

Single and multiple helicity states in the Reversed Field Pinch

S. Cappello¹, F. D'Angelo¹, D.F. Escande^{1,2}, R. Paccagnella¹, D. Bénisti¹

¹Consorzio RFX, Corso Stati Uniti 4, 35127 Padova, Italy

²UMR 6633, case 321, Av. Escadrille Normandie-Niemen, F-13397 Marseille, France

Abstract

This work deals with the single helicity (SH) ohmic states of the RFP in the framework of the resistive MHD model in cylindrical geometry with and without pressure, and with their connection with the multiple helicity (MH) states. The related magnetic diffusion properties are also studied.

1. Calculation of single helicity ohmic states

The helically symmetric ohmic equilibria are calculated starting from a Grad-Shafranov equation (GSE) for the helical flux function χ (see for example [1]):

$$\Delta_h \chi = \frac{1}{f} \frac{\partial}{\partial r} \left(f \frac{\partial \chi}{\partial r} \right) + \frac{1}{rf} \frac{\partial^2 \chi}{\partial u^2} = \left(c - \frac{dg}{d\chi} \right) g(\chi) - \frac{dp}{d\chi} (m^2 + k^2 r^2)$$

where r is the radial coordinate, $u = m\theta + kz$ is the helical angle, $f = r/(m^2 + k^2 r^2)$, $c = 2km/(m^2 + k^2 r^2)$, $p(\chi)$ is the pressure profile and $\lambda(\chi) = dg/d\chi$ is the normalised parallel component of the current density. In addition, the helical equilibrium is required also to satisfy Ohm's law:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{j}$$

which implies [2]:

$$E_o \langle B_z \rangle_\chi = \eta (\chi) \lambda(\chi) \langle B^2 \rangle_\chi$$

(where $\langle \rangle_\chi$ means flux surface average). This defines η , and, for finite η values, imposes the constraint (OLC) that $\langle B_z \rangle_\chi$ and $\lambda(\chi)$ must reverse at the same flux surface $\chi = \chi^*$. Note that the satisfaction of the OLC is influenced implicitly by the pressure through the χ solution and the flux averages.

In order to solve the GSE we use two different techniques: a spectral approach based on a polynomial dependence of λ on χ , and a perturbative numerical approach applied to an axisymmetric configuration. The latter takes advantage of the fact that the marginal MHD Newcomb equation is obtained by linearising the helical Grad-Shafranov equation [3]. The two approaches are found to agree for moderate amplitudes of the helical part of χ . In the first approach we look for a solution with λ specified as a polynomial expansion ($\lambda(\chi) = \sum_{n=0}^N \lambda_n \chi^n$), and for the Fourier expansion of χ only the first two terms are self-consistently computed. The convergence of this scheme has been tested by checking the amplitude of the higher order harmonics. This yields two nonlinear second order coupled differential equations for the axisymmetric and the considered helical part of the helical flux.

The GSE was found to have two basins of solutions. In the first one the axisymmetric part of the flux χ_o has a local maximum in the plasma region (at the resonance radius), while χ_o is a monotonic function of r in the second one. We considered λ functions increasing with

χ which therefore have a maximum at the same position. The two basins correspond to a resonant or non-resonant helical term respectively. It is worth noting that λ 's with a non central maximum have been found in various RFP's in the MH state [4,5].

We present here an ohmic solution in the resonant case with pressure (in this case $N=3$ and $\lambda_0=3.17$, $\lambda_1=11.19$, $\lambda_2=-145.46$, $\lambda_3=-508.09$, $\chi_0(0) = 0$, $\chi_1'(0) = 0.067$). **Figure 1a** displays the normalised axisymmetric B_z , λ and p (p amplified by a factor 30) as a function of r for $\Theta=2$, $F=-0.13$ and a volume averaged $\beta=0.17$ where Θ and F are the usual Pinch and Reversal parameters for RFP configurations. The plasma radius a is defined as the value where χ_1 vanishes. For the pressure we took a linear dependence on the helical flux function, $p(\chi) = p_o + p_1 \chi$, where $p_o = -p_1 \chi_o(a)$ since p must vanish at the plasma boundary.

