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Ion Turbulence in Tokamak Plasmas

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A new approach, different from P.I.C. or fluid simulation, based on a direct numerical solution of the Vlasov equation, is proposed to study ion turbulence in tokamaks. This so-called semi-lagrangian method[1], allows good phase space and time resolution together with low noise level. It is inconditionnally stable and free from CFL conditions. The present work focuses on trapped ion driven modes which are expected to produce a significant transport because of their large radial scales. They are driven through the resonant interaction between the wave and the trapped ion precession motion and a kinetic treatment must be done. Averaging the kinetic equation over both cyclotron and bounce motions allows to reduce the number of independent variables in phase space. The distribution function averaged over the fast motion depends only on 2 variables (precession angle ϕ_3 and poloidal flux ψ) and the energy. The final problem is therefore 2D, parametrized by the particle energy, allowing for an efficient parallelization of the code.

I Equations of the model

We assume that the plasma pressure is small, so that the analysis can be restricted to electrostatic modes, whose self-consistency is ensured by the quasi-electroneutrality constraint. Electron density fluctuations are assumed here to be adiabatic, whereas ion density fluctuations have to be calculated with a kinetic equation (passing ions are neglected).

The motion of a charged particle in a tokamak equilibrium field is separable between the motion of the guiding center and the ion cyclotron motion and it is integrable and quasi-periodic. Thus, it is convenient to use a set of angular and action variables[2] to describe this motion; the ion response is given by a gyrokinetic equation, gyroaveraged over both the cyclotron motion and the bounce motion. Consequently we have reduced the 6D Vlasov equation into a set of 2D Vlasov equations acting on a 2D phase space ψ, ϕ_3 parametrized by the energy E. The self-consistency constraint $\tilde{n}_e = \tilde{n}_i$ is expressed by using the adiabatic electron response $\tilde{n}_e / n_{eq} = e\tilde{U} / T_{eq}$. With little algebra[2], one obtains the perturbed density together with the self-consistency constraint

$$\frac{2}{a_{tr}}\frac{e}{T_{eq}}\tilde{U} = \int_1^{1-2\varepsilon} \frac{d\lambda}{4a_{tr}\overline{\omega}_b} \int_0^{+\infty} \frac{2}{\pi^{1/2}} E^{1/2} \; dE\left(J_0 . f_E\right) - 1$$

where $a_{tr} = \pi / \left(2 (2\varepsilon)^{1/2} \right)$ is the fraction of trapped particle, ε the aspect ratio and J_0 the gyroaverage operator. Morover, f_E, the distribution function associated to a given energy E, is the solution of the gyrokinetic equation

$$\frac{\partial}{\partial t} f_E + \left(\omega_d E + \frac{\partial}{\partial \psi} \left(J_0 \tilde{U} \right) \right) \frac{\partial}{\partial \phi_3} f_E - \frac{\partial}{\partial \phi_3} \left(J_0 \tilde{U} \right) \frac{\partial}{\partial \psi} f_E = 0 \qquad (1)$$

The quasi-electroneutrality constraint[3], with appropriated conditions, is reduced to

$$C\tilde{U}(\phi_3, \psi, t) = \frac{2}{\sqrt{\pi}} \int_0^\infty \sqrt{E} \; J_0 \; f_E(\phi_3, \psi, t) dE - 1 \qquad (2)$$

II Stability analysis

The trapped ion driven instability is characterised by a temperature gradient threshold. This critical value is determined using the linear dispersion relation and looking for real frequency solutions. The dispersion equation is obtained by linearizing Eq.(1), assuming $f = f_0 + f_1$, $\tilde{U} = \tilde{U}_0 + \tilde{U}_1$ with a particular equilibrium solution which depends on the temperature gradient $\Delta\tau$. Assuming f_1 of the form $f_{1n}(\psi)\, e^{i(n\,\phi_3 - \omega t)}$, the quasi-neutrality equation (2) then reads

$$C\tilde{U}_{1n}(\psi) = \frac{2}{\sqrt{\pi}} \frac{\Delta\tau}{\omega_d} \int_0^\infty dE \frac{\sqrt{E}\, e^{-E} \left(E - \frac{3}{2} \right)}{E - \frac{\omega}{n\,\omega_d}} J_0^2 \; \tilde{U}_{1n} \qquad (3)$$

There are two possibilities.

