

Modelling different régimes of particle transport in strong electrostatic turbulence

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Abstract

We have studied the transport of test particles moving in the Hasegawa Mima (HM) field and the role of the equilibrium density gradient, which determines the strength of the linear dispersive term in the HM equation. As anisotropy arising from the latter increases, 'strange kinetics' emerge: the poloidal particle transport undergoes a sharp qualitative transition from diffusive, through super diffusive, to ballistic.

1. The Hasegawa Mima model

It is important to understand the consequences of different forms of strong plasma turbulence for particle transport in fusion plasmas. For magnetically confined plasmas, a relatively simple and flexible two-dimensional model for the turbulent electrostatic field in the (x, y) plane perpendicular to the magnetic field (z axis) is provided by the Hasegawa Mima (HM) equation [1], which also has many wider physical applications [2]. The ions are cold, with negligible inertia parallel to \mathbf{B} . The electrons are assumed to have an immediate adiabatic response, with Boltzmann distribution; their background density depends only on y , equivalent to the radial direction in a tokamak, $n_0 \equiv n_0(y)$. The HM equation is then written (in convenient units) [1 - 3]

$$\frac{\partial}{\partial t} (\phi - \nabla^2 \phi) - \{\phi, \nabla^2 \phi\} - \beta \frac{\partial \phi}{\partial x} = 0; \quad (1)$$

here ϕ is the electrostatic potential, $\{A, B\} = \partial_x A \partial_y B - \partial_y A \partial_x B$ is the Poisson bracket, and $\beta = \partial_y \{\ln[n_0(y)]\}$ is a parameter measuring the anisotropy of the background density. The linear limit of eq.(1) is equivalent to the evolution of independent drift waves each obeying the dispersion relation $\omega_k = \beta k_x / (1 + k^2)$. We have implemented [3] eq.(1) in a computational box that is finite in the y direction and periodic in the direction of propagation of drift waves, x , equivalent to the poloidal direction in a tokamak. To the right hand side of eq.(1) we have added a dissipation term, both for high and low wavenumbers, for reasons of numerical stability, and also a forcing term in order to reach a quasi-stationary state. Increasing β enhances the anisotropy of the fields, and our aim is to study the influence of anisotropy (a linear dispersive effect) on the poloidal transport of particles.

2. Particle transport

Recall that for normal diffusion (a classical random walk), the mean squared displacement is proportional to time: $\langle \Delta x^2 \rangle \sim t$. If transport is anomalous ('strange kinetics' [4]), this becomes $\langle \Delta x^2 \rangle \sim t^\mu$: for $0 < \mu < 1$, sub diffusion; $1 < \mu < 2$, super diffusion; $\mu = 2$, ballistic motion - particles move with constant velocity. In order to analyse the transport of test particles moving in the HM field, we create an ensemble of 3000 test particles whose motion is given by the $\mathbf{E} \times \mathbf{B}$ drift,

$$\frac{d\mathbf{r}}{dt} = \mathbf{B} \times \frac{\nabla \phi}{B^2}; \quad (2)$$

here ϕ is the potential resulting from the solution of the HM equation, and we neglect the polarisation drift. The test particles are noninteracting and without inertia, we consider only their guiding center motion and not the Larmor radius effects (but compare ref.[3]). We initially distribute the particles randomly within a narrow vertical band. To study the effect of anisotropy on transport, we have run the HM test particle code for a range of values of β . When $\beta = 0$, i. e. the background density is isotropic, the diffusion appears normal, see fig.(1). When β is increased the particles start to super diffuse, for example for $\beta = 0.05$ the exponent μ is about 1.7, see fig.(2). If β is increased up to 0.25 or beyond, the motion of test particles is ballistic, see fig.(3).

The random walk theory of normal diffusion leads to a distribution of particles that is Gaussian at large times; the poloidal distribution of test particles in our model is plotted for the cases $\beta = 0$ (left of fig.(4)), where a Gaussian is superimposed, and $\beta = 0.25$ (right of fig.(4)). The distribution for $\beta = 0$ is similar to a Gaussian, whereas for $\beta = 0.25$, it is much flatter. A measure of the proximity of a distribution to a Gaussian is its kurtosis (peakedness) $K = \langle \Delta x^4 \rangle / 3 \langle \Delta x^2 \rangle^2$. A Gaussian distribution has $K = 1$, whereas if the particles start from a central band and all have ballistic motion, they should end up by having a rectangular distribution with $K = 0.6$. In our simulations, in the case $\beta = 0$, we find $K \simeq 1.2$, so that the distribution is slightly