Figure 1b shows the poloidal contour plots of the helical flux function $\chi(r,u) = \chi_o(r) + \chi_1(r) \cos(u)$: one of them is dashed and refers to the surface χ^* defined by the OLC constraint.

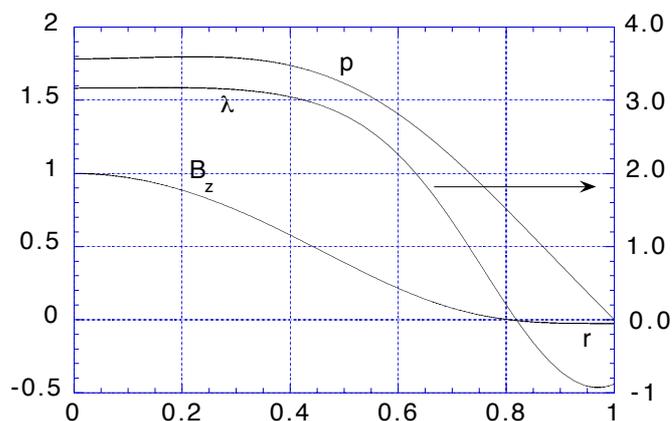


Fig 1a

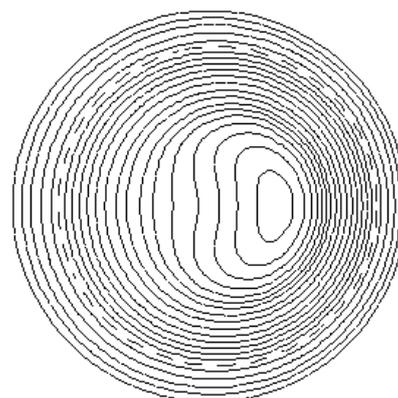


Fig.1b

2. Single and multiple helicity states revisited in 2D and 3D dynamical simulations: bifurcations and related magnetic diffusion properties.

RFP dynamics within helical symmetry has been extensively explored in early nonlinear MHD numerical studies [2,6,7,8,9,10]. As a function of the various parameters defining the plasma, these simulations displayed equilibria with and without a magnetic separatrix, which is a signature of a bifurcation in the magnetic topology. A solution without a separatrix is shown in **Fig.2**, which displays the SH equilibrium computed in one of our 2D simulations where $\Theta=1.9$, the Lundquist number $S=3 \times 10^3$, $m/n=1/-12$, and the Prandtl number $P=2/3$ (ratio of resistive to viscous decay times).

Full 3D dynamics, revealed the existence of a bifurcation between SH and MH states [2,11,12,13], which, in particular, can be controlled by the Prandtl number. These studies had a Lundquist number $S \approx 1 \div 3 \times 10^3$. We recently checked the occurrence of this bifurcation at a more realistic value of this number: $S=3.3 \times 10^4$ (present day experiments work at $S \approx 10^5 \div 10^7$ with $P^{\parallel} \approx 10^6$ and $P^{\perp} \approx 1$ classical values). Simulations at $P=1, 5, 20, 100$, and 1000 were performed, starting from MH initial conditions. They show that a SH state settles spontaneously for $P=1000$ (**Fig.3a**: $m=1$ magnetic mode energy evolution - SH: $m/n = 1/-$

11). This happens already at $P=10$, for S ten times smaller[11,12]. This shows that the bifurcation point is quite sensitive to the Lundquist number.

Two basins of attraction were recovered for a smaller P . Indeed, when starting a new simulation at $P=100$ from the SH state obtained at $P=1000$, the system settles in a quasi SH (QSH) regime, where two new different (1/-6,1/-16) helicities saturate with an amplitude two orders of magnitude below the dominant one (see **Fig.3b**).

