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DETERMINATION OF DISPERSION RELATION FOR THE LOWEST MODE OF ION-ACOUSTIC INSTABILITIES IN A BOUNDED HIGH-CURRENT PLASMA

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1 Introduction

In a non-isothermal plasma created by the high-current low-pressure discharge in cylindrical tubes, the low-frequency oscillations at frequencies $\omega \lesssim 10^{-2}\omega_{pi}$ (ω_{pi} is the ion plasma frequency) simultaneously can arise together with the ion-acoustic oscillations at high frequencies $\omega \lesssim \omega_{pi}$ [1–7]. According to [7] the above oscillations are excited by the longitudinal discharge current. The current threshold of instabilities depends on the gas pressure, discharge diameter, and distribution of neutral gas density over the discharge. The increase of current above the threshold can result in the destruction of discharge tubes [3] and appearance of other accompanying effects, for example, to limit an output power of CW ion lasers [6].

At low frequencies a spectrum of the oscillations of argon plasma in discharge tubes with channel diameters of $5 \div 30$ mm consists of separate narrow peaks in the range of $0.1 \lesssim f \lesssim 2$ MHz ($f = \omega/2\pi$). As the excesses of current threshold increases, the number of peaks grows. As shown in [8], the observed frequencies are the lowest-order normal modes of long-wavelength ion-acoustic oscillations in the bounded plasma. However, dispersion relations for the oscillations were not measured in the experiment, although these relations are important characteristics for understanding and identification of instabilities. In this report, which is continuation of [7, 8], the frequency-wavenumber spectrum of the lowest oscillation mode was determined by means of correlation and spectral analysis of spontaneous emission out of two small areas of plasma. The dispersion obtained is compared with available theoretical models.

2 Experimental set-up and results

The experimental set-up was analogous to [8]. The argon plasma was created by the stationary high-current discharge in a tube 1 m long with a channel diameter of 16 mm. The tube had the cold arc cathode with self-heating refractory bushing and was made up of water-cooled aluminium oxide-coated sections [6]. Two sections had lateral windows as a vertical slit $4 \div 16$ mm for emission output from the plasma. These sections were placed near the cathode region of tube where the oscillations being investigated were the most intense. The distance between windows was $d = 15$ cm. The current threshold values were $300 \div 400$ A at filling argon pressure of $0.2 \div 0.4$ Torr (pressure in the discharge is more lower, especially near the cathode region). In our experiment the internal parameters of plasma were [6]: electron density $n_e \simeq 10^{14}$ cm^{-3} and temperature $T_e \simeq 5$ eV, ion

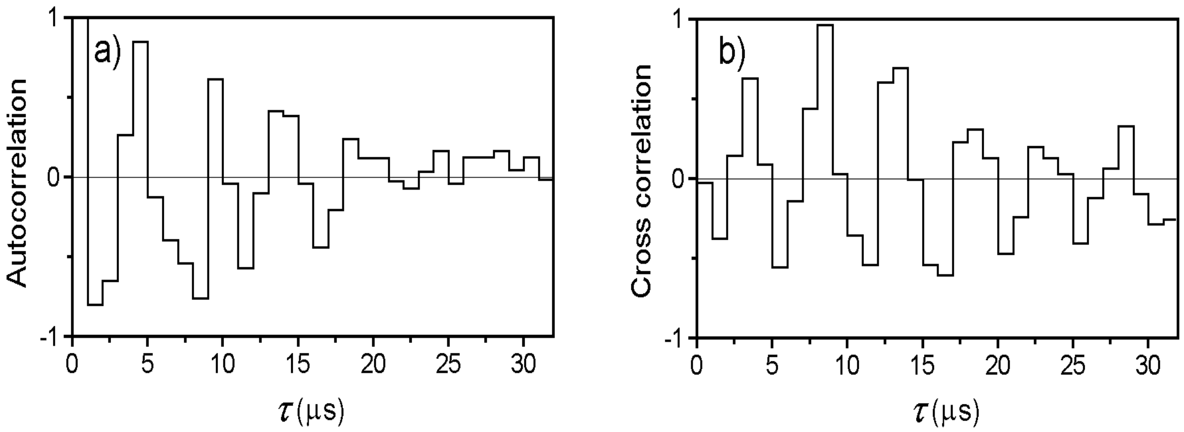

Figure 1: *Autocorrelation (a) and cross-correlation (b) functions of oscillations*

temperature $T_i \simeq 2$ eV, presence of double-charged ions $\simeq 10\%$ (on discharge axis). The oscillation spectrum was similar to one shown in fig. 3 of ref. [8].

