26^{th} EPS Conf. on Contr. Fusion and Plasma Physics, Maastricht, 14 - 18 June 1999 ECA Vol.23J (1999) 1289 - 1292

The Fast Wave Dissipation And Current Drive In Tokamak Plasmas

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Nowadays, fast waves (FW) with frequency of the order or higher then the ion cyclotron frequency ($\omega > \omega_{ci}$) are widely used for heating and current drive in modern tokamak experiments like JET and D-III-D (see, for example [1,2]). In this frequency range, the fast waves dissipate via electron Landau, transit time magnetic pumping (TTMP) and ion cyclotron resonance, what is beneficial to heat the plasma core. For efficient current drive, to avoid undesirable ion cyclotron dissipation, Fish and Karney[3] have suggested using low frequency fast waves $\omega < \omega_{ci}$; but parasitic mode conversion heating near plasma boundary may appear because of the Alfvén continuum. For large parallel phase velocities ($\omega/k_\parallel \approx 2 - 5v_{Te}$), dissipation on α-particles and on trapped electrons is practically absent in reactor plasmas and the current drive efficiency is very high[3] but FW Landau and TTMP damping may be exponentially small. Moreover, untrapped electrons, which move along tokamak magnetic field, suffer velocity modulation because of the poloidal inhomogeneity of the magnetic field. As a result, the Landau and TTMP damping may be modified by bounce resonance, $\omega_{b,p} \approx (p+nq_t)\sqrt{2\epsilon}v_T/(R_0\, q_t)$, dissipation of untrapped particles[4]. The effect of weak collisions is also important[5] because it may destroy phase correlation and can change the value of FW dissipation. In reality, this fast wave current drive is not yet studied experimentally.

Here, using the small toroidicity, $\epsilon = r/R_0 \ll 1$, we study the effect of weak collisions on bounce resonance dissipation of fast waves with the large phase velocity in tokamaks. The possibility of applications of fast wave current drive for Tokamak Chauffage Alfvén wave experiment in Brazil (TCABR) parameters[6] is proposed. We consider the pseudo-toroidal geometry, $R = R_0(1 + \epsilon\cos\theta), Z = r\sin\theta$, which is symmetric along toroidal coordinate ζ, in the Vlasov equation with the Landau-Fokker-Planck collisional operator. The distribution function of any plasma species α is represented as a sum of the Maxwell distibution F_0 and the oscillating part, $f_\alpha \sim \exp[\mathrm{i}\,(n\zeta - \omega\, t)]$, which is perturbed by a radiofrequency field (RF) with toroidal wavenumber n and frequency ω.

Because of small Larmour radius and Hall parameter $v_{Te}/\omega_{ce}, \omega/\omega_{ce} \ll 1$, we consider the drift approach of the Vlasov equation for electrons with accuracy of order ϵ,

$$\begin{aligned} &-\mathrm{i}\,\omega\bar{f}_s + sk_0v\sqrt{(1+\epsilon\cos\Theta-\lambda)}\left(\frac{\partial\bar{f}_s}{\partial\Theta} + \mathrm{i}nq_t\bar{f}_s\right) - \hat{C}_{e,i}[\bar{f}_s] = \\ &= -\frac{e\,vF_M}{T_e}\left[s\sqrt{1+\epsilon\cos\Theta-\lambda}E_3 - \frac{v\,\lambda}{2\omega_{ce0}}\left(\frac{1}{r}\frac{\partial}{\partial r}(rE_b) - \hat{k}_bE_r\right)\right] \end{aligned} \tag{1}$$

where $\hat{k}_b(E_j) = B_\zeta/rB\partial E_j/\partial\theta - \mathrm{i}nB_\theta E_j/(RB)$, $e = |e|$ is the modulus of the electron charge, $k_0 = B_\theta/rB$, and $q_t = rB_\zeta/R_0B_\theta$ is the tokamak stability parameter. In the above equation, we introduce a new variable related to the invariance of magnetic moment[4−5],

$$\lambda = \sin^2\gamma(1+\epsilon\cos\theta),\;\; \cos\gamma = s\sqrt{1-\lambda/(1+\epsilon\cos\theta)}$$

where the sign $s = \pm 1$ means positive or negative direction of a velocity of charged particles along magnetic field, $s = 1$ for $0 \le \gamma \le \pi/2$, and $s = -1$ for $\pi/2 \le \gamma \le \pi$. In

