

Regeneration of Wave Perturbation in the Ionosphere due to Plasma Kinetic Effects

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During RF ionospheric modification experiments when a powerful radio wave from a Earth-based radio transmitter, operating in the high frequency (HF) range, is injected vertically into the overhead overdense ionospheric plasma, the level of ion acoustic and Langmuir waves is strongly enhanced near the RF reflection point where the HF matches the local ionospheric plasma frequency. However, sometimes a weaker enhancement is observed at a higher altitude, on the far side of the ionospheric layer where the RF frequency matches the local plasma frequency. The observations were ascribed (Ganguly et al., 1983; Isham et al., 1990) to a coupling from the injected *O*-mode to the *Z*-mode which propagates from the lower matching height to the higher one and there is strong enough to resonantly generate ion acoustic and Langmuir waves. We would like to point out that there might be an alternative explanation of this wave regeneration effect. This explanation is based on the result obtained by *Lisitchenko and Oraevskii* (1972) who demonstrated the possibility of Langmuir wave regeneration on the far side of a rectangular far side of barrier via a kinetic plasma effect even if the barrier is dense enough that regular barrier wave tunneling be totally negligible. Carrying this a few steps further, *Erokhin and Moiseev* (1983) pointed out the possibility of an electric plasma wave perturbation, impinging upon an inhomogeneous, overdense plasma barrier. The kinetic tunnelling effect is due to the trapping of electrons inside the electrostatic well formed by the barrier and can, in a way, be seen to be akin to an echo effect. *Erokhin and Moiseev* did, however not consider a specific shape of the electrostatic potential associated with the barrier, and, hence, did not determine what electrostatic potentials, or, alternatively, what plasma density profile for which such a wave field regeneration would be possible. Let us consider an inhomogeneous plasma barrier in the ionosphere and assume that its associated balancing electrostatic potential is given by

$$\phi(x) = \phi_0 \left[1 - \left(\frac{x^2}{L^2} \right)^\alpha \right], \quad 0 < \alpha < \infty \quad (1)$$

where $\phi_0 = \text{const}$ and L is the characteristic spatial scale of the inhomogeneity. The ions are heavy enough to be considered to be immobile in this confining potential, and constitute a neutralizing background only. If we are restricting ourselves to the one-dimensional spatial problem and neglect collisions, we describe the electron distribution function $f(x, v, t)$ by the 1D Vlasov equation

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} - \frac{e}{m} E \frac{\partial f}{\partial v} = 0 \quad (2)$$

Here, $-e$ and m are the charge and mass of the electron, respectively, and E is the electric field, which in the general case is a sum of the electrostatic field $E_0 = -d\phi/dx$, the self-consistent field E^s and the external perturbation field E^{ext} . Assuming the external field E^{ext} to be a small perturbation, we express f as $f = f_0 + \delta f$, where f_0 is the equilibrium electron distribution

function without the field E^{ext} and δf is a small perturbation of f_0 due to the external field. Then, for an equilibrium state, Eq.(2) takes the form

$$v \frac{\partial f_0}{\partial x} + \frac{e}{m} \frac{d\phi}{dx} \frac{\partial f_0}{\partial v} \quad (3)$$

with the general solution $f_0 = F(\epsilon)$, where F is an arbitrary function of its argument, $\epsilon = mv^2 - e\phi(x)$.

Let the plasma in the vicinity of the point $x = 0$ be characterized by a Maxwell distribution with constant temperature T and density N . Then, taking Eq.(3) into account, we have

$$f_0(\epsilon) = \frac{n_0(x)}{\sqrt{2\pi}v_T} \exp\left[-\frac{v^2}{2v_T^2}\right] \quad (4)$$

with

$$n_0(x) = N \exp\left[-\frac{e\phi_0}{T} \left(\frac{x^2}{L^2}\right)^\alpha\right], \quad v_T^2 = \frac{T}{m} \quad (5)$$

Let E_1 and ω_1 be the amplitude and frequency of the external perturbation. As the electric field of this external perturbation, we choose the model $E^{ext}(x, t) = E_1 \delta[k(x + x_1)] \cos \omega_1 t$. Here $x = \pm x_1$ are the points of plasma resonance defined by the condition $\omega_1 = \omega_{pe}(\pm x_1)$, with ω_{pe} the plasma frequency, and k is a factor that characterizes the spatial scale of the electric field in the vicinity of the plasma resonance points $\pm x_1$. This choice of external perturbation models, e.g. local plasma oscillations driven in the vicinity of the plasma resonance in the case of the oblique incidence of an electromagnetic wave polarized in the plane of incidence.

