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PARALLEL DIELECTRIC PERMITTIVITY OF ELONGATED TOKAMAKS FOR THE TOROIDICITY-INDUCED ALFVÉN EIGENMODES

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The problems of plasma stability, heating and current drive in large-size tokamaks by using the radio-frequency waves should be analyzed by solving the two-dimensional Maxwell's equations with dielectric tensor, accounting the bounce resonant wave particle interactions there. In this paper, we analyze the parallel dielectric permittivity of a collisionless toroidal plasma with elliptic magnetic surfaces in the frequency range of toroidicity-induced Alfvén eigenmodes[1] (TAEs). The concrete computer calculations are carried out for the JET-parameters. As is well known,[2,3] the electron Landau damping of TAEs is defined by the imaginary part of the parallel permittivity (or the real part of the wave conductivity). This dielectric characteristic can be evaluated by solving the drift-kinetic equations for trapped and untrapped particles/electrons in the concrete two-dimensional plasma model. To describe the tokamak with elliptic magnetic surfaces, we use the variables (r, θ') instead of the quasi-toroidal coordinates (ρ, θ):

$$r = \rho\sqrt{\frac{a^2}{b^2}\sin^2\theta + \cos^2\theta}\ , \qquad \theta' = \arctan\left(\frac{a}{b}\tan\theta\right)\ .$$

As a result, the modulus of an equilibrium magnetic field, $\mathbf{H}$, is

$$H = \sqrt{H_{\phi0}^2 + H_{\theta0}^2}\frac{\sqrt{1+\lambda\cos^2\theta'}}{1+\epsilon\cos\theta'} \approx \sqrt{H_{\phi0}^2 + H_{\theta0}^2}\left(1 - \epsilon\cos\theta' + \frac{\epsilon^2}{1+\epsilon}\cos^2\theta' + \frac{\lambda}{2}\cos^2\theta'\right),$$

where $H_{\phi0}$ and $H_{\theta0}$ are the toroidal and poloidal projections of $\mathbf{H}$ for a given magnetic surface, at the points $\theta = \pm\pi/2$. Here

$$\epsilon = \frac{r}{R}, \qquad \lambda = h_\theta^2\left(\frac{b^2}{a^2} - 1\right) = \frac{\epsilon^2}{q^2+\epsilon^2}\left(\frac{b^2}{a^2} - 1\right), \qquad q = \epsilon\frac{h_\phi}{h_\theta},$$
$$h_\theta = H_{\theta0}/\sqrt{H_{\phi0}^2 + H_{\theta0}^2}, \qquad h_\phi = H_{\phi0}/\sqrt{H_{\phi0}^2 + H_{\theta0}^2},$$

R is the major tokamak radius; b and a are the major and minor semiaxes of the external elliptic magnetic surface. In this plasma model, all magnetic surfaces are similar each other with the same elongation equal to b/a. The steady-state distribution function is given as maxwellian with the particle density N, temperature T, and mass M. The evaluation of the parallel (longitudinal) permittivity is presented in Ref. [4]. In contrast to Ref. [5] related to it, we describe the untrapped and trapped particles taking into account the quadratic corrections over the inverse tokamak aspect ratio, ϵ. These corrections allow us to describe more correctly the finite-ϵ effects in tokamaks with $a/R \sim 1/3$.

In a tokamak with elliptic magnetic surfaces, in the general case, there is one interesting feature. Together with the usual untrapped and t-trapped particles, two additional groups of d-trapped particles can appear at such magnetic surfaces where $\lambda > \epsilon$ (or $b/a >$

$\sqrt{1+\epsilon+q^2/\epsilon}$ in other terms). As easily to see, this criterion is not satisfied for JET-parameters, as well as for any tokamak with the small elongation $b/a < 2$ and $q > 1$. Therefore we present the contribution of untrapped and t-trapped electrons to the parallel permittivity in the case when the d-trapped particles are absent in the plasma.