Eq.(1), the diffusion form of the collision operator over the velocity modulus v is assumed only because, due to the large wave phase velocity, the electron distribution function is strongly perturbed in the region $v \approx \omega/k_\parallel \gg v_{Te} = \sqrt{T_e/m_e}$,

$$\hat{C}_{e,i}[f_e] \approx \frac{\nu_e\, v_T^2}{v^2}\frac{\partial}{\partial v}\left(\frac{v_T^3}{v}\frac{\partial f}{\partial v} + f\right) \approx \frac{\nu_{ef}\, v_T^2}{v}\frac{\partial^2}{\partial v^2}\,(v\, f) \tag{2}$$

where $\nu_{ee} = 4\,\pi e^4 \Lambda N_0/\sqrt{m_e T_e^3}$ is electron collision frequency, Λ is the Coulomb logarithm, n_α and T_α are, respectively, the density and temperature of electrons or ions.

Following Refs.[4-5], we expand the parallel components of the electric field $\bar{E}_3$ and of the oscillating current $\bar{j}_3$ in Fourier series over Θ, which harmonics have the form,

$$j_{3,p} = j_{3,p}^u + j_{3,p}^t = -\frac{e}{2}\sum_s s\int_0^\infty v^3 \mathrm{d}v \int_{-\pi}^{\pi} \exp(-\mathrm{i}\,p\,\Theta)\left[\int_0^{1-\epsilon} \bar{f}_s^u \mathrm{d}\lambda + \int_{1-\epsilon}^{1+\epsilon\cos\Theta} \bar{f}_s^t \mathrm{d}\lambda\right]\mathrm{d}\Theta \tag{3}$$

where the current component is divided between the untrapped and trapped electrons.

The solving procedure of Eq.(1) depends on whether the electrons or ions are untrapped (circulating along magnetic field lines over magnetic surface of a tokamak) or trapped. Because of the large phase velocity, we will analyze only the untrapped part of the distribution function $\bar{f}_s^u$. This function is defined in the whole interval of the variable Θ, $-\pi \le \Theta \le \pi$ and the boundary conditions will be periodic, $\bar{f}_s^u(\pi) = \bar{f}_s^u(-\pi)$. In the case of weak collisions, the boundary conditions for Eq.(1) will be the same as in the collisionless case because electrons have many bounce-resonance oscillations before being scattered. Introducing the Jacobi functions as variables (see [4-5]), which are periodic, the above periodic conditions of $\bar{f}_s^u$ are fulfilled automatically over $w(\Theta) = \int_0^\Theta \mathrm{d}\theta/\sqrt{(1-\kappa^2\sin^2(\theta/2)}$ and $\kappa = \sqrt{2\epsilon/(1-\lambda+\epsilon)}$, the untrapped oscillating current in Eq. (3) can be rewritten in the form,

$$j_{3,p}^{(u)} = -4\pi\epsilon e\sum_s s\int_0^\infty v^3\mathrm{d}v\int_{\kappa_0}^1 \bar{f}_s^u\frac{\mathrm{d}\kappa}{\kappa^3} = -\frac{i\Omega}{4\pi}(\varepsilon_{33,p,j}^{(u)}E_{3,m} + \varepsilon_{31,p,j}^{(u)}E_{r,m} + \varepsilon_{32,p,j}^{(u)}E_{b,m}) \tag{4}$$

where the integral is realized over the untrapped particle region, and κ_0 is $\sqrt{2\epsilon}/\sqrt{1+\epsilon}$. Then, the factor in the right hand side of Eq. (1) is expanded into Fourier series,

$$\sum_{p=-\infty}^{\infty} C_{p,l,s}^{(u)}\exp[\mathrm{i}\ p\theta] = \exp\left[\mathrm{i}\,s\sqrt{2\epsilon}\frac{r\,\omega\tilde{Z}(w)}{\kappa v h_\theta} - 2\mathrm{i}\,nq_t[\tilde{\mathrm{am}}(w) - \epsilon\,\mathrm{sn}(w)\mathrm{cn}(w)] + \mathrm{i}\frac{l\pi\, w}{K(\kappa)}\right] \tag{5}$$

