

Generation of high-energy tail electron populations from broadband Alfvén wave-spectrum interaction

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1. INTRODUCTION

As indicated by numerous spacecraft observations, most astrophysical plasmas are found to have velocity distribution functions exhibiting non-Maxwellian superthermal tails [1]. Velocity space distributions of magnetospheric and coronal electron and proton spectra are well represented by a power law in particle speed and are commonly modeled by a kappa-distribution [2]. Such non-thermal features may result from an acceleration mechanism by wave-particle interaction due to the presence of broadband lower hybrid or Alfvén wave turbulence. In the latter context, the problem of wave induced particle energization is formulated within a Fokker-Planck approach, where the velocity space diffusion is induced by Landau interaction [3]. From an initially Maxwellian electron equilibrium distribution the time evolution of the distribution function due to Alfvén wave-particle interaction is simulated numerically, where major attention is drawn to the specific shape of the diffusion operator, generating energetic electron tails in velocity space. In addition, collisional drag can act as a limiting factor for the saturation of the particle energization. It is demonstrated that in particular the interaction of a broadband Alfvén wave-spectrum generates a superthermal particle population, exhibiting high-energy tails consistent with the family of kappa-distributions.

2. THEORY

The general equation for the collective, time dependent development of the velocity space distribution in response to a wave spectrum affecting the particles via Cherenkov interaction, as well as in response to particle collisions and external electric fields reads

$$\frac{\partial f}{\partial t} = \left[\frac{\partial f}{\partial t} \right]_{waves} + \left[\frac{\partial f}{\partial t} \right]_{collisions} + \left[\frac{\partial f}{\partial t} \right]_{E-field} \quad (1)$$

Here $f(v, t = 0)$ shall be represented by a starting one-dimensional Maxwellian equilibrium distribution function $f(v, 0) = N/(\pi^{1/2}v_{th}) \exp[-v^2/v_{th}^2]$, where N denotes the particle density and $v_{th} = (2k_B T/m)^{1/2}$ is the thermal speed.

Neglecting external electric field contributions, we model the wave-particle interaction by a diffusive process [4] and consider the time evolution of the parallel velocity distribution function $f(v)$, with respect to the magnetic field \mathbf{B}_0 , being regulated by the Fokker-Planck equation as

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v} \nu(v) \left[v f + v_{th}^2 \frac{\partial f}{\partial v} \right] + \frac{\partial}{\partial v} D(v) \frac{\partial f}{\partial v} \quad (2)$$

Here $\nu(v)$ and $D(v)$ denote the velocity dependent collision and diffusion operators, respectively. The first term on the right hand side of equation (2) restores the distribution function to a Maxwellian. The consideration of a uniform magnetic field \mathbf{B}_0 allows with regard to axial symmetry a reduction of the three dimensional problem to two dimensions. The reduction to a reasonable one-dimensional approach can be achieved by assuming low collisionality such that changes in the pitch angle are negligible and we note that the main dynamics of Alfvén wave-particle energy exchange due to Landau interactions is regulated in parallel direction.

The quasi-linear, one dimensional diffusion operator due to Cherenkov wave-particle interaction [5] may be written as

$$D(v) = \left(\frac{e}{m} \cos \theta\right)^2 \int \frac{d\Omega dK}{2\pi} S_E(K, \Omega) \pi \delta(\Omega - Kv \cos \theta) \quad (3)$$

where $S_E(K, \Omega)$ is the spectral energy density of the wave turbulence and the Dirac term describes the wave-particle resonance. The $\cos \theta$ dependence is a consequence of the wave parallel electric field component where θ is the angle between the wave vector and the magnetic field \mathbf{B}_0 .

Next, we adopt a broadband spectrum $R(k)$ of Alfvén waves, trapped inside an envelop of extension $L_{\parallel, \perp}$ parallel and perpendicular to the magnetic field \mathbf{B}_0 , characterized by their wavenumbers k and frequencies $\omega(k)$. Performing the resonant interaction between the wave turbulence and the plasma electrons in the wave packet frame and using a Gaussian shape for the wave envelope, the diffusion coefficient for parallel electron acceleration can be written as

$$D(v) = \frac{L_{\perp} A^2 \cos \theta}{\sqrt{8\pi} av} \sum_k |R(k)|^2 g(a, \gamma) \exp(-a\gamma^2) \quad (4)$$

where A is the amplitude of the wave packet and a denotes a parameter determining over which part of the wave packet the diffusion acts. Here $g(a, \gamma)$ enters from the consideration of spatially confined turbulence where $\gamma = ikL_{\perp} [\omega(k) - kv \cos \theta] / 2akv \cos \theta$.

Equation (4) permits in combination with the Fokker-Planck approach from equation (2) the simulation of wave-particle energy exchange due to a broadband Alfvén wave packet with $\cos \theta \simeq 1$, obeying the dispersion relation for shear-kinetic Alfvén waves

$$\omega^2 = k_{\parallel}^2 v_A^2 \frac{1 + k_{\perp}^2 r_{gi}^2 (3/4 + T_e/T_i)}{1 + k_{\perp}^2 c^2 / \omega_{pe}^2} \quad (5)$$

where ω_{pe} is the electron plasma frequency. Usually the parallel wavelength is much larger than the ion gyroradius $r_{gi} = m_i v_{th} c / (eB_0)$ wherefore the condition $k_{\parallel} \ll k_{\perp} \propto r_{gi}$ holds. The shear-kinetic Alfvén wave described by the dispersion relation (5) contains the ion thermal corrections and additionally takes care of the inertia of the electron background plasma. For Alfvén waves the main damping mechanism is Landau damping by electrons since the electron thermal speed is comparable to the Alfvén velocity for high temperature space plasmas. Thus, Alfvén waves absorbed in this context can resonantly interact to create an extended tail in the particle velocity space distribution.

