

COLLISIONLESS MAGNETIC RECONNECTION

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Abstract

The nonlinear evolution of collisionless magnetic field line reconnection is investigated numerically in plasma regimes where the effects of the electron and of the ion temperatures modify the structure of the current and vorticity layers that are formed during the development of the reconnection instability. Here the results of investigations in a two-dimensional periodic configuration in regimes of large ion radius, and the long time evolution of collisionless reconnection in a two-dimensional Harris-type configuration is presented. In addition we show preliminary results on nonlinear three-dimensional magnetic reconnection.

Collisionless magnetic reconnection, in regimes where the decoupling of the plasma motion from the magnetic field is caused by the effect of electron inertia, can be described in the framework of a two-fluid model [1] which takes into account electron and ion temperature effects. The governing equations normalized on the Alfvén time, τ_A and on the equilibrium scale length are:

$$\frac{\partial F}{\partial t} + [\varphi, F] = \varrho_s^2 [U, \psi] + \frac{\partial \varphi}{\partial z} \quad (1)$$

$$\frac{\partial U}{\partial t} + [\varphi, U] = [J, \psi] - \frac{\partial J}{\partial z} \quad (2)$$

$$F = \psi + d_e^2 J \quad (3)$$

$$J = -\nabla^2 \psi \quad (4)$$

$$U - \varrho_i^2 \nabla^2 U = \nabla^2 \varphi \quad (5)$$

where ψ is the magnetic flux, φ is the stream function (which is proportional to the electrostatic potential) and $[A, B] = \mathbf{e}_z \cdot \nabla A \times \nabla B$. Here a strong uniform magnetic field is assumed along the z -direction and only modes with phase velocity above the ion thermal velocity are considered.

Electron inertia enters the governing equations through the electron skin depth, $d_e = c/\omega_{pe}$, and the electron and the ion temperatures through the sound Larmor radius, $\varrho_s = (m_e c^2 T_e / e^2 B^2)^{1/2}$ and the ion Larmor radius $\varrho_i = (m_i c^2 T_i / e^2 B^2)^{1/2}$, respectively. Equation (1) describes the motion of the electrons along the field lines, while Eq.(2) originates from the continuity equation and U is related to the density variation. In the cold ion limit, $\varrho_i \rightarrow 0$, Eq. (5) coincides with the usual definition of the vorticity as the Laplacian of the stream function of the plasma. Here we will also consider the opposite

limit by taking large values of ϱ_i , in which case φ is still related to the $\mathbf{E} \times \mathbf{B}$ drift of the electrons and is directly proportional to U through Eq. (5) where the first term on its l.h.s. can be neglected.

In Ref. [2] the “early” nonlinear phase, defined by the inequalities $d_e \leq 2w \leq L_{cell}$, where L_{cell} is the size of the flow cell and w is the magnetic island halfwidth, was studied in a two-dimensional doubly periodic magnetic configuration in the cold ion limit. It was shown that cross shaped structures, related to the Casimirs (conserved fields) of the system, are formed when ϱ_s effects are taken into account.

The aim of this paper is to extend these results to more realistic regimes in which additional geometric and dynamical effects are considered.

First of all, we analyze the effect of the ion temperature on the 2D evolution ($\partial/\partial z = 0$) of the reconnection process. It is well known that in linear theory [3] ϱ_i and ϱ_s enter the expression of the linear growth rate, γ_L through their geometric mean, $\varrho = (\varrho_i^2 + \varrho_s^2)^{1/2}$ and that the current density is largely unaffected by the inclusion of the ion temperature effects, while the pattern of the stream function (i.e. of the electrostatic potential) changes significantly.

The simulations have been carried out with a spectral code which uses standard Fourier transforms to calculate the spatial derivatives. The equations are integrated in the domain $[-L_x, L_x] \times [-L_y, L_y]$ with periodic boundary conditions in both directions. In order to follow the nonlinear evolution we have introduced diffusive terms into Eqs. (1,2), with corresponding viscosity coefficients $\nu_i = 4 * 10^{-4}$ and $\nu_e = 3 * 10^{-4}$, such that the characteristic dissipative scalelength is smaller than d_e . The equilibrium configuration is given by $\psi_{eq} = \cos(x)$, $\varphi_{eq} = 0$, with $L_x = \pi$, $\epsilon = L_x/L_y = 0.5$, $d_e = 0.08L_x$. We choose $\varrho_s = 0$ and $\varrho_i = 3d_e$ which corresponds to the value of ϱ_s used in Refs. [2], i.e. to the same value of the linear growth rate. In fig. 1 we draw the contour plots of ψ , φ , J at $t = 38$ and the time behavior of the reconnected flux, as measured by $\delta\psi_X \equiv \psi(0, 0; t) - \psi_{eq}(0)$. We see that also in the nonlinear phase the stream function evolves with a different shape in comparison to the case $\varrho_i = 0$. In the limit of large ion gyroradius Eq. (5) gives $U \sim -\varphi/\varrho_i^2$, and the contour lines of U tend to coincide with those of φ . In this limit the advection term in Eq. (2) vanishes identically and Eq. (2) reduces simply to $\partial\varphi/\partial t = -\varrho_i^2[J, \psi]$. Indeed the macroscopic flow cells are more localized in the region around the X -point. This can account for the stronger localization of the current density sheets on the separatrix of the magnetic flux.