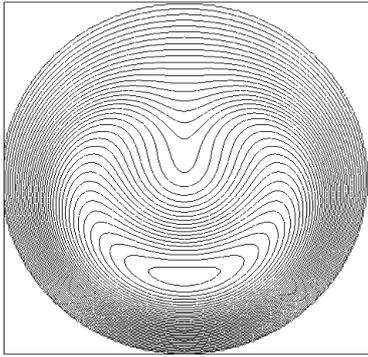


Fig. 2

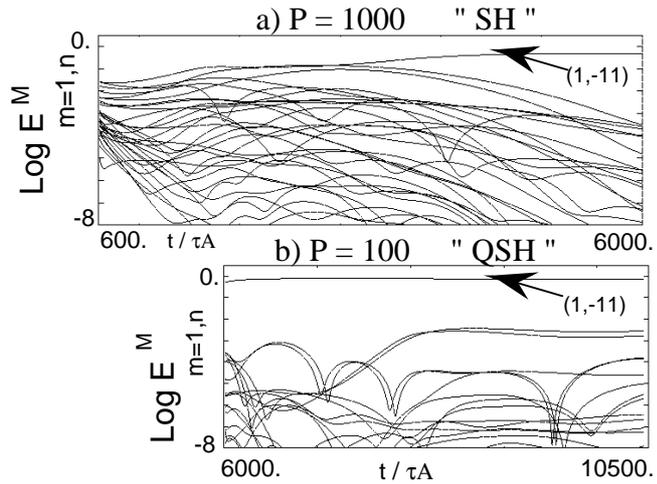


Fig. 3 a,b

Also, within full 3D dynamics, some sensitivity to the Θ parameter is found by comparing the evolution of the spectra for $\Theta=1.9$ and $\Theta=1.5$ ($P=1$, $S=3.3 \times 10^4$) [14], as shown in **Fig.4a,b**. At high Θ there is more energy separation, on average, between one dominant mode and the other modes. In addition it is possible to observe spontaneous transient QSH regimes to occur especially in high Θ simulations. In **Fig.4a** one of the most interesting time intervals, $t/\tau_A=1200-1600$, is highlighted: in this case the dominant mode $(m,n)=(1,-9)$ is by a factor of four larger than the second largest mode (1,-13), similarly to findings reported in different experiments, (both for amplitude and duration) [15,16,17,18].

Finally, we discuss the magnetic diffusion properties of the above discussed configurations that is related to the bifurcation of a magnetic separatrix. We use the magnetic spectrum corresponding to the state in **Fig.3a** to perform a Poincaré surface of section analysis. There is a dominant helicity with some remnant of the 3D spectrum. When setting to zero the other helicities, the magnetic pattern has no separatrix like cases shown in **Figs.1b,2**. Since chaos always originates in the vicinity of separatrices, we expect it to be of more limited extent in this case. Indeed **Fig.5a** shows the Poincaré plot of the full configuration where the stochastic region is confined into a bean-shaped internal region. Conversely, we expect, when reintroducing the separatrix for the dominant helicity, the stochasticity to spread over the volume. This is shown to happen in **Fig.5b**, which is obtained by reducing the amplitude of the dominant helicity by a factor of ten. Note that, contrary to the traditional wisdom, the fully stochastic configuration, **Fig.5b**, is therefore obtained with a fluctuating magnetic amplitude which is lower (by roughly a factor ten) than in the case of the partially stochastic configuration **Fig.5a**.

Therefore we can conclude that experimental configurations where the dominant helicity alone yields magnetic surfaces without a separatrix are good candidates for explaining the

robustness of magnetic islands observed by SXR measurements [19] in situations where the amplitude of perturbing helicities is not very small.

