26th EPS Conf. on Contr. Fusion and Plasma Physics, Maastricht, 14 - 18 June 1999 ECA Vol.23J (1999) 1281 - 1284

Impurity Radiation in the Regimes with Bursty Plasma Transport

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Abstract

An impact of bursty cross field heat transport on impurity radiation loss is considered. A simple model of anomalous transport based on "delayed diffusion" is employed. Radiation losses with and without bursty effects are compared and the results of numerical modeling are discussed.

Introduction

Cross field plasma transport plays a crucial role in many tokamak phenomena. In particular, it determines such important process as an impurity radiation loss in the peripheral plasmas [1]. However, macroscopic plasma transport models employed in [1] to study an impact of cross field plasma transport were solely based on "laminar" diffusive processes. Meanwhile, theoretical developments [2] and experimental observations [3] indicate that the edge plasma transport is characterized by large bursts in both heat flux and plasma parameters. Since there is no first principals theory describing bursty transport in plasmas we use a model equation [4] resulting in a strong fluctuations of the "plasma temperature" and the flux.

Transport model and stability analysis

To study the effects of bursty transport on impurity radiation we consider 1D nonlinear transport equation based on "delayed diffusion" which results in bursty transport phenomena [4]. With impurity radiation loss, 1D "delayed diffusion" transport model for the plasma temperature T(x,t) ($T(|x| = L, t) = 0$) is

$$\partial_t T = \partial_x \left(\left\langle \left| \partial_x T(t-\tau) \right|^{\alpha} \right\rangle \partial_x T \right) + S - R(T), \qquad (1)$$

where $-L \le x \le L$, S and R(T) describe the heat source and impurity radiation sink. Following [4] we employ nonlinear plasma heat diffusivity of the form $D(x,t) = \left\langle \left| \partial_x T(t-\tau) \right|^{\alpha} \right\rangle$, where α is an adjustable parameter, τ is a small time scale, and $\langle (\ldots) \rangle \equiv \frac{k_0}{2} \int \exp\left(-|x - x'| k_0\right)(\ldots)\, dx'$, $k_0 L >> 1$. Notice that τ can be interpreted as inverse growth rate of the plasma instability which results in nonlinear plasma heat diffusivity D having a time delay $\sim \tau$ with respect to the temperature gradient. Although τ can be small, ($\tau << L^2/D \sim \tau_E$) it causes small scale, $kL >> 1$, perturbations to be unstable for $R(T)=0$ [4].

We consider stability of Eq. (1) assuming a small scale perturbation $\tilde{T} \propto \exp(-i\omega t + ikx)$, where $\omega \equiv \omega_0 + i\gamma$. After some algebra we find

$$i\Omega = \Omega_D\{1 + \alpha_k \exp(i\Omega)\} - \Omega_R, \qquad (2)$$

where $\Omega = \omega_0\tau + i\gamma\tau \equiv \Omega_0 + i\Gamma$, $\alpha_k = \alpha/\left(1 + (k/k_0)^2\right)$, $\Omega_R = \tau\gamma_R \equiv \tau(-dR(T)/dT)$, and $\Omega_D = |\partial_x T_0|^\alpha k^2\tau \equiv Dk^2\tau$. Separating the real and imaginary parts in Eq. (2) we have

$$F(\Omega_0) \equiv \frac{\sin\Omega_0}{\Omega_0}\exp\left(\frac{\Omega_0}{\mathrm{tg}\Omega_0}\right) = \frac{\exp(\Omega_R)}{\alpha_k}\frac{\exp(-\Omega_D)}{\Omega_D}, \qquad (3)$$

$$1 - \frac{\Omega_R}{\Omega_D} + \alpha_k \exp(-\Gamma)\cos\Omega_0 = -\frac{\Gamma}{\Omega_D}. \qquad (4)$$

Then, from Eq. (4) we immediately find that instability ($\gamma>0$) can only occur when

$$\frac{\Omega_R}{\Omega_D} + \alpha_k > 1. \qquad (5)$$

The case $\alpha_k << 1$, $\Omega_R > \Omega_D$ ($\gamma_R > Dk^2$) corresponds to the radiation driven instability. Radiation driven instability usually has a macroscopic character resulting in a bifurcation of temperature profile and does not cause a bursty transport. Small scale instability which can result in a bursty transport [4] corresponds to the case $\alpha_k > 1$, $\Omega_R < \Omega_D$ ($\gamma_R < Dk^2$). To show the details of this instability we assume $\Omega_R = 0$.