To first perturbative order, Eq.(2), takes the form

$$\frac{\partial \delta f}{\partial t} + v \frac{\partial \delta f}{\partial x} - \frac{e}{m} E_0 \frac{\partial \delta f}{\partial v} = ev(E^{ext} + E^s) = \frac{df_0}{d\epsilon} \quad (6)$$

Since we are interested in layer penetration due to the preservation of the memory of an external perturbation of the form of ballistic oscillations of the electron distribution function (Van Kampen modes), the self-consistent field may be neglected. Fourier transforming in time, we find that in the ballistic approximation the characteristics of Eq.(6) give

$$v(x, \epsilon) = \sqrt{\frac{2}{m}[\epsilon + e\phi(x)]}, \quad \frac{d\delta f_\omega}{dx} = i\frac{\omega}{v} \delta f_\omega + eE_\omega^{ext} \frac{df_0}{d\epsilon} \quad (7)$$

In the linear approximation, the electrons moving in the negative and positive directions of the x axis are independent of each other. They are superimposed on each other but do not interact and we can consider the perturbations separately. The response of the electrostatic field to a distribution function perturbation is obtained from the equation

$$\frac{\partial \delta E}{\partial t} = -4\pi \delta j, \quad \delta j(x, t) = -e \int_{-\infty}^{\infty} v \delta f(x, v, t) dv \quad (8)$$

Here δj is the current density. We find the linear perturbations δf^- and δf^+ of the distribution function which corresponds to the electrons moving with negative and positive velocities, respectively, from Eq.(8). Introducing these expressions into equation for current density (8), taking into account the fact that external field modulates only electrons with energies $\epsilon \geq \epsilon_p \equiv -e\phi(\pm x_1)$, and taking the inverse Fourier transform, we can obtain the linear

response of the electrostatic field to the external perturbation at the plasma resonance point $x = x_1$ as

$$\delta E(x_1, t) = -i \frac{2\pi e^2 E_1}{m T k \omega_1} e^{-i\omega_1 t} \int_{\epsilon_p}^{\infty} f_0(\epsilon) (e^{2i\omega_1 h_1} - 2e^{2i\omega_1 h_2} + e^{2i\omega_1 h_3}) d\epsilon + c.c. \quad (9)$$

with

$$h_1(\epsilon) = \int_0^{x_r} \frac{dx}{v(x, \epsilon)} + \int_{x_1}^{x_r} \frac{dx}{v(x, \epsilon)}, \quad h_2(\epsilon) = \int_0^{x_r} \frac{dx}{v(x, \epsilon)}, \quad h_3(\epsilon) = \int_0^{x_1} \frac{dx}{v(x, \epsilon)} \quad (10)$$

where $\pm x_r(\epsilon)$ are the turning points, defined as the points where the electron with total energy ϵ has a vanishing kinetic energy, $\epsilon = -e\phi(\pm x_r)$.

To calculate the integrals in Eq.(9) we use the stationary phase method. The stationary phase point, $\epsilon = \epsilon_1$, in the first term in Eq.(9) is calculated from the condition $h_1' = 0$ where the prime denotes a derivative with respect to the argument. From the analysis of this equation for the stationary phase point we conclude that the first term in the RHS of Eq.(9) has a stationary phase point only for profiles (1) with $\alpha > 1$. The corresponding electric field excited at the right-hand plasma resonance point has the form

$$\delta E(x_1, t) = \frac{4\pi^{3/2} e^2 E_1}{m T k \omega_1^{3/2}} \frac{f_0(\epsilon_1)}{|h_1''(\epsilon_1)|^{1/2}} \cos[\omega_1 t - 2\omega_1 h_1(\epsilon_1) - \frac{\pi}{4}], \quad \alpha > 1 \quad (11)$$

It follows from the analysis of the second term in the RHS of Eq.(9) that the derivative of the function h_2 does not vanish for any value of ϵ if $\alpha \neq 1$. In the particular case $\alpha = 1$, the function h_2 is a constant and the integral is easy to calculate. The result is

$$\delta E(x_1, t) = \frac{8\pi e^2 E_1}{m k \omega_1} f_0(\epsilon_p) \sin(\omega_1 t - 2\omega_1 h_2), \quad h_2 = \frac{\pi L}{2} \sqrt{\frac{m}{2e\phi_0}} = const, \quad \alpha = 1 \quad (12)$$

The stationary phase point for the third term in the RHS of Eq.(9) is defined through the condition $h_3'(\epsilon_3) = 0$, and the electric field excited at the right-hand plasma resonance point for profiles (1) with $\alpha < 1$ has the form

$$\delta E(x_1, t) = \frac{4\pi^{3/2} e^2 E_1}{m T k \omega_1^{3/2}} \frac{f_0(\epsilon_3)}{|h_3''(\epsilon_3)|^{1/2}} \sin[\omega_1 t - 2\omega_1 h_3(\epsilon_3) + \frac{3\pi}{4}], \quad \alpha < 1 \quad (13)$$

