

Plasma cross-B diffusion and its measurement by collective scattering

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1 Introduction

Collective scattering (CS) is a common diagnostics of plasma turbulence that provides informations on plasma irregularities and motions that are especially valuable for magnetized ionosphere as well as fusion plasmas. Plasma cross-B diffusion coefficients can be readily obtained from the CS signal spectrum analysis³. These informations from the CS signal however are not only constructed from macroscopic plasma parameters but from the particle distribution analysis at a given spatial scale. Plasma turbulence is often found to extend from very large scale, that of the plasma size, to small scale of the order of the ion Larmor radius. In this later case, the CS signal observed at this scale should be expected to be also shaped by small scale particle trajectories characteristics.

This analysis is the objective of the present contribution. Two different results will be presented. One is a theoretical examination of the CS physical process and a calculation of the magnetized, collisional charged particle dynamics contribution to the CS signal. The second is a series of CS measurements in a magnetized toroidal plasma. These two results will be compared and discussed.

2 Mesoscopic and microscopic contribution to the scattering signal

The collectively scattered electric field is a made of a sum over the individual field scattered from each elementary particle. For fusion plasma at the centimeter scale, the elementary building blocks are the dressed ions. The total scattered E-field is obtained as the sum over all observed particles

$$\vec{E}_s(R, t) = \frac{r_0}{R} \vec{E}_i(0, t) \sum_{j=1}^{j=N} e^{i\vec{k} \cdot \vec{r}_j(t)} \quad (1)$$

where $\vec{E}_i(0, t)$ is the incident field complex amplitude at a reference origin near to the scatterer position \vec{r}_j , r_0 is the classical Thomson radius, R is the distance to the observer, and the scattering analyzing wave-vector \vec{k} is the difference between incident \vec{k}_i

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and scattered \vec{k}_s wave-vectors

$$\vec{k} = \vec{k}_s - \vec{k}_i \quad (2)$$

To form the field at time $(t + \tau)$, the particle trajectories are split in two terms, the mean fluid displacement $\vec{\Delta}(\tau)$ (defined as the mean displacement over a small cloud of neighbouring particles) and the additional individual particle displacement $\vec{\delta}_j(\tau)$ in the local "cloud" plasma frame

$$\vec{r}_j(t + \tau) = \vec{r}_j(t) + \vec{\Delta}(\vec{r}_j(t), \tau) + \vec{\delta}_j(t, \tau) \quad (3)$$

The product of this field times the initial field at time t provides the scattered field time correlation function. If statistical independence is assumed between density, mesoscopic displacement and also with microscopic displacements,

$$C(\tau) \cong \left| \frac{r_0}{R} \vec{E}_i(0, t) \right|^2 NS(\vec{k}) \langle e^{-i\vec{k} \cdot \vec{\Delta}(\tau)} \rangle \langle e^{-i\vec{k} \cdot \vec{\delta}_\tau} \rangle \quad (4)$$

where $S(\vec{k})$ is the "form factor". The time correlation function is thus made of the product of two different "statistical characteristics". One corresponds to the mean, "mesoscopic scale" motion along \vec{k} , $\Delta_{\vec{k}, \tau}$, and the second one to the microscopic motion along \vec{k} , $\delta_{\vec{k}, \tau}$. If these displacements are gaussian random processes,

$$\langle e^{-i\vec{k} \cdot \vec{\Delta}(\tau)} \rangle = e^{-\frac{1}{2}k^2 \langle \Delta_{\vec{k}, \tau}^2 \rangle} \text{ and } \langle e^{-i\vec{k} \cdot \vec{\delta}_\tau} \rangle = e^{-\frac{1}{2}k^2 \langle \delta_{\vec{k}, \tau}^2 \rangle} \quad (5)$$

The mesoscopic (turbulent motion) position random mean square can be described by a Ornstein function ⁴

$$\langle \Delta_{\vec{k}, t}^2 \rangle = 2D_{meso}\tau_c \left(\frac{t}{\tau_c} - 1 + e^{-\frac{t}{\tau_c}} \right) \quad (6)$$

where D_{meso} and τ_c are the turbulent fluid diffusion coefficient and correlation time resp. The long time displacement behaviour is that of random motion and consequently that of the "characteristic" is a decreasing exponential at the rate $k^2 D_{meso}$.

