

## Phase Locking Analysis in 3D MHD Simulations

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### Introduction

In many RFP's the formation of a rotating localized helical magnetic structure has been observed: the so called "Slinky Mode" [1, 2, 3, 4, 5]. The Slinky Mode some times locks to static error fields, causing strong interaction between the first wall and the plasma [6]. The consequent degradation of the plasma is one of the major limits to performing high plasma current discharges. There are many experimental evidences which relate the slinky mode formation to the phase locking in time and space of the  $m = 0$  (low  $n$ ) and  $m = 1$  (dominant) modes [1, 2].

Our numerical study on slinky mode formation is performed by using the 3D MHD code SPECYL [7] which solves the visco-resistive MHD equations in cylindrical geometry with ideal wall boundary conditions at  $r = a$ . In this work we study in detail the phase locking behavior of both  $m = 0$  (low  $n$ ) and  $m = 1$  (dominant) modes for different values of the pinch parameter  $\Theta = B_\theta(a)/\langle B_z \rangle$  ( $\langle \dots \rangle$  is the volume average) by analyzing two simulations which differ only in the  $\Theta$  value ( $\Theta = 1.7$  and  $\Theta = 1.9$ ). Our analysis shows a good agreement with both the experimental observations and previous numerical works [8, 9] and for essential aspects, also with the theoretical model discussed in reference [10].

### Phase Locking Analysis

RFP sustainment, in ordinary conditions, involves quasiperiodical relaxation events characterized by an intense nonlinear dynamics [11, 12]. Therefore our attention focused on temporal windows containing one relaxation event (the rapid decrease of the reversal parameter  $F$  as shown in figure 1), having in mind the nonlinear nature of the phase locking phenomenon. In figure (2) we plot the perturbed toroidal field as a function of  $z$ , for a fixed  $\hat{t} = \tau/\tau_A$  ( $\tau_A$  is the Alfvén time) and for  $(r, \theta) = (a, 0)$  ( $r, \theta, z = 2\pi R\phi$  usual cylindrical coordinates are used; here  $a$  and  $R$  are the minor and the major radius of the torus respectively). The presence of a toroidally localized structure around  $z = z_{lock}$  (the Slinky mode) is observed indicating that the phases of the modes are locked together [8, 9]. In both low and high  $\Theta$  cases, a good temporal correlation between the relaxation events and the formation of the slinky mode has been found (as evidenced in figure 1), confirming that the nonlinear interaction between MHD modes underlies phase locking process [8, 9]. Typically we observe that the slinky mode formation slightly precedes the beginning of the reversal parameter crash, in agreement with MST experimental observations [4].

A spectral analysis of the slinky mode shows that both  $m = 0$  ( $n = 1 \dots 5$ ) and  $m = 1$  ( $n = 7 \dots 14$ ) modes are locked in phase at the same toroidal position: in figure 3 we show an example of the temporal evolution for the different modes. The phases of the modes

$\Phi_{m,n}$  and their dispersion  $\sigma_m$  are defined as

$$\Phi_{m,n}(z_{lock}, \hat{t}) = \frac{1}{\pi} \arctan \left( \frac{Im(b_{m,n}^z(z_{lock}, \hat{t}))}{Re(b_{m,n}^z(z_{lock}, \hat{t}))} \right) \quad (1)$$

$$\sigma_m(z_{lock}, \hat{t}) = \frac{1}{1 + 2 + \dots + N} \sum_{j=1}^{n_{max}-1} \sum_{k=j+1}^{n_{max}} \left| \sin \left( \frac{\Phi_{m,j}(z_{lock}, \hat{t}) - \Phi_{m,k}(z_{lock}, \hat{t})}{2} \right) \right| \quad (2)$$

where  $z_{lock}$  is the toroidal position of the slinky mode,  $N = n_{max} - n_{min} - 1$  and  $n_{max}$ ,  $n_{min}$  represent the maximum and the minimum toroidal mode numbers examined [3].

A comparison between the high  $\Theta$  and low  $\Theta$  simulations does not show a substantial difference of the degree of phase locking ( $\sigma_m$ ), neither for the  $m = 0$  nor for the  $m = 1$  modes. We observe that the two  $m = 1$  dominant modes drive the other  $m = 1$  modes to phase lock (Fig. 3 a, b, c) as discussed in [9]. In accordance with the experimental observations on the slinky mode rotation [2, 3, 4], in some cases we observe that the  $m = 1$  dominant modes, which are phase locked, rotate together in the same direction or in the opposite direction ( as for example in figure 3 a ) with respect to the plasma current, with an angular velocity of the order of  $\omega \sim 10^4 \div 10^5 rad/sec$ . Instead the  $m = 0$  mode phase locking typically shows a quasi-stationary behavior near the phase value  $\Phi = -\pi$  with respect to the  $z_{lock}$  position (Fig. 3 e). It is also interesting to notice that, at least for high  $\Theta$ , the  $m = 0$  modes phase locking always appears with a delay of about  $100\tau_A$  with respect to the phase locking of the  $m = 1$  modes. This observation is consistent with the picture in which the non linear coupling of  $m = 1$  modes, driving the  $m = 0$  modes, is the basic mechanism underlying the relaxation events and the phase locking phenomenon.[9, 14].