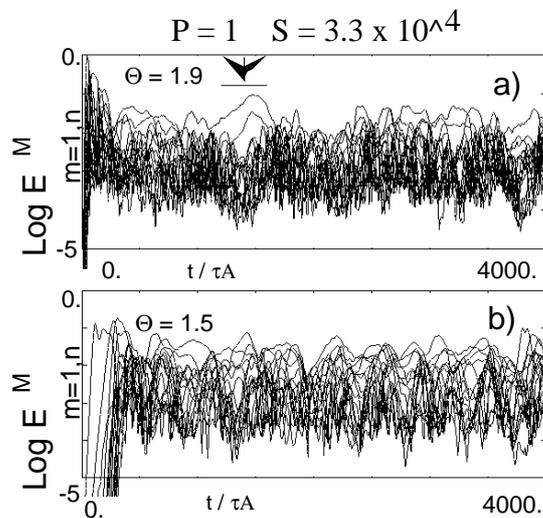


Fig. 4a,b

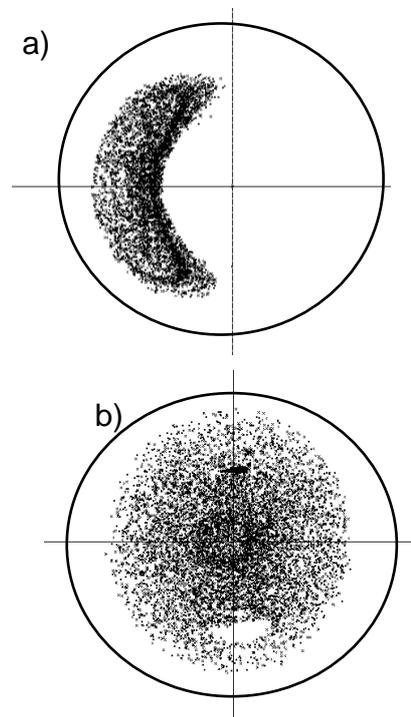


Fig. 5a,b

Acknowledgement.

The authors would like to thank the effort of Chiara Marchetto for the Poincaré surface of section analysis.

References.

- [1] BISKAMP, D., Nonlinear Magnetohydrodynamics, Cambridge University Press, Cambridge, Great Britain, 1993
- [2] FINN, J.M., NEBEL, R.A., BATHKE, C., Phys.Fluids B4 (1992) 1262.
- [3] MILLER G., Phys. Fluids B1 (1989) 384.
- [4] ANTONI V. et al., Proc. of Work. on Mirror-Based and Field Reversed Approaches to Fusion, Varenna Italy, (1983) 107.
- [5] NORDLUND P., Phys. Plasmas 1 (1994) 2945.
- [6] CARAMANA, E.J., NEBEL, R.A., SCHNACK, D.D., Phys. Fluids 26 (1983) 1305
- [7] SCHNACK, D.D., CARAMANA, E.J., NEBEL, R.A., Phys. Fluids 28 (1985) 321
- [8] HOLMES J.A., CARRERAS B.A., HENDER T.C., et al., Phys Fluids 28 (1985) 261
- [9] KUSANO K., SATO T., Nucl. Fusion 26 (1986) 1051.
- [10] MONTGOMERY, D., PHILLIPS, L., THEOBALD, L., Phys. Rev. A 40 (1989) 1515
- [11] CAPPELLO, S., PACCAGNELLA, R., Proc. " Theory of Fusion Plasmas ", Varenna Italy, J.Vaclavik,F.Troyon and E.Sindoni SIF Bologna, (1990) 595
- [12] CAPPELLO S., PACCAGNELLA, R., Phys. Fluids B 4 (1992) 611
- [13] CAPPELLO S., BISKAMP D., Nuclear Fusion , 36 (1996) 571
- [14] CAPPELLO S., BISKAMP D., Proc.International Conference on Plasma Physics (1996) - Nagoya, Ed.H.Sugai, T. Hayashi, Vol.1 p.854
- [15] BRUNSELL, P.R., YAGI, Y., HIRANO, Y.,et al., T., Phys. Fluids B5 (1993) 885
- [16] NORDLUND, P., MAZUR, S., Phys. Plasmas 1 (1994) 4032
- [17] HIRANO, Y., MAEJIMA, Y., SHIMADA, T., et al., Y., Nucl. Fus. 36 (1996) 721
- [18] MARTIN, P., Plasma Physics and Controlled Fus. 41 (1999) A247
- [19] MURARI, A., et al, "Tomographic imaging of RFX plasmas in various confinement regimes", This conference.