3 Magnetized ion motion and the scattering signal

The microscopic motion is the cyclotron motion at angular frequency ω , with random ion-neutral collisions at frequency ν . Using an appropriate microscopic, individual particle model, the microscopic diffusion can be calculated⁵. When normalized to the ion cyclotron radius ρ the microscopic mean square random motion (diffusion) across the magnetic field is found as

$$\frac{\langle \delta_{\vec{k}, t}^2 \rangle}{\rho^2} = 2 \frac{\frac{\omega^2}{\nu^2}}{\left(1 + \frac{\omega^2}{\nu^2}\right)^2} \left\{ \nu t - 1 + (1 + \nu t) \frac{\omega^2}{\nu^2} + e^{-\nu t} \left[\left(1 - \frac{\omega^2}{\nu^2}\right) \cos(\omega t) - 2 \frac{\omega}{\nu} \sin(\omega t) \right] \right\} \quad (7)$$

⁴see e.g. S.Chandrasekhar, Reviews of Modern Physics, Vol.15 pp.1-89, 1943

⁵B.P.Tomchuk, PhD thesis, Ecole Polytechnique, 1999

Variation of this mean squared normalized displacement as a function of time (normalized with the cyclotron frequency) (ωt) and of the collision rate (ν/ω) is shown in Figure 1. In the collisionless case, the rms displacement is seen to oscillate at the cyclotron period. This modulation is kept as long as the collision rate is small. For large collision frequency instead, the rms displacement behavior is that of Brownian motion, linearly increasing with time. The microscopic factor in the time correlation function is obtained from Eqs.5 and 7. Modulation at the cyclotron frequency will be especially sensible for large k such as $k\rho > 1$.

4 Collective scattering observation and plasma cross-B diffusion

The collective scattering process analysis was completed with a specific experiment led with an infrared laser scattering device set on a toroidal magnetically confined discharge plasma. The scattering device is made a CW, one watt, CO_2 laser source with heterodyne detection of the forward scattered light. The optical setup is tuned to observe fluctuation wavelength from 2.5 to 7 mm. The "Torix" plasma device is made of a toroidal vacuum chamber of 0.2 m poloidal and 0.6 m toroidal diameters. The toroidal DC magnetic field can be changed from 0.11 to 0.4 Tesla. Argon gas is introduced at pressure of 0.7 milli-Torr. Discharge is maintained by a hot filament biased at -100 Volts with respect to walls, emitting 0.1 Ampere D.C. The plasma density is in the range of $10^{17}m^{-3}$, and is continuously maintained. The scattering device is observing the plasma column along a vertical chord, centered in the poloidal plane. The analyzing wavevector \vec{k} is set perpendicular to the \vec{B} field (along which plasma fluctuations are aligned). The detected heterodyne signal is digitally recorded as a sequence of 500,000 datas sampled at a 1 MHz rate. On the same plasma, at a magnetic (center line) field of 0.28 T, recordings of the scattered signal each corresponding to a different wavelength have been obtained and analyzed. Their time correlation functions are shown in Figure 2. They are seen to fit with decaying exponentials, the larger is k the quicker is the decay. This is expected from Eq.5 and 6 from which a diffusion coefficient can be obtained. This diffusion coefficient is shown in Fig.3 for different k values. Since Fig.3 shows this coefficient does not depend on k , this result validates the signal interpretation along Eq.6.

5 Discussion

The ion (charge exchange) collision frequency to cyclotron (angular) frequency ratio is very small (5.10^{-3}) and the ion cyclotron radius is large enough for ($0.5 < k\rho < 1$). The microscopic part (Eqs.5 and 7) should be modulating the signal at the ion cyclotron period of 10 microseconds. This is not seen however in Fig.2, where decay times are of the order or shorter of this cyclotron period. The decay time should thus be attributed to "mesoscale" turbulent motion as described by Eq.6, from which a diffusion coefficient is obtained at a value of $0.04m^2/sec$. As a comparison, the Bohm diffusion coefficient in this experiment is $D_{Bohm} = 0.045m^2/sec$, where $D_{Bohm} = \frac{1}{2}\rho^2\omega$. This is significantly close to the above value : it means the (turbulent, equivalent) collision frequency is close to the ion cyclotron angular frequency.