Second, we solve our two-fluid model in the 2D ($\partial/\partial z = 0$) cold ion limit, over an integration domain corresponding to the Harris pinch equilibrium with a strong superimposed uniform field B_z , i.e. $\mathbf{B}_{eq} = B_{z0}\mathbf{e}_z + \mathbf{B}_{yeq}(x)\mathbf{e}_y$, where $\mathbf{B}_{yeq}(x) = \tanh(x/L)$ and L is the equilibrium scale length. The domain along the x -direction has an infinite extent. This new configuration allows us to study the long time evolution of the reconnection process where the magnetic island width can become comparable to L . By contrast in Ref. [2], the long term evolution, being affected by the periodic boundary conditions, was not considered. Simulations have been carried out with a code which uses a finite difference method in the inhomogeneous x direction and Fourier transforms in the periodic y direction. The instability parameter is $\Delta' = 2(1/k - k)$, where $k = m\pi L_y/L$ and m is the mode number. Since we were interested in the numerical integration of the Eqs. (1-5) in the full nonlinear evolution, when the nonlinear energy injection on the “small” scales becomes very efficient, we added two hyperviscosity terms instead of increasing the value

of the diffusive coefficients which would lead to dissipative scalengths comparable to d_e . In particular we added the terms $\mu_e \nabla^4 F$ and $\mu_i \nabla^4 U$ into Eqs. (1) and (2), respectively, with $\mu_i = 10^{-5}$ and $\mu_e = 10^{-5}$. We choose the following values for the simulation parameters: $-14.8L < x < 14.8L$, $-L_y < y < L_y$, $L = 1$, $L_y = 4\pi$ and $\varrho_s = d_e = 0.2$. In this way we obtain $\Delta' d_e = 1.5$ for the unstable mode $m = 1$, which is comparable to $\Delta' d_e = 2$ used in ref. [2]. The results of the simulations are summarized in fig. 2. Here ψ, φ, J, U in a later stage of the reconnection process and the time behavior of $\ln \delta\psi_X$ are shown. First of all we confirm the formation of the typical cross structure in the vorticity and current density layers in the early nonlinear phase, as found in Ref. [2]. Therefore we conclude that this structure does not depend on the boundary conditions. A similar conclusion was also found for a forced reconnection problem in Ref. [4]. Moreover the cross structure persists in the full nonlinear phase. We can see that the nonlinear growth is faster than exponential up to time of the order of $150\tau_A$, when the magnetic island halfwidth has reached amplitudes of the order of $17d_e$. After this time a saturation mechanism develops. Convection cells start to grow inside the magnetic island, near the O - *points* of the magnetic flux. This new structure of the velocity field is clearly evident also from the vorticity patterns that develop along the separatrix. At saturation the velocity cells are confined within the island separatrix which therefore ceases to grow.

Finally, we are presently considering the evolution of a three dimensional perturbation in a three dimensional periodic box. At this stage we are performing preliminary tests adopting an equilibrium of the type used in Ref. [2], where $\mathbf{B}_{eq} = B_{z0}\mathbf{e}_z + B_0\nabla\psi_{eq}(x) \times \mathbf{e}_z$ with $\psi_{eq} = \cos(x)$. We used as initial conditions a magnetic perturbation with two different helicities corresponding to $\exp[i(k_y y \pm k_z z)]$. In the adopted periodic equilibrium configuration this corresponds to four rational surfaces, located at $\mathbf{k} \cdot \mathbf{B} = k_z + k_y \sin(x) = 0$. Here we choose $B_z/B_0 = 1$, but we notice that this ratio can always be included in a coordinate transformation along the third direction, $z \rightarrow B_z/B_0 z$. From the simulations we observe that four current sheets with different inclinations, depending on the presence of two helicities, are formed at the location of the null lines of the magnetic field. After 70 Alfvén times these structures start to interact. In particular the current density layers of the same sign start to attract each other collapsing in a single current sheet.