In order to study the relationship between mode evolution and plasma flow, which is a key ingredient in the theoretical model proposed in [10, 15], we investigated the behavior of the toroidal rotation velocities,  $v_{rot}^{m,n}$  of both  $m = 0$  and  $m = 1$  modes, compared with the plasma toroidal flow at their resonant surfaces  $v_{plasma}^z(r_{m,n}^{res})$ . We remind here that, in our simulations, plasma flow originates only through nonlinear action of the model equations [13]. We find that, in average,  $v_{rot}^{1,n} \sim 20 \div 50 v_{plasma}^z(x_{1,n}^{res})$ , both at low and high  $\Theta$ , showing that a no-slip condition for  $m = 1$  modes is not satisfied, which could be expected in a stochastic plasma where island structures are destroyed. On the other hand we observe that the toroidal rotation velocity of the mode ( $m = 0, n = 1$ ) reduces substantially and becomes comparable with the toroidal plasma flow at the reversal surface, whenever its energy is sufficiently large. A comparison between the high  $\Theta$  simulation and the low  $\Theta$  one, shows that this phenomenon is triggered whenever the magnetic energy of the (0,1) mode exceeds a common threshold value (Fig. 4). This observation is in accordance with the idea of phase locking in RFPs as a process in which  $m = 0$  modes switch from a suppressed island state to a fully reconnected island state, which might be related to the persistence of  $m = 0$  structures even in a RFP with stochastic core [16].

We can conclude that the phase locking phenomenon can be understood in the framework of the 3D nonlinear visco-resistive MHD; in fact it is an effect of the  $m = 1$ , driving the  $m = 0$ , mode nonlinear interaction, which is responsible of the evolution of mode phases.

## References

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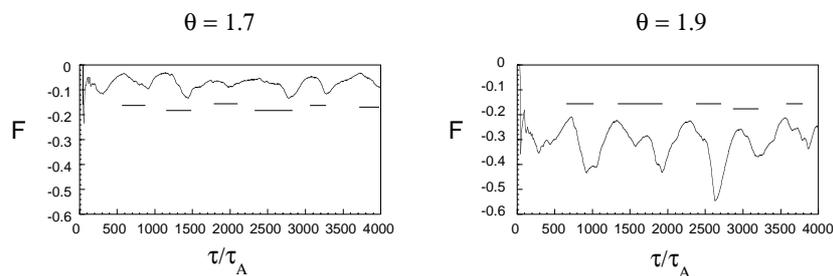


Figure 1: *Reversal parameter time evolution for  $\Theta = 1.7$  and  $\Theta = 1.9$ . The horizontal lines indicate the Slinky Mode duration.*

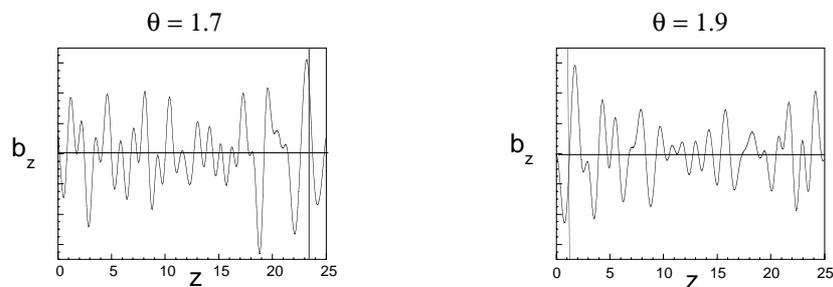


Figure 2: *Toroidal magnetic field at  $r = a$ ,  $\theta = 0$  and for a fixed  $t$  ( $t = 2000$  and  $t = 1400\tau_A$  for low and high  $\Theta$  respectively) as a function of  $z$ . The vertical line indicates the toroidal position  $z_{lock}$  of the Slinky Mode.*