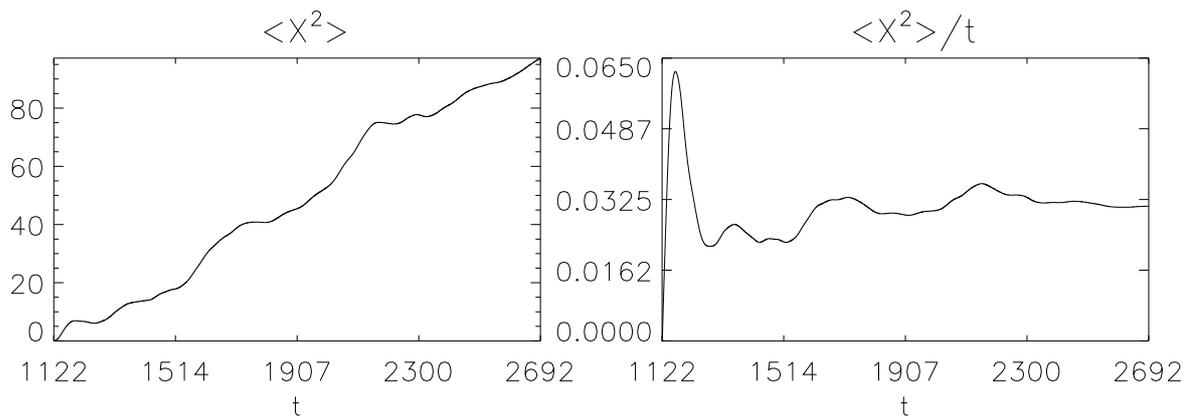


Figure 1: Transport of test particles for $\beta = 0$: normal diffusion. Left: time evolution of the mean squared displacement. Right: mean squared displacement divided by the time.

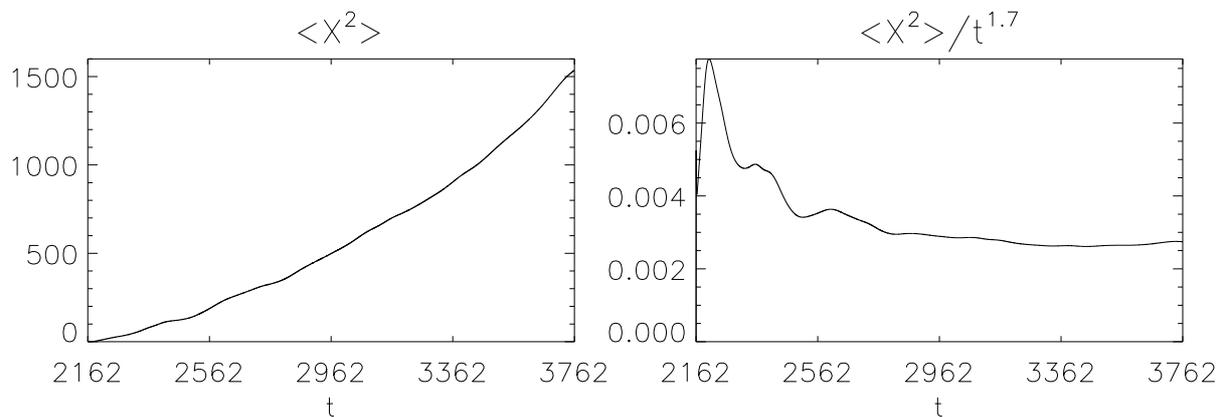


Figure 2: Transport of test particles for $\beta = 0.05$: super diffusion. Left: time evolution of the mean squared displacement. Right: mean squared displacement divided by $t^{1.7}$.