First we consider Eq. (3). A positive part of the function $F(\Omega_0)$ is shown in Fig. 1.

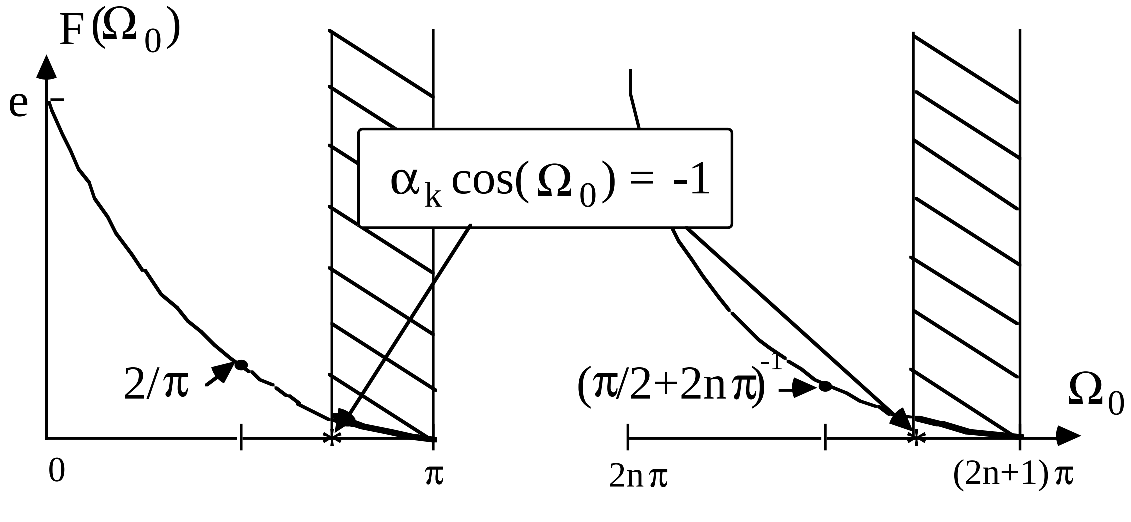

Fig. 1. Unstable solutions of Eqs. (3), (4) are only possible in shadowed regions of Ω_0.

As one sees from Fig.1 and Eqs. (3) and (4), unstable solutions only exist in the ranges of Ω_0 satisfying the inequalities $1+\alpha_k \cos\Omega_0<0$ and $\sin\Omega_0>0$ which result in

$$\arccos(1/\alpha_k) + 2n\pi \le \Omega_0 \le (2n+1)\pi, \qquad (6)$$

where n is an integer number. Therefore, to satisfy Eq. (3) and (4) within the range (6) the following inequality must be satisfied

$$\frac{\exp(-\Omega_D)}{\Omega_D} \le \frac{\sqrt{\alpha_k^2 - 1}}{(2n+1)\pi - \arccos(1/\alpha_k)} \exp\left(- \frac{(2n+1)\pi - \arccos(1/\alpha_k)}{\sqrt{\alpha_k^2 - 1}} \right). \qquad (7)$$

Notice that multiple unstable solution of Eqs. (3) and (4) are possible when Eq. (7) is satisfied for more than one integer number n.

Results of numerical modeling

To study nonlinear regime of "delayed diffusion" instability and associated effects of impurity radiation loss we solve Eq. (1) numerically.

