field case. As can be seen, Eq. (7) has as limit the corresponding results in Ref. [5] if $T_\parallel = T_\perp$. The approach developed to evaluate the longitudinal permittivity elements is suitable as well to study the wave-particle interaction in the range of electron/ion cyclotron frequencies and their harmonics. The information related to the basic cyclotron resonance effects is in the transverse dielectric tensor components, which can be derived by solving Eq. (1) for $l = \pm 1$, taking into account the cyclotron and bounce oscillations.

Since the proton cyclotron instability is an important contributor to cool proton heating in the Earths radiation belts, it is possible (and interesting) to develop a two-dimensional numerical code in order to describe this process in magnetospheric plasma models with new dielectric tensor components, accounting the bounce-resonance effects.

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