The dielectric tensor elements can be derived after the Fourier expansions of the current density and electric field over the angle $\tilde{\theta} = \theta' + 0.125\lambda \sin 2\theta'$:

$$j_{\parallel}(\tilde{\theta})(1+\epsilon\cos\tilde{\theta} - \nu\cos^2\tilde{\theta}) = \sum_{m=-\infty}^{+\infty} j_{\parallel}^{(m)} \exp(im\tilde{\theta}), \qquad E_{\parallel}(\tilde{\theta}) = \sum_{m'=-\infty}^{+\infty} E_{\parallel}^{(m')} \exp(im'\tilde{\theta}).$$

As a result, the harmonics $j_{\parallel}^{(m)}$ and $E_{\parallel}^{(m')}$ are connected by the parallel permittivity elements:

$$\frac{4\pi i}{\omega} j_{\parallel}^{(m)} = \sum_{m'=-\infty}^{+\infty} \epsilon_{\parallel}^{m,m'} E_{\parallel}^{(m')} = \sum_{m'=-\infty}^{+\infty} \left(\epsilon_{\parallel,u}^{m,m'} + \epsilon_{\parallel,t}^{m,m'}\right) E_{\parallel}^{(m')},$$

where $\epsilon_{\parallel,u}^{m,m'}$ and $\epsilon_{\parallel,t}^{m,m'}$ are the separate contributions of untrapped and t-trapped particles, respectively, to the parallel permittivity, $\epsilon_{\parallel}^{m,m'} = \epsilon_{\parallel,u}^{m,m'} + \epsilon_{\parallel,t}^{m,m'}$, (see details in Refs.[4,5]).

The contribution of untrapped particles to the parallel (longitudinal) permittivity is

$$\epsilon_{\parallel,u}^{m,m'} = \frac{2\omega_{po}^2 r^2}{h_\theta^2 v_T^2 \pi^3} \sum_{p}^{\pm\infty} \int_0^1 \frac{(1+0.5\lambda)K(\kappa)\left[1+2u_p^2+2i\sqrt{\pi}u_p^3 W(u_p)\right] A_p^m A_{p+m-m'}^{m'}}{(m+nq+p)^2\sqrt{1-\mu(1-\nu)+\sqrt{\epsilon^2\mu^2+4\nu\mu(1-\mu)}}} \frac{d\mu}{d\kappa} d\kappa. \quad (1)$$

Here, we used the following definitions:

$$\mu(\kappa) = \frac{\kappa^2+\nu(8-8\kappa+\kappa^2)-(2-\kappa)\sqrt{\epsilon^2\kappa^2+16\nu^2(1-\kappa)}}{\kappa^2(1+\nu)^2-\epsilon^2(2-\kappa)^2+32\nu^2(1-\kappa)}, \qquad \omega_{po}^2 = \frac{4\pi N e^2}{M},$$

$$u_p = \frac{\omega r(1+0.25\lambda)\sqrt{2\kappa}K(\kappa)}{h_\theta|m+nq+p|v_T\pi\left[\epsilon^2\mu^2+4\nu\mu(1-\mu)\right]^{0.25}}, \qquad K(\kappa) = \int_0^{\pi/2} \frac{d\eta}{\sqrt{1-\kappa\sin^2\eta}},$$

$$A_p^m = \int_{-K(\kappa)}^{K(\kappa)} \exp\left[i\frac{p\pi w}{K(\kappa)} - 2i(m+nq)\chi_u(w) + 2inq\psi_u(w)\right] \frac{\sqrt{1+\beta}\,\mathrm{dn}(\kappa,w)}{1+\beta\,\mathrm{cn}^2(\kappa,w)} dw,$$

$$\chi_u(w) = \arctan\left(\frac{1}{\sqrt{1+\beta}}\frac{\mathrm{sn}(\kappa,w)}{\mathrm{cn}(\kappa,w)}\right) - \frac{\pi w}{2K(\kappa)}; \qquad \beta = \frac{4\nu}{\epsilon-2\nu+\sqrt{\epsilon^2+4\nu(1-\mu)/\mu}},$$

$$\psi_u(w) = \sqrt{1+\beta}\frac{\mathrm{sn}(\kappa,w)\,\mathrm{cn}(\kappa,w)}{1+\beta\,\mathrm{cn}^2(\kappa,w)}\left[\epsilon+\zeta\frac{(2+\beta)\mathrm{cn}^2(\kappa,w)-1}{1+\beta\,\mathrm{cn}^2(\kappa,w)}\right], \qquad v_T = \sqrt{\frac{2T}{M}},$$

where $\mathrm{sn}(\kappa,w)$, $\mathrm{cn}(\kappa,w)$, $\mathrm{dn}(\kappa,w)$ are the Jacobi elliptic functions.