where $\tilde{Z}(w), \tilde{\mathrm{am}}(w), \mathrm{sn}(w)$, $\mathrm{cn}(w)$ are the Jacobi functions and K is first kind elliptic integral. Finally, Eq.(1) can be solved by the Laplace method[5] and the expression for the Fourier expansion of the parallel components of the dielectric tensor appears as:

$$\begin{aligned}\varepsilon_{33(u)}^{p,m} &= 4\frac{\epsilon\sqrt{2\epsilon}}{\pi}\omega_{pe}^2\sum_{r=-\infty}^{\infty}\int_{\kappa_0}^1\frac{K\,\mathrm{d}\kappa}{\kappa^4}\frac{C_{p,r}^{(u)}C_{m,r}^{(u)}}{[\bar{\omega}_{bu}^r]^2}\{1\\ &+\mathrm{i}\frac{\omega}{|\bar{\omega}_{bu}^r|}\int_0^\infty \mathrm{d}\eta(1-\eta^2)\exp\left[\frac{\mathrm{i}\,\omega\eta}{|\bar{\omega}_{bu}^r|} - \frac{\nu_{ef}\,\eta^3}{3|\bar{\omega}_{bu}^r|} - \frac{\eta^2}{2}\right]\bigg\}\end{aligned} \tag{6}$$

where $\omega_{pe} = \sqrt{4\pi N_0 e^2/m_e}$ is the electron plasma frequency, $\bar{\omega}_{bu}^r = \bar{\omega}_{bu}\,(r + n\,q_t)$ and $\bar{\omega}_{bu}$

are the bounce frequencies of untrapped particles when $v = v_T$. Using Jacobi function q-series[4] $C^u_{m,r} \approx J_r(Qb_m)$ in the collisionless case, the imaginary part of Eq.(6) is obtained,

$$\operatorname{Im}\varepsilon^{p,m}_{33(u)} = \int_{\kappa_0}^{1} \mathrm{d}\kappa \sum_{l=-\infty}^{\infty} \frac{4^4\sqrt{2\pi}\pi^5 e^2 \epsilon^6 q_t^5 R_0^5 \omega^3 n_e \,(b_m + 4\,l)^2}{v_T^5 h_\zeta^5 m_e K^4 \kappa^7 \,|nq_t + l + m|\, \Phi^4\, b_m^2} J_l(Q\,b_m)^2 \exp\left(\frac{-4\pi^2\eta^2 x \Omega^2 q_t^2 R_0^2}{(K\,v_T h_\zeta \kappa \Phi)^2}\right) \tag{7}$$

where Q, Φ and b_m are given by expressions:

$$Q = q + 2q^5 + 15q^9 + ...; \;\; q = 0.5[1-(1-\kappa^2)^{1/4}]/[1+(1-\kappa^2)^{1/4}];$$

$$b_m = 4\,nq_t - \frac{4\,\epsilon q_t n \pi^2}{\kappa^2 K^2} + \Phi + 4\,m, \;\; \Phi - \frac{2\,\pi^2 \epsilon\,(nq_t + l + m)}{K^2\,(\kappa^2 - \epsilon + \epsilon E/K + \epsilon\kappa^2/2)} \tag{8}$$

In this case, the criteria of collisionless dissipation[5] is

$$\nu_{ef} = \nu_{ee}(v_{Te}/v_{ph})^2 \ll \omega^6/|\bar{\omega}^{(m)}_{bu}|^5 \exp(-\omega^2/\bar{\omega}^{(m)\,2}_{bu}) \tag{9}$$

We have also carried out numerical calculations of the imaginary part of ε^{mm}_{33}, followed by a comparison between our result (toroidal geometry) and the cylindrical result. This comparison was done for typical values of the toroidicity and of the parallel and poloidal wave numbers and different ratios between the phase and thermal velocities. The corresponding results are presented in Fig.1 where $\operatorname{Im}(\varepsilon^{mm}_{33})$ is normalized to the cylindrical Landau damping $\epsilon = 0$.

