

Mirnov signal reconstruction from numerical simulations of toroidally coupled tearing modes

^αR. Coelho¹, M. Maraschek², H.Zohm³, F. Serra¹

¹Associação Euratom-IST, Centro de Fusão Nuclear, 1049-001, Lisboa, Portugal

²Max-Planck-Institut für Plasmaphysik, EURATOM Association, D-85748 Garching, Germany

³Institut für Plasmaforschung, Universität Stuttgart, D-70569 Stuttgart, Germany

I – Introduction

The onset and growth of both ohmic and neo-classical resistive tearing modes in many tokamak discharges [1,2] have confirmed the strong influence that these modes have on global stability, transport and discharge operational limits (e.g. β -limit). The toroidal coupling of tearing modes with neighbouring helicity ((m,n) and (m \pm 1,n)), can destabilise significantly marginally stable modes, as shown using a non-linear Δ' numerical code developed for the study of the non-linear time evolution of toroidally coupled tearing modes [3].

A detailed analysis of the available experimental data can therefore be of great advantage for the understanding of how coupled rotating islands evolve. Amongst several possible diagnostics, the Mirnov magnetic signal proves to be a powerful diagnostic when regarding both the experimental inferences of existing magnetic island width and frequency [4]. When two modes are toroidally coupled, one cannot infer both islands width from one single magnetic signal (using a large poloidal array makes it feasible [4]) since the mode frequencies are equal and the amplitude of the signal is not expected to be the superposition of the two separate modes.

In this paper, a reconstruction of the Mirnov signal expected to be observed when two toroidally coupled rotating tearing modes are identified is made. This signal reconstruction is derived from the analytical expressions for the flux functions of both interacting modes. A comparison with data from ASDEX Upgrade density limit discharges is made.

II – Analytical derivation of simulated Mirnov signal

In Ref. 3, a numerical model for the study of the non-linear time evolution of island width and frequency of toroidally coupled tearing modes is presented, where the effect of mode coupling is approximated by adopting a model of localised helical currents of appropriate magnitude and phase, located at the rational surfaces (labelled r_{s1} and r_{s2}). A similar treatment is used to account for the resistive wall effect (located at a minor radius \mathbf{d}

^α presently at Istituto di Fisica del Plasma – CNR, Milan

and with resistive time τ_R), through appropriate boundary conditions imposed when solving the tearing mode equation of each of the interacting modes.

The flux function of both modes is thus determined by solving the tearing mode equation in the several domains of interest: plasma – vacuum – wall and imposing suitable boundary conditions. The expressions for the flux function of both the inner and outer modes (the inner mode has the lower poloidal mode number – m_1), calculated at the wall radius (which is assumed to be the Mirnov coils location), can be worked out to yield, respectively,

$$\begin{aligned} \psi_1(d) = e^{i\phi_1} \left(\frac{r_{s1}}{d} \right)^{m_1} & \left[\psi_{s1} \frac{1 - i(\omega_1 \tau_{w1})}{1 + (\omega_1 \tau_{w1})^2} + \psi_{s1} \left(\frac{r_{s1}}{d} \right)^{2m_1} \frac{(\omega_1 \tau_{w1})^2 + i(\omega_1 \tau_{w1})}{1 + (\omega_1 \tau_{w1})^2} + \right. \\ & \left. + C_\alpha e^{i(\phi_2 - \phi_1)} \psi_{s2} \frac{1 - i(\omega_2 \tau_{w1})}{1 + (\omega_2 \tau_{w1})^2} + C_\alpha e^{i(\phi_2 - \phi_1)} \psi_{s2} \left(\frac{r_{s1}}{d} \right)^{2m_1} \frac{(\omega_2 \tau_{w1})^2 + i(\omega_2 \tau_{w1})}{1 + (\omega_2 \tau_{w1})^2} \right] \end{aligned} \quad (1)$$

and

$$\psi_2(d) = e^{i\phi_2} \left(\frac{r_{s2}}{d} \right)^{m_2} \left[\psi_{s2} \frac{1 - i(\omega_2 \tau_{w2})}{1 + (\omega_2 \tau_{w2})^2} + \psi_{s2} \left(\frac{r_{s2}}{d} \right)^{2m_2} \frac{(\omega_2 \tau_{w2})^2 + i(\omega_2 \tau_{w2})}{1 + (\omega_2 \tau_{w2})^2} \right] \quad (2)$$

where C_α is a geometrical constant due to the coupling interaction, $\tau_{wk} = \tau_R / k$, $\psi_{sk} = \psi_k(r_{sk})$ is the value of the flux function of mode k at its rational surface (r_{sk}) and ϕ_k is the mode phase satisfying $\frac{d\phi_k}{dt} = \omega_k$. The associated Mirnov signal can therefore be modelled by

$$M(t) = -\frac{d}{dt} \left[\text{Re} \left(\frac{1}{d} (m_1 \psi_1(d) + m_2 \psi_2(d)) \right) \right] \quad (3)$$

The amplitude of the Mirnov signal is dependent on both mode frequency and mode amplitude. In fact, from the expression for the simulated signal it can be noted that both modes contribute to the signal and that, due to coupling, the width of the modes cannot be directly resolved from a single Mirnov signal with a single dominant frequency.