On the one hand, a marginal steady solution corresponding to a real frequency ω can be found using $\omega / n = (3/2)\, \omega_d$. Eq.(3) then leads to a differential equation the solutions of which give the critical gradient $\Delta\tau = \Delta\tau_c$. On the other hand, introducing $\Delta\omega = (\omega / n) - (3/2)\omega_d$, one gets the dispersion relation

$$\frac{\Delta\omega}{\omega_d} \left[1 + \sqrt{\frac{3}{2} + \frac{\Delta\omega}{\omega_d}} \;\; Z\left(\sqrt{\frac{3}{2} + \frac{\Delta\omega}{\omega_d}} \right) \right] = \frac{(\Delta\tau - \Delta\tau_c)}{2\,\Delta\tau_c} \qquad (4)$$

where Z is the well-known plasma dispersion function.

III Numerical results: linear and saturation stage

The phase space (ϕ_3, ψ) is divided into 128*64 mesh points for the precession angle and the poloidal flux respectively, the normalized banana width δ_b is 10^{-2}, the level of the perturbation on the first mode is 10^{-4}, and 55 values of energy E are retained. In the linear stage, according to (4), it appears that the growth rate (imaginary part of $\Delta\omega$) depends both on the quantity $\Delta\tau - \Delta\tau_c$ and the precession pulsation. Figure 1 shows the growth rates of the first mode for differents values of $\Delta\tau - \Delta\tau_c$. A good agreement is found between the simulations and the theorical values given by the dispersion relation (4) which gives confidence in our code. Furthermore, the solution at the threshold has been tested succesfully.

Figure 2 shows the behaviour of electric potential fluctuations in phase space. Small structures appear as a consequence for the non linear mode coupling. These structures coalesce leading to a saturated state dominated by large scale structures (figure 3), where the energy is transfered to low wavenumbers. This phenomena has already been observed in previous studies[4,5]. Figure 4 shows the time average wavenumber spectrum of the electric potential fluctuations for two values of $\Delta\tau - \Delta\tau_c$. Excepting large k's (which may be affected by aliasing effects due to numerical cut off), the spectrum is close to a straight line. Its slope is approximatively equal to -3. This is similar to the spectra obtained in simulations of 2D fluid turbulence[6,7].

It is also interesting to compute the turbulent heat flux .This turbulent flux can be compared to a quasi-linear estimate[8]. For $\Delta\tau - \Delta\tau_c = 0.02$ and $\delta_b = 0.01$, we find $\chi = 6.10^{-2}\, m^2\, s^{-1}$. This weak value is not surprising since it corresponds to a low value of $\Delta\tau - \Delta\tau_c$, i.e. to a situation close to the threshold. It is expected to reach a value of order $1 m^2 s^{-1}$ for larger values of the gradient.