All measurements described below were made at small excesses of the current threshold, when the spectrum consisted of two first peaks. Since the oscillations have a spatial mode structure [8], it was possible to adjust the optical parts so that to measure only the lowest mode with the peak at 205 kHz and a width $\simeq$ 80 kHz. A signal from a photomultiplier, which acted as a photon counter, entered on the correlator. The correlator with the maximum clock frequency of 160 MHz [9] measured autocorrelation and cross-correlation (case of two windows) functions of the electric signals. The measuring scheme used did not contain any frequency filters which could limit correlator pass-band. The sampling time was established to $\tau_0 = 0.5 \div 1.0$ μs. Registered emission was determined mainly by the blue-green lines of excited argon ions with a density n_i^*. Under small exceedings of the current threshold, the ground-state ion density contains a small time-oscillated component $n_i(t) = n_i + \tilde{n}_i(t)$. In the experimental conditions $n_i^*(t) \propto n_i(t)$, at least for frequencies $f < \beta_i$ (where β_i is the ionization rate) [6]. The cross-correlation function measured at points z' and $z' + d$ on the discharge axis ($z > 0$) is given by

$$G(\tau) = n_i^2 + \lim_{T\to\infty} \frac{1}{T} \int\limits_0^T \tilde{n}_i^*(t, z') \cdot \tilde{n}_i^*(t+\tau, z'+d)dt, \tag{1}$$

where τ is the delay time and T is the averaging time. The normalized autocorrelation and cross-correlation functions of oscillations obtained at the discharge current of $I = 320$ A are shown in fig. 1. The time shift $\Delta\tau$ of the cross correlation with respect to the autocorrelation testifies that the oscillations axially propagated towards anode. The group velocity is defined as $\Delta\tau/d$ and was equal to $(1.23 \pm 0.25) \cdot 10^6$ cm/s in measurements at different values of τ_0 and I. The dispersion relation of oscillations was found using the spectral analysis of measured cross-correlation functions [10]. The first-order time correlation function and power spectrum of a stationary random process are connected by Fourier transform under the Wiener-Khinchine theorem. The cross spectrum $S(\omega)$ of $\tilde{n}_i(t, z')$ and $\tilde{n}_i(t, z'+d)$ is determined by Fourier transform of equation (1). The

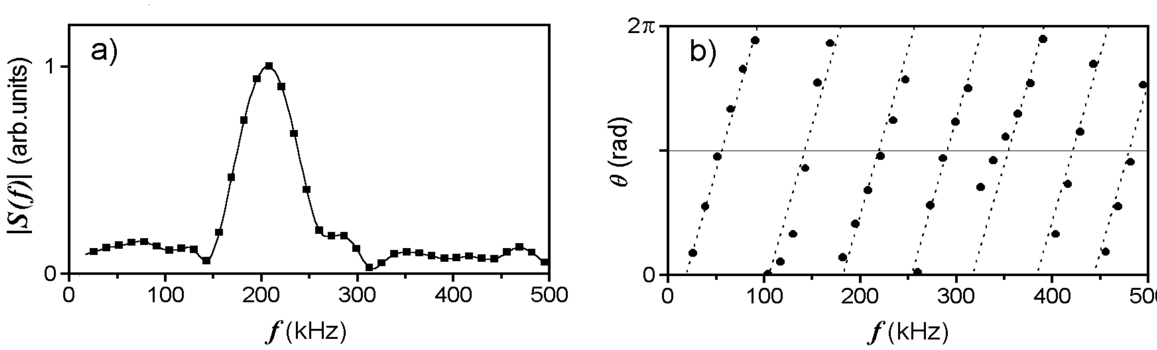

Figure 2: *Module (a) and phase angle (b) of the cross spectrum S(f)*

dependence $\tilde{n}_i(t,z)$ may be represented by a wave packet, travelling along the discharge axis