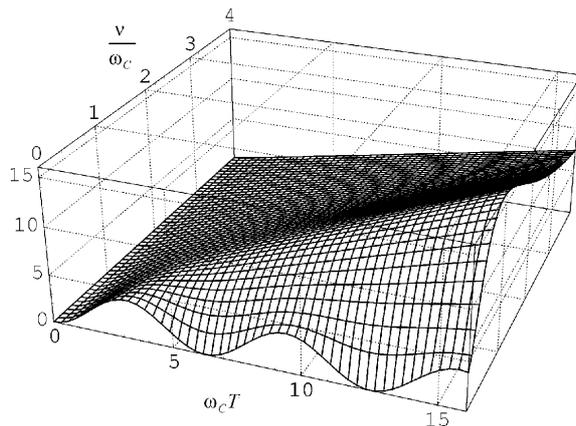


Figure 1: $\frac{\langle \delta_{i,x}^2 \rangle}{\rho^2}$ - Cross-B diffusive motion as a function of time (normalized with the cyclotron frequency) and of the collision rate (ν/ω).

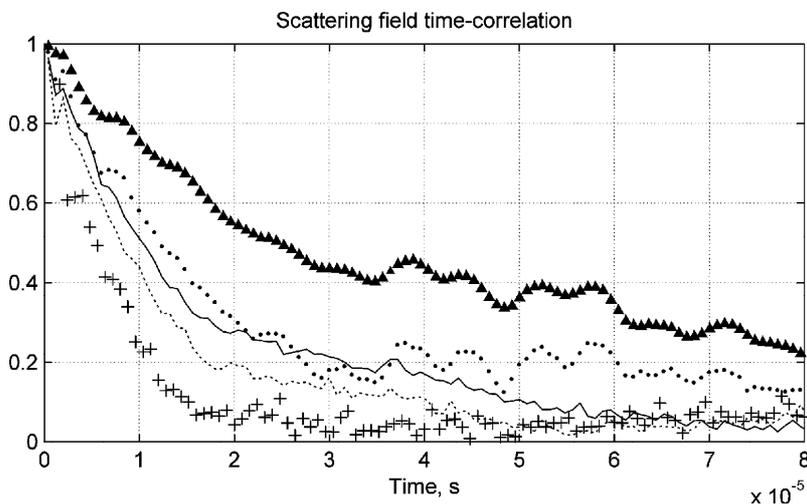


Figure 2: Scattering field time-correlation for different wavevectors (triangle: $1.01 \times 10^3 \text{m}^{-1}$, dot: $1.19 \times 10^3 \text{m}^{-1}$, solid line: $1.36 \times 10^3 \text{m}^{-1}$, dashed line: $1.48 \times 10^3 \text{m}^{-1}$, plus: $1.72 \times 10^3 \text{m}^{-1}$)

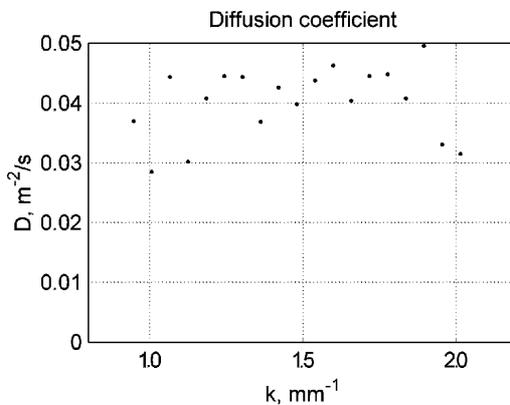


Figure 3: Diffusion coefficient (averaged: $4.01 \times 10^{-2} \text{m}^2/\text{s}$)