References

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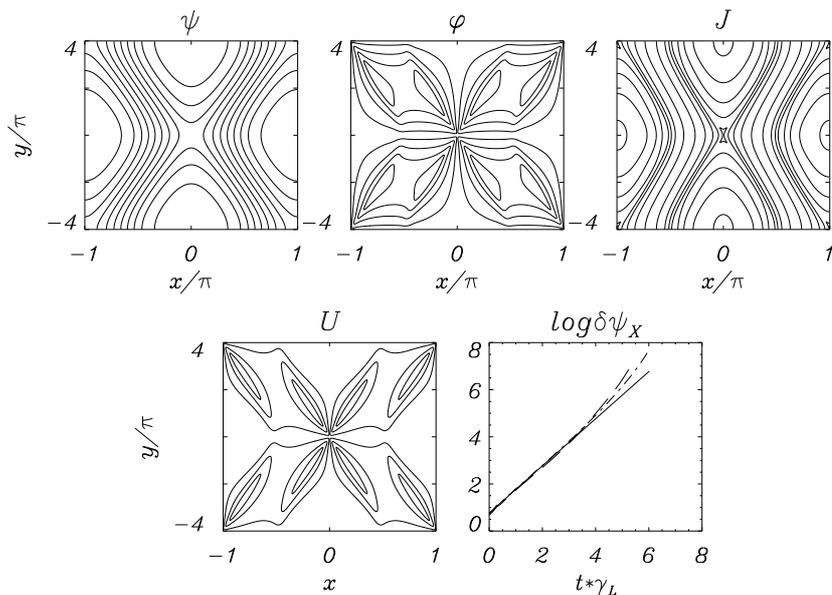


Figure 1: Simulation of collisionless reconnection with ion Larmor radius effect: contour plots of ψ , φ and J , at time $t = 39\tau_A$, corresponding to a magnetic island halfwidth $w = 8.5d_e$. In the last subplot the $\ln(\delta\psi_X)$ versus the normalized time, $t * \gamma_L$ is drawn for different value of $\rho_{i,s}$. Dashed line: $\rho_i = 3d_e$ and $\rho_s = 0$; dot-dashed line: $\rho_i = 0$ and $\rho_s = 3d_e$: the continuous line represents the fit over the linear phases

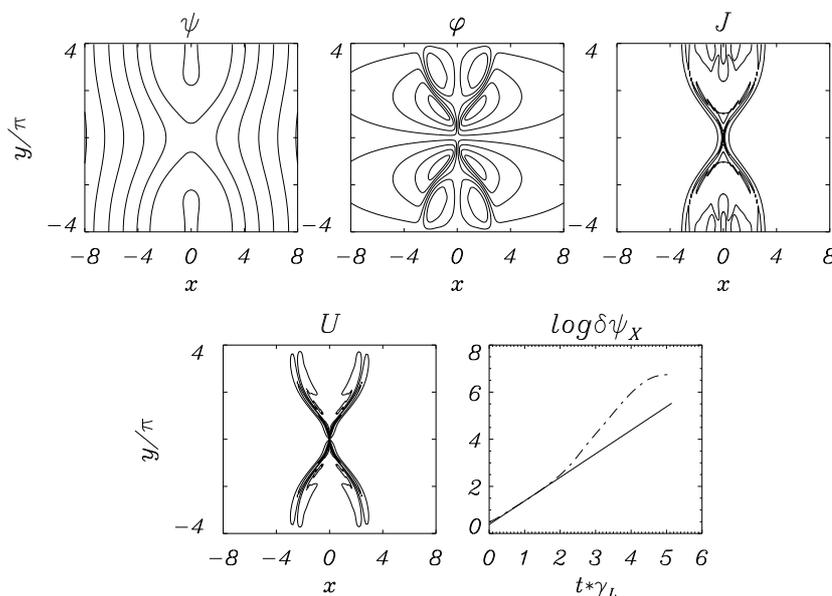


Figure 2: Simulation of collisionless reconnection of a Harris pinch equilibrium: contour plots of ψ , φ , J and U , at time $t = 150\tau_A$, corresponding to a magnetic island halfwidth $w = 17d_e$. In the last subplot the $\ln(\delta\psi_X)$ versus the normalized time, $t * \gamma_L$ is drawn. The continuous line represents the fit over the linear phase