The contribution of t-trapped particles to the parallel (longitudinal) permittivity is

$$\epsilon_{\parallel,t}^{m,m'} = \frac{\sqrt{8}\omega_{po}^2 r^2}{h_\theta^2 v_T^2 \pi^3}(1+0.5\lambda)\sum_{p=1}^{\infty}\int_0^1 \frac{K(\kappa)\left[1+2v_p^2+2i\sqrt{\pi}v_p^3 W(v_p)\right] B_p^m B_p^{m'}}{p^2\left[\epsilon^2\tau^2(\kappa)+4\nu\tau(\kappa)(1-\tau(\kappa))\right]^{1/4}}\frac{d\tau(\kappa)}{d\kappa}d\kappa, \quad (2)$$

$$\tau(\kappa) = \frac{1+\nu(8\kappa^2-8\kappa+1)-(2\kappa-1)\sqrt{\epsilon^2+16\nu^2\kappa(\kappa-1)}}{(1+\nu)^2-\epsilon^2(2\kappa-1)^2+32\nu^2\kappa(\kappa-1)},$$

$$B_p^m = \int_{-2K(\kappa)}^{2K(\kappa)} \exp\left[\frac{ip\pi w}{2K(\kappa)} - 2i(m+nq)\chi_t(w) + 2inq\psi_t(w)\right] \frac{\sqrt{\kappa(1+\delta)}\mathrm{cn}(\kappa,w)}{1+\delta\ \mathrm{dn}^2(\kappa,w)} d\ w,$$

$$\chi_t(w) = \arcsin\left(\sqrt{\frac{\kappa\ \mathrm{sn}^2(\kappa,w)}{1+\delta\ \mathrm{dn}^2(\kappa,w)}}\right), \qquad v_p = \frac{2\sqrt{2}\omega r(1+0.25\lambda)K(\kappa)}{h_\theta p v_T \pi\left[\epsilon^2\tau^2 + 4\nu\tau(1-\tau)\right]^{1/4}},$$

$$\psi_t(w) = \sqrt{\kappa(1+\delta)}\frac{\mathrm{sn}(\kappa,w)\ \mathrm{dn}(\kappa,w)}{1+\delta\ \mathrm{dn}^2(\kappa,w)}\left[\epsilon + \zeta\frac{(2+\delta)\mathrm{dn}^2(\kappa,w)-1}{1+\delta\ \mathrm{dn}^2(\kappa,w)}\right],$$

$$\delta = \frac{4\ \nu}{\epsilon - 2\nu + \sqrt{\epsilon^2 + 4\nu(1-\tau)/\tau}}, \qquad W(z) = \exp(-z^2)\left[1 + \frac{2i}{\sqrt{\pi}}\int_0^z \exp(t^2)dt\right].$$

The phase coefficients A_p^m and B_p^m do not depend on the wave frequency that allowed us to derive $\epsilon_{\parallel,u}^{m,m'}$ and $\epsilon_{\parallel,t}^{m,m'}$ by the summation of the bounce resonance terms including the plasma dispersion function $W(z)$. Note, the expressions (1),(2) have a natural limit to the corresponding results for plasmas with circular magnetic surfaces, if $b = a$.