From the point of view of wave penetration of plasma layers, it is important to know the coefficient of regeneration for a wave or external perturbation on the far side of an opaque region. We restrict ourselves from now on to a consideration of an amplitude regeneration coefficient $K(\alpha)$, defined as the ratio between the regenerated field amplitude and the initial field amplitude. First of all, let us find the amplitude regeneration coefficient for the case of a purely parabolic potential ($\alpha = 1$). Inserting expression for the equilibrium distribution function into Eq.(11), we obtain

$$K(\alpha = 1) \equiv \frac{|\delta E(x_1, t)|}{E_1} = \sqrt{\frac{2}{\pi}} \frac{\omega_1}{k v_T} \quad (14)$$

It should be remembered that k^{-1} is the spatial scale of the electric field amplitude in the vicinity of the plasma resonance point. This scale is determined by the effective electron collision frequency or the resonance excitation of plasma waves. From the condition of validity

of the perturbative approach, we find that $(\omega_1/kv_T) < 1$. Thus, the amplitude regeneration coefficient (14) may be of order of unity or less. This is understandable in view of the fact that in the particular case of a parabolic potential, the electrons will move along elliptic trajectories in the phase plane. As a consequence, all electrons that extract energy from the external perturbation at the left-hand plasma resonance point, arrive simultaneously at the right-hand plasma resonance point and there irradiate their energy coherently in the form of a similar perturbation. It worthwhile to note that we carry out our study of the penetration of an inhomogeneous plasma layer in the ballistic approximation. Hence, the expressions obtained may be used only for obtaining a rather rough estimate of the regeneration coefficient. In order to calculate this coefficient more correctly, one has to take both the self-consistent electric field and an electron collision term in Eq.(2) into account.

For this reason, in the case of potential profiles with $\alpha \neq 1$, we consider the amplitude regeneration coefficient normalized to $K(\alpha = 1)$. In other words, this normalized coefficient, which we call β , equals the ratio between the field amplitudes (11) or (13) and the field amplitude (12). Thus, we obtain

$$\beta_i = \frac{K(\alpha \neq 1)}{K(\alpha = 1)} = \sqrt{\frac{\sqrt{2}\alpha\pi v_T}{x_1\omega_1}} \left(\frac{\epsilon_i + e\phi_0}{T}\right)^{1/2} \left(\frac{\epsilon_i - \epsilon_p}{T}\right)^{3/4} \exp\left(-\frac{\epsilon_i - \epsilon_p}{T}\right) \quad (15)$$

where $i = 1$ for $\alpha > 1$ and $i = 3$ for $\alpha < 1$.

From a qualitative analysis of Eq.(15) it follows that the amplitude regeneration coefficient $K(\alpha)$ for layers with $\alpha \neq 1$ is smaller than the coefficient for a parabolic layer, Eq.(14), when the stationary phase points ϵ_1 and ϵ_3 lie in the high energy region, i.e., when $\epsilon_i - \epsilon_p \gg T$. If the stationary phase point ϵ_i lies near ϵ_p , i.e. if $\epsilon_i - \epsilon_p \sim T$, then β_i may have a value of order of unity.

In conclusion, the effect of a linear kinetic wave regeneration on the far side of an overdense plasma inhomogeneity was investigated for a class of plasma profiles described by electrostatic potentials of the form (1). The external perturbation was chosen so as to model a peak of an electrostatic field which appears in the vicinity of the plasma resonance point for the case of an oblique incidence of an electromagnetic wave, polarized in the plane of incidence, upon an overdense plasma layer. Expressions for the electric field regenerated on the far side of the plasma layer at a symmetric point of plasma resonance was obtained. For the case of a confining potential with parabolic shape, $\alpha = 1$, we were able to confirm the conclusion, made by *Erokhin and Moiseev* (1983), that a phase focusing conditions satisfied only for electrons reflected once at the turning point. We demonstrated that for layers confined by an electrostatic potential of non-parabolic shape, $\alpha \neq 1$, the phase focusing condition is satisfied either during the direct passing of the electrons between the plasma resonance points, or after two reflections of the electrons by the walls of the potential well. Estimates of the amplitude regeneration coefficients $K(\alpha)$ and its dependence on the shape parameter α was made. In the case of a parabolic confining potential, $K(\alpha)$ may be of order unity. As for the transmission through the plasma layer, one must take into account the linear transformation ratio between transverse and longitudinal waves. It follows from the analysis that the ratio β_i may range in magnitude from being exponentially small to the order of unity. Consequently, the linear kinetic regeneration effect exists formally for any value of the parameter α which defines the shape of the electrostatic potential well, but the regeneration coefficient may vary between being exponentially small to be of the order of unity, depending on α .