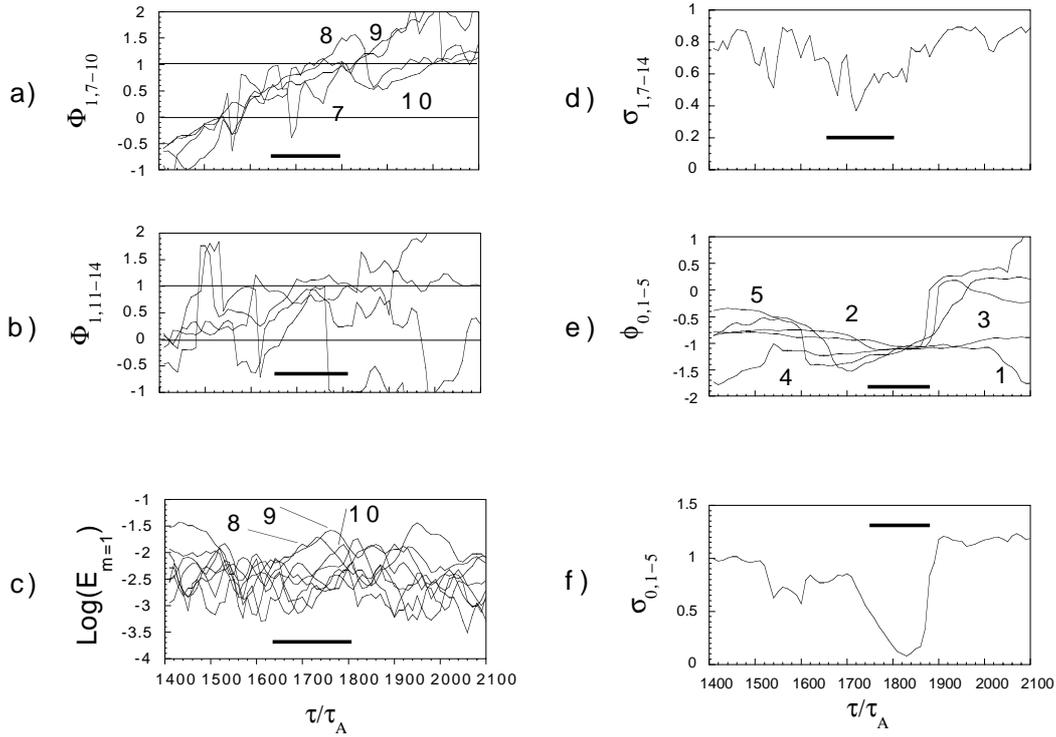


Figure 3:  $\Theta = 1.9$ . a) and b) are  $(m = 1, n = 7 \div 10)$  and  $(m = 1, n = 11 \div 14)$  mode phases respectively; c) are the  $m = 1$  mode energies; d) is the  $(m = 1, n = 7 \div 14)$  phase dispersion; e) are the  $(m = 0, n = 1 \div 5)$  mode phases; f) is the  $(m = 0, n = 1 \div 5)$  phase dispersion. The horizontal lines are the phase locking duration. When  $m = 1, n = 8, 9$  modes, which are in phase, become dominant,  $10 \div 14$  mode phases lock to them ( $\tau/\tau_A \simeq 1700$ ).  $m = 0$  mode phase locking takes place about  $100\tau_A$  later.

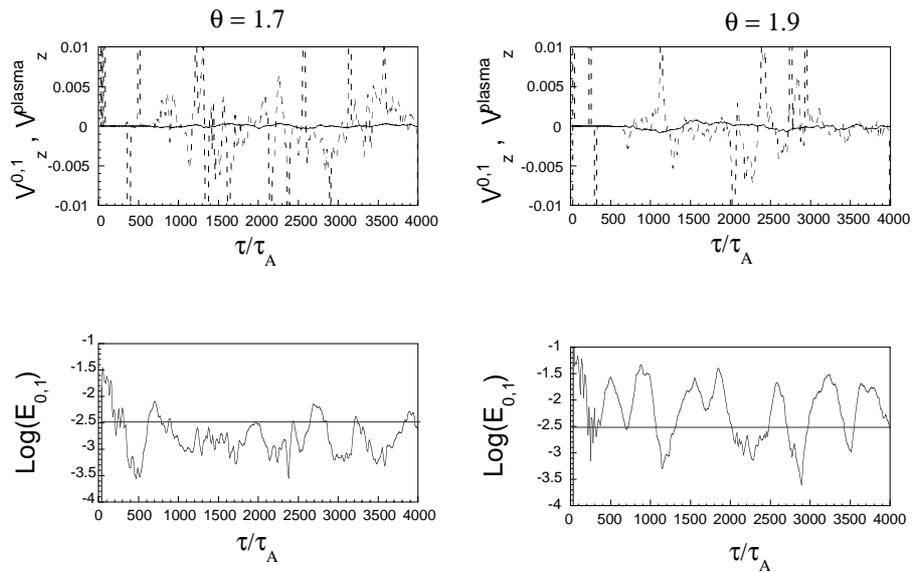


Figure 4: Rotation velocity of the  $(m = 0, n = 1)$  mode (dashed line) locks to the toroidal plasma flow at the reversal surface (solid line). The energy threshold value is  $E_{mag}^{0,1} \simeq 10^{-2.5}$ .