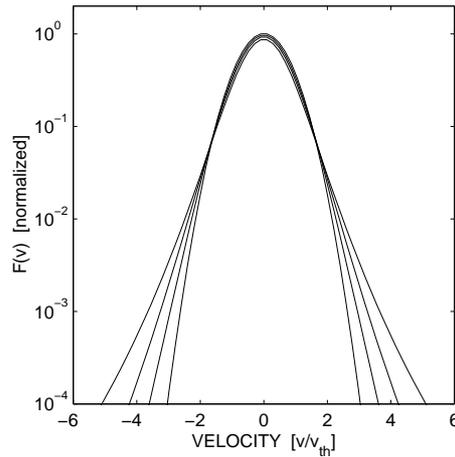


Figure 1: A family of kappa-distributions for $\kappa = 3, 5, 10$ and ∞ . The outermost curve exhibits with $\kappa = 3$ pronounced non-thermal tails, the innermost curve represents with $\kappa = \infty$ an isotropic Maxwellian.

Finally, let us consider the generally observed high energy tails in the particle distributions from a one dimensional analytical form of the family of kappa-distributions as [6]

$$F(v) = \frac{N}{v_{th}\sqrt{\pi}} \frac{\Gamma(\kappa + 1)}{\kappa^{3/2}\Gamma(\kappa - 1/2)} \left(1 + \frac{v^2}{\kappa v_{th}^2}\right)^{-(\kappa+1)} \quad (6)$$

hence, representing a power law in particle speed, see Fig. 1.

The parameter κ shapes predominantly the superthermal tails of the distribution, Γ denotes the standard Gamma function and as $\kappa \rightarrow \infty$, $F(v)$ approaches a Maxwellian. The advantage in using the function (6) instead to model the energetic particles by a power law at high energies was outlined in numerous studies as [7], [8], [9], trying to fit accurately spacecraft observations, wherefore it is highly desirable to search for appropriate generation mechanisms of superthermal tails in velocity space.

3. RESULTS AND CONCLUSION

The numerical simulation of the time evolution of the electron distribution due to a diffusive process of wave-particle interaction and collisions is treated within the Crank-Nicholson implicit scheme. As initial condition a Maxwellian equilibrium velocity distribution is introduced. For magnetospheric-auroral conditions the basic parameters are chosen as $N = 25 \text{ cm}^{-3}$, $T_e = 1 \times 10^5 \text{ K}$ and $B_0 = 2200\gamma$, noting that a simultaneous change of the parameters produce similar results for different physical systems. A generation mechanism representing the family of kappa-distributions accurately enough in the context of wave-particle interaction due to a spectrum of Alfvén waves provides a possible theoretical explanation for the formation of persistent kappa-like distribution functions in astrophysical plasmas.

The left panel of Fig. 2 illustrates the velocity dependence of the diffusion function evaluated from equation (4) with respect to the dispersion relation (5) for a broadband Alfvén wave spectrum, where $L_{\perp} \ll L_{\parallel}$ is assumed.

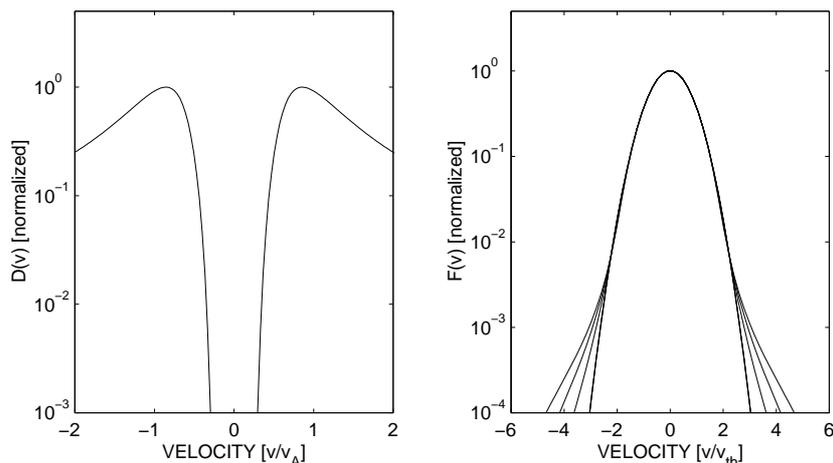


Figure 2: Left panel: normalized diffusion function. Right panel: Time sequence of the formation of a superthermal electron population due to an Alfvén wave packet. Non-thermal features reproduce well those of the kappa-distribution family.

The right panel of Fig. 2 demonstrates impressively that kappa-like superthermal tails in the electron distribution, exhibiting a smooth onset in velocity space can be generated by a broadband Alfvén wave-particle energy exchange process. Starting from a Maxwellian, equidistant snapshots in time are presented, to be compared with the family of kappa-distributions in Fig. 1. Adopting a non-zero collision frequency ν in equation (2) it can be shown that the electron energization due to Alfvén wave turbulence may generate a saturated stage at a balancing level between energy input and collisional thermalization.

It turns out that the specific shape of the diffusion operator in velocity space, as calculated from the wave power spectrum and dispersion relation, enters as dominant model parameter. Smooth kappa-like superthermal tail electron distributions can be generated accurately by broadband Alfvén wave spectra, wherefore a justification for the use of the analytical family of kappa distributions for a variety of space plasma studies is provided.

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