III – Comparison with experimental data

Several L-mode Asdex Upgrade discharges are terminated by density limit disruptions, where a dominant (2,1) tearing mode precursor is observed to be destabilised (in cases where the $q=2$ surface is close to the edge, increased edge cooling is thought to be the cause of destabilisation) and evolves towards locking. In addition, from comparison of the ratio between inboard and outboard magnetic probe amplitudes, one may infer that the variation of the amplitude ratio indicates the development of coupled modes on neighbouring resonant surfaces [4].

In this paper a low q_{95} case is analysed ($q_{95} \approx 3.2$), where a dominant background (2,1) mode with a frequency around 3.4 kHz is observed to be destabilised ≈ 12 ms before locking (see figure 1).

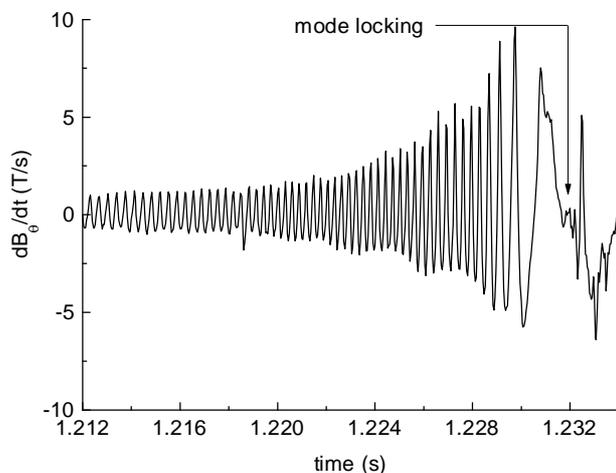


Figure 1 – Outboard midplane magnetic signal of a typical low q_{95} density limit discharge (shot #11826) dominated by a (2,1) component, showing the fast growth of the mode before locking at ≈ 1.232 s.

In the simulations, the time evolution of the (2,1) island width was modelled in order to give an approximate fit to the experimental time evolution of the integrated magnetic signal, which shows that a slow mode growth (characteristic of current gradient driven tearing modes) lasting till $t \approx 1.228$ is followed by a very fast growth up to locking, when the mode is thought to saturate. An indication of the island width at the time of the locking of the mode, given by ECE temperature measurements, yielded ≈ 7 cm. The (2,1) mode is considered to interact via toroidicity with a stable (3,1) mode, destabilising this mode to a width which does not exceed ≈ 4 cm, and the experimentally inferred q -profile, plasma density and temperature are used. The resistive wall time is set to 1 ms and from the q profile one has $r_{s1}/a \approx 0.9$, $r_{s2}/a \approx 0.97$ and $d/a \approx 1.4$ (a is the minor radius of the q_{95} surface).

The simulated magnetic signal (for $t > 1.221$ s) is presented in figure 2. Although the time when mode locking occurs and the shape of the magnetic signal are very similar to the experimental observations, the magnitude of the simulated signal is some 40 % less than the experimental one in the later stages before mode locking. This may suggest that the amplitude of the (2,1) mode was undervalued (larger final width) or that additional modes should be taken into consideration. In fact, in these low q_{95} discharges, a (1,1) component is also important [4] and could in principle contribute to some extent to the magnetic signal.

In addition, one may note from Eq. (1), (2) and (3) that the magnitude of the magnetic signal is very sensitive to the value of the tokamak vessel minor radius (d). Since the tokamak and plasma cross sections are not circular, the value used in the simulation should be considered an average value, lower values leading to a magnetic signal of larger amplitude.

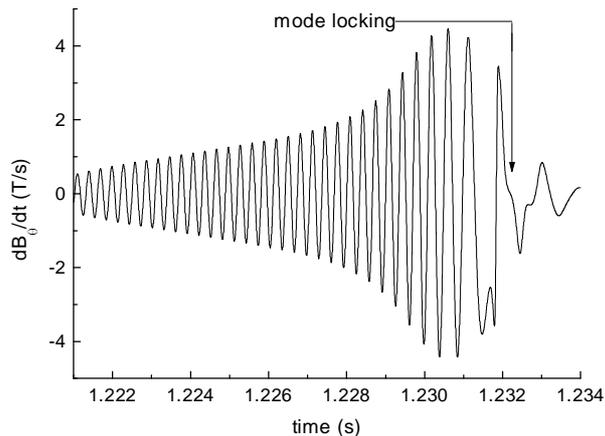


Figure 2 – Simulated magnetic signal showing the evolution of two coupled (2,1) and (3,1) modes towards locking. The outer (3,1) mode represents $\approx 25\text{-}40\%$ of the total magnetic signal.

The decrease of the simulated signal amplitude near locking is also observed experimentally and it is a result of the very fast variation of the mode frequency in the vicinity of mode locking. The fast signal variations after mode locking (see figure 1, $t \in [1.232, 1.234]$) are not reproduced in the simulation since they are an effect of a minor disruption. In the simulation, the signal obtained after the locking of the modes results from the oscillation of the (3,1) mode around the (2,1) mode (if nonetheless such oscillations were fast enough, spikes would appear in the magnetic signal after the locking of the modes).

IV – Conclusions

An analytical expression for the Mirnov signal resultant of the interaction of toroidally coupled tearing modes with a resistive wall was investigated and applied to the analysis of experimentally observed coupled mode locking before the onset of density limit disruptions in the tokamak Asdex Upgrade. The shape and amplitude of the signal strongly depend on the mode growth and on the tokamak wall location. A relatively good agreement is found between simulated and experimental signals, nonetheless an extension of the model should be made in order to include the contribution of additional modes (such as the (1,1)).

References

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