Figure 1

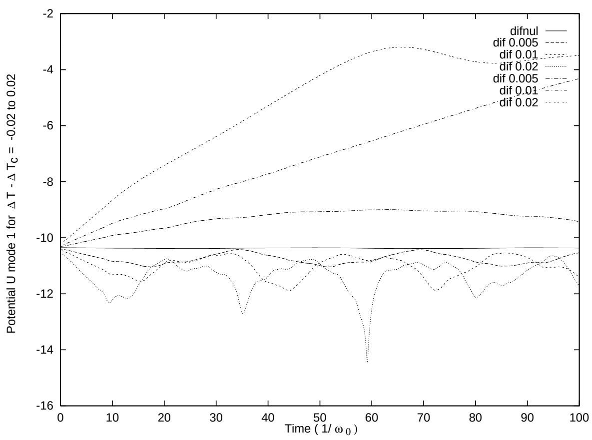


Figure 4

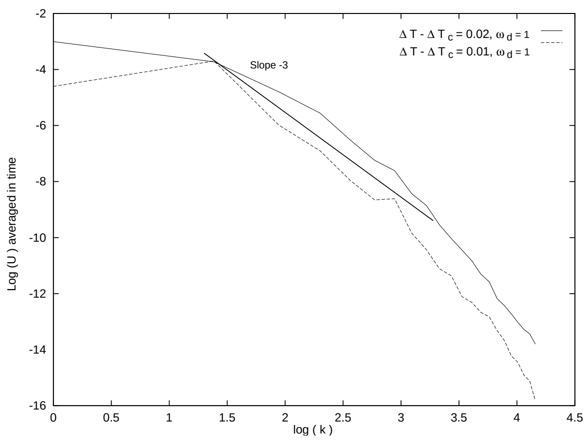


Figure 2

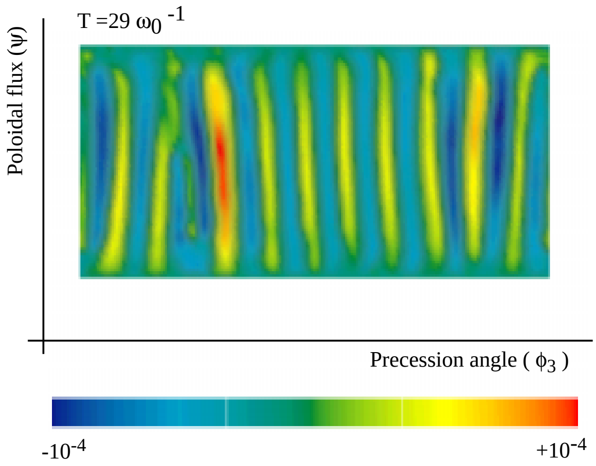


Figure 3

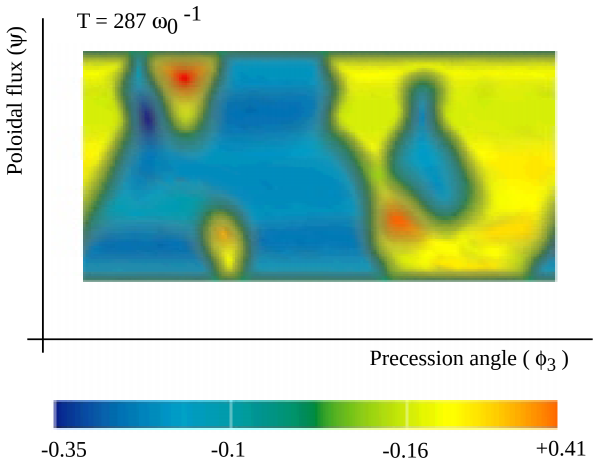
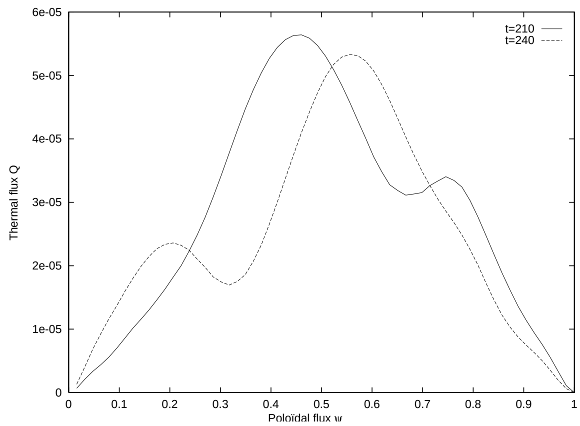


Figure 5



IVConclusion

Vlasov simulations have been performed and compared successfully with analytical predictions concerning both the instability threshold and the growth rate in the linear regime. In the non linear regime, after saturation, a turbulent spectrum is fully developed with a tendancy to condensate into large structures. Therefore, our code is a good candidate to study trapped ion turbulence driven by ion temperature gradient.

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