As shown in Ref. [1], TAEs can be excited in toroidal plasmas by the energetic ions, e.g., by the fusion-born alpha particles. Of course, in the cold plasmas with the nonuniform safety factor $q = q(r)$, the MHD theory allows us to describe the strong coupling of two neighboring poloidal harmonics of the electromagnetic field with m and $m-1$ at such magnetic surfaces where $|m-1+nq(r)| = |m+nq(r)|$. The stabilization of TAEs, in the hot collisionless plasmas, is defined by their electron Landau damping. The corresponding analytical analysis is developed in Refs. [2,3] for tokamaks with circular magnetic surfaces, by solving the wave equations for two neighboring harmonics of an electric potential Φ_{m_o} and Φ_{m_o-1} : $\Phi = \sum_{m=m_o}^{m_o-1} \Phi_m \exp(-i\omega t + in\phi + im\theta)$. At the present time, there is a large progress[6,7] to study numerically the TAEs structure by using the PENN code. As some restriction, the bounce resonance effects are not included there. It should be noted, due to the magnetic field nonuniformity, the whole spectrum of E-field is present in the given m-harmonic of the current density components. This feature should be taken into account to predict the possible damping of TAEs in the hot plasmas. As a result, the dissipated power, $P = \mathrm{Re}\mathbf{E}\cdot\mathrm{Re}\mathbf{j}$, of TAEs (as well as any other eigenmodes) in tokamaks by the trapped/untrapped electrons can be estimated by

$$P = \frac{\omega}{8\pi}\sum_m\sum_{m'} \mathrm{Im}\epsilon_\parallel^{m,m'}\left[\mathrm{Re}E_\parallel^{(m)}\mathrm{Re}E_\parallel^{(m')} + \mathrm{Im}E_\parallel^{(m)}\mathrm{Im}E_\parallel^{(m')}\right], \qquad (3)$$

where $\mathrm{Im}\epsilon_\parallel^{m,m'} = \mathrm{Im}\epsilon_{\parallel,u}^{m,m'} + \mathrm{Im}\epsilon_{\parallel,t}^{m,m'}$. In the simplest case, describing the coupling of two harmonics with $m = m_o - 1, m_o$, the terms with $m' = m_o, m_o - 1$ should be also accounted in Eq. (3) in order to estimate the electron absorption of TAEs. Of course, for the one-mode (quasi-cylindrical) approximation, when $m = m' = m_o$, Eq. (3) is reduced to the well known expression $P = (\omega/8\pi)\mathrm{Im}\epsilon_\parallel^{m,m}|E_\parallel^{(m)}|^2$.

In this paper we do not solve the wave equations. Our purpose is to calculate numerically the parallel permittivity elements, Eqs. (1,2), suitable for elongated tokamaks with $b/a < 2$ and $q > 1$, and valid in the frequency range of TAEs and ellipticity-induced Alfvén eigenmodes.[8,9] The estimations of $\mathrm{Im}\epsilon_{\parallel,u}^{m,m'}$ and $\mathrm{Im}\epsilon_{\parallel,t}^{m,m'}$ are carried out for TAEs with mode numbers $n = 2$, $m_1 = -3, m_2 = -4$ and $f = \omega/2\pi = 165$ kHz, which can be excited in JET-plasma [$R = 3$ m, $b/a = 1.7$, $a/R = 0.36$, $H_{\phi 0} = 3.37$ T, $q = 1.23/(1-2r^2/3a^2)$, $N = 3\times 10^{19}(1-0.9r^2/a^2)^{0.5}\mathrm{m}^{-3}$, $T = 6(1-0.995r^2/a^2)^{0.7}$ keV] at the magnetic surfaces $r_x/a = 0.67$, where the parallel wave vectors of both modes are equal to each other: $|m_1 + nq(r_x)| = |m_2 + nq(r_x)|$. The eigenfrequency at this point is $f_{Ax} = C_A(r_x)/(4\pi Rq(r_x)) = 165$ kHz, so that $f = f_{Ax}$. The corresponding TAEs gap is $(1 \pm r_x/R)f_{Ax} = 145 \div 185$ kHz. The plots of Im $\epsilon_{\|,u}^{m,m}$ and Im $\epsilon_{\|,t}^{m,m}$ versus r/a are presented in Fig.1a and Fig.1b for $m = -3$ and $m = -4$, respectively. By the direct comparison with the local cylindrical approximation Im $\epsilon_{\|,c}^{m,m}$ we see that Im $\epsilon_{\|,u}^{m,m}$ + Im $\epsilon_{\|,t}^{m,m} \sim$ Im $\epsilon_{\|,c}^{m,m}$ for the given plasma-wave parameters. As to the non-diagonal elements, Im $\epsilon_{\|,u}^{m,m-1}$ = Im $\epsilon_{\|,u}^{m-1,m} <<$ Im $\epsilon_{\|,u}^{m,m}$ and Im $\epsilon_{\|,t}^{m,m-1}$ = Im $\epsilon_{\|,t}^{m-1,m} <<$ Im $\epsilon_{\|,t}^{m,m}$ in the range of $r = r_x$.