We solve Eq. (1) for L=0.5, $S=1$, $\alpha=2$, and k_0=25 with and without radiation loss and for different values of time delay τ. We employ the following form of impurity radiation function

$$R(T) = R_0 \exp\left\{ -\left((T - T_R)/\delta T_R \right)^2 \right\}. \qquad (8)$$

This form of $R(T)$ resembles impurity radiation in coronal approximation. In what follows we use $\delta T_R = T_R = 0.1$.

In Fig. 2 we show the time history of the heat flux, q_b, through the boundaries x=$\pm$0.5 for the case τ=0.28 and no impurity radiation loss, $R_0=0$.

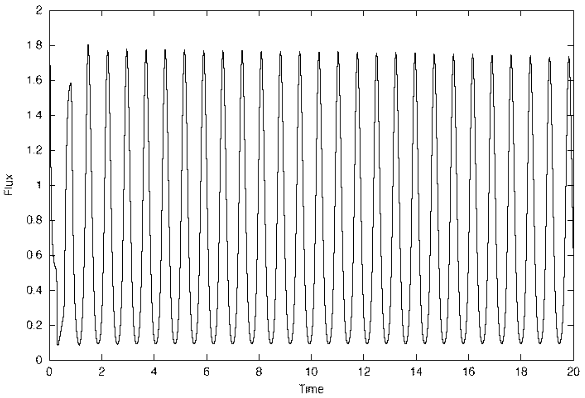

Fig. 2. *Heat flux* q_b *calculated for* τ*=0.28,* R_0 *=0.*

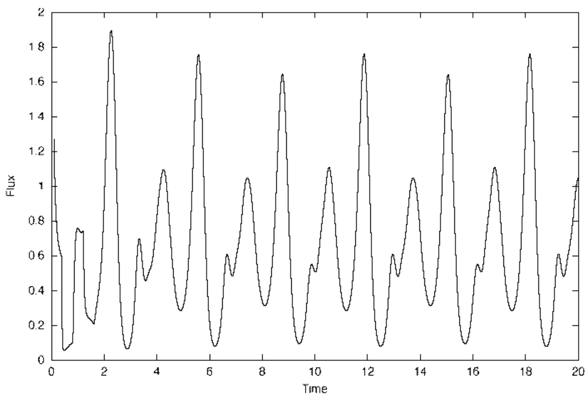

Fig. 3. *Heat flux* q_b *calculated for* τ*=0.4,* R_0 *=0.8.*

We find that in accordance with our linear analysis "delayed diffusion" instability develops and results in a strong fluctuations of both temperature profile and the heat flux through the boundaries.

In Fig. 3 we show the time history of q_b for the case τ=0.4 and with impurity radiation loss, R_0=0.8. We find that time averaged impurity radiation loss in this case is $\approx$0.45 out of total energy source equal to 1. Interestingly enough that total impurity radiation loss for steady state case with R_0=0.8 and τ=0 is $\approx$0.45. Therefore, we may conclude that in spite of "delayed diffusion" instability significantly modulate the heat flux through the boundaries it does not alter averaged radiation loss. This counter-intuitive result needs to be verified for other values of τ and R_0 parameters.

We notice that numerical solution of Eq. (1) requires large computer memory to store temperature profiles back to the time t-τ, while for a reasonable accuracy a time step of the order of 10^{-6} must be used for the parameters L, S, α, k_0, and R_0 discussed here.

Conclusions

Cross field plasma transport plays a crucial role in many tokamak phenomena. Recent theoretical developments and experimental observations indicate that the edge plasma transport is characterized by large bursts in both heat flux and plasma parameters. To study the effects of bursty transport on impurity radiation we consider 1D nonlinear transport equation based on "delayed diffusion" which results in bursty transport phenomena .

We found that in spite of "delayed diffusion" instability significantly modulate both temperature profile and the heat flux through the boundaries it does not alter averaged radiation loss. However, this counter-intuitive result needs to be verified for other values of delay time τ and the magnitude of radiation loss R_0.

Acknowledgments

Work performed under the project 2410P-E CONACyT at Universidad Nacional Autonoma de Mexico, Mexico, and by U. S. DOE grant DE-FG02-91ER-54109 at MIT

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