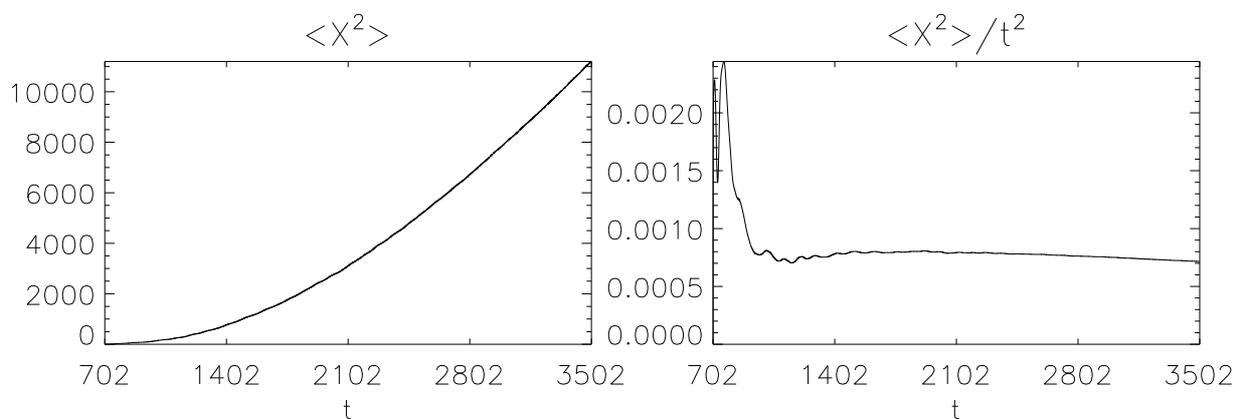


Figure 3: Transport of test particles for $\beta = 0.25$: ballistic motion. Left: time evolution of the mean squared displacement. Right: mean squared displacement divided by t^2 .

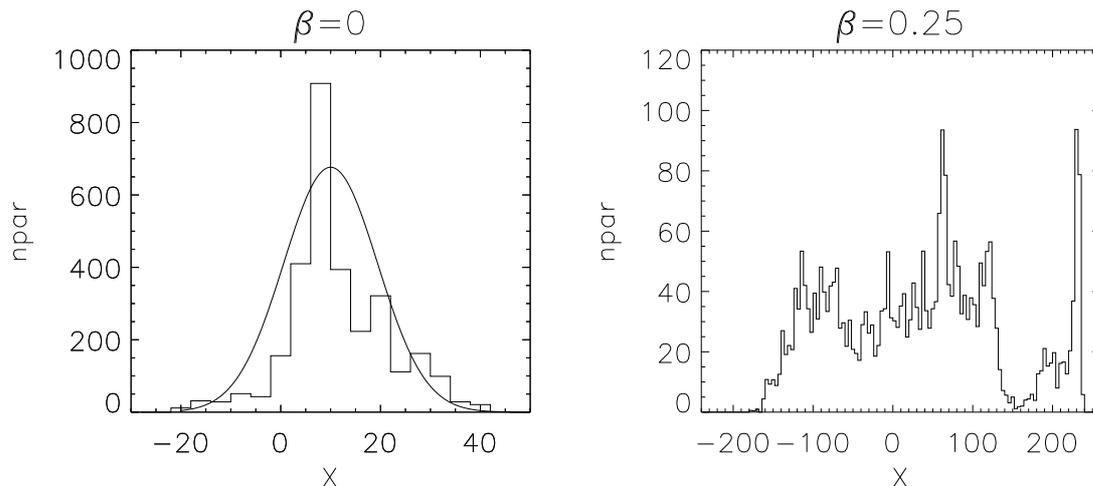


Figure 4: Distribution of test particles at large times. Left: $\beta = 0$, a Gaussian of the same width is superimposed. Right: $\beta = 0.25$.

more peaked than a Gaussian, (fig.(4)); however for $\beta = 0.25$, $K \simeq 0.73$, much closer to a rectangular distribution.

3. Conclusions

We have studied the transport of test particles in turbulent HM fields along the direction of propagation of drift waves (poloidal, in a tokamak). When the background density of electrons is isotropic, there is approximately normal diffusion, whereas increasing β (the density gradient and coefficient of the linear dispersive effects in the HM equation) from zero leads to ‘strange kinetics’ [4]: first super diffusion, then ballistic motion. Particle transport in the perpendicular direction (radial, in a tokamak) has already been studied in [3], where the main result is that the combination of linear and nonlinear effects suppresses diffusion. We conclude from the present study and ref.[3] that the presence of a background density gradient leads to enhanced ‘strange kinetic’ transport of particles in the poloidal direction, but suppresses particle transport in the radial direction.

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References

- [1] A. Hasegawa and K. Mima, Phys. Fluids **21**, 87 (1977)
- [2] A. Hasegawa, Advances in Physics **34**, 1 (1985)
- [3] G. Manfredi and R.O. Dendy, Phys. Plasmas **4**, 628 (1997)
- [4] M.F. Shlesinger, G.M. Zaslavsky and J. Klafter, Nature **363**, 31 (1993)