It should be noted that the excitation of TAEs with $n = 2$, $m = -2, -3$ and the closed JET-parameters is described in Ref. [6]. There was shown that $f = 200$ kHz is the resonant frequency for TAEs, which can be excited in the region of $r/a = 0.15$.

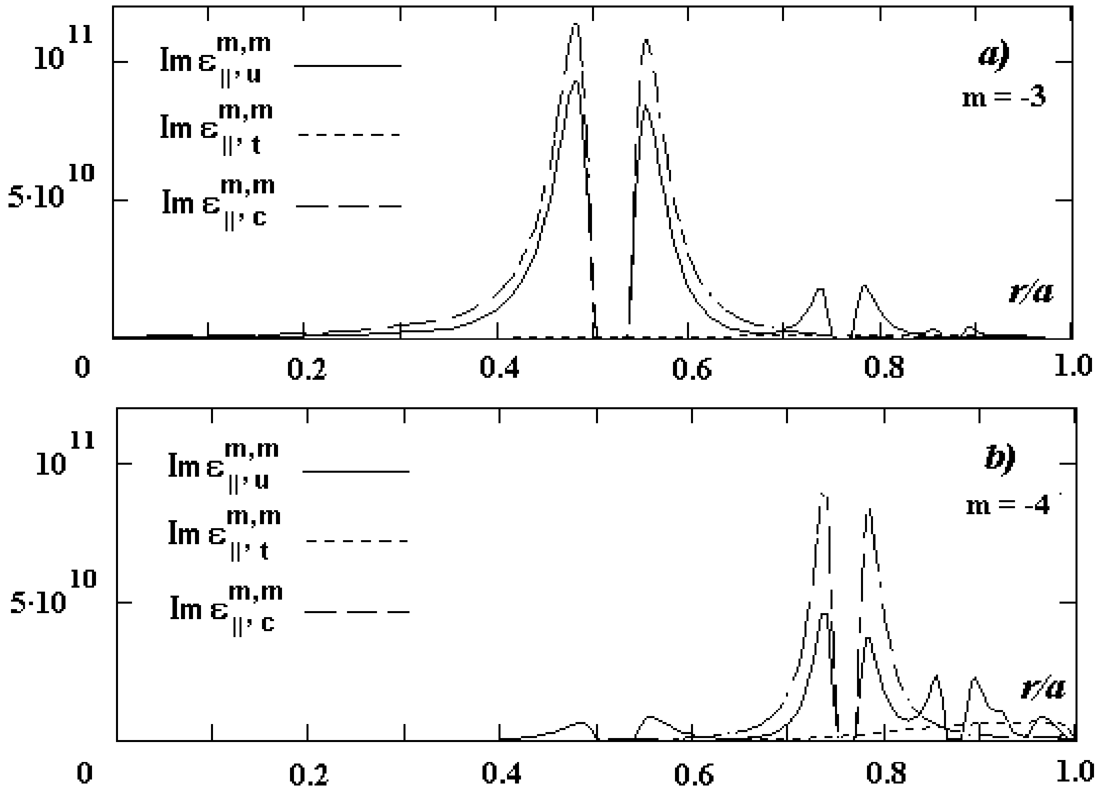

FIG.1. Contribution of trapped/untrapped electrons to Im$\epsilon_{\|}^{m,m}$ for TAEs in the JET-plasma.

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