$$\tilde{n}_i(t,z) = \int_0^\infty \phi(\omega) exp[ik_z(\omega)z - i\omega t]d\omega, \tag{2}$$

where $\phi(\omega)$ is the smoothly varying function and $k_z(\omega)$ is the relation between longitudinal component of the wave vector $\vec{k}$ and frequency. The phase angle $\theta(\omega)$ of the complex function $S(\omega)$ (see fig. 2) is related to propagation of wave packet from z' to $z' + d$.

The local dispersion relation for equation (2) takes the form $k_z(\omega) = \theta(\omega)/d$, where $\theta(\omega)$ is defined experimentally. The obtained dispersion curve for the lowest mode of oscillations is plotted in fig. 3.

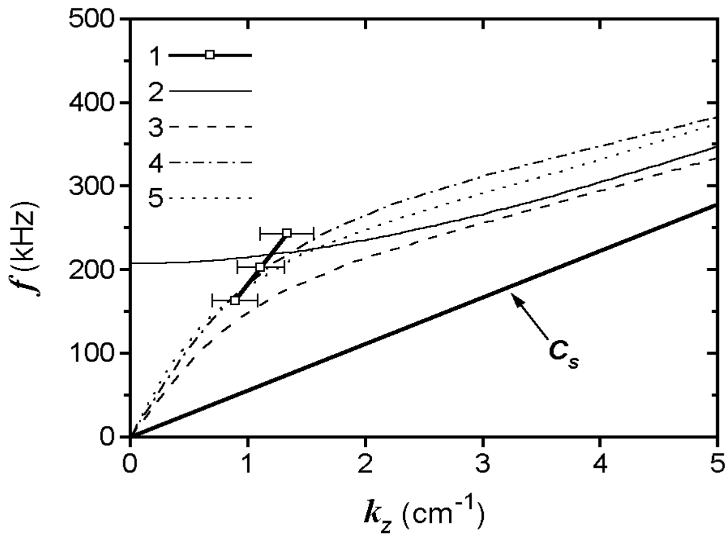

Figure 3: *Dependence of oscillation frequency f versus k_z. 1 — Experimental points of the present study; the calculated curves: 2 — mode (1,1) [8], 3 and 4 — modes (0,1) and (1,2) [12], 5 — mode (0,1) [13], mode numbers are indicated in notations of the corresponding references*

The phase velocity was found to be $(1.15 \pm 0.15) \cdot 10^6$ cm/s, that is considerably larger than $c_s = \sqrt{T_e/M_i} = 3.5 \cdot 10^5$ cm/s (M_i — ion mass). To take T_i into consideration in our conditions can increase c_s no more, than 1.5 times. The group velocity was found as $d\omega/dk = (1.25 \pm 0.28) \cdot 10^6$ cm/s. Measurements of spatial radial structure of the oscillations [11] have shown that $k_z \simeq |\vec{k}| = 2\pi/\lambda$ ($\lambda-$ oscillation wavelength) in present conditions.

3 Discussion

Woods was among the first to deduce the dispersion relation for ion-acoustic waves in a bounded low-pressure positive column using the hydrodynamic approximation in view of ion loss to the walls — see [12], where the lowest-order radial and azimuthal modes were obtained. In ref. [13] the dispersion relation was modified taking into account the collisions between particles in a volume, drift of particles and influence of an external longitudinal magnetic field. The results of both these papers have negligible differences in the low-frequency region (see fig. 3). The authors of [12, 13] do not explain the appearance of limitations on low frequencies which are observed in experiments. The approximate model [8] explains these frequencies. Boundary conditions for the frequency cut-off follows also from the "plasma in a waveguide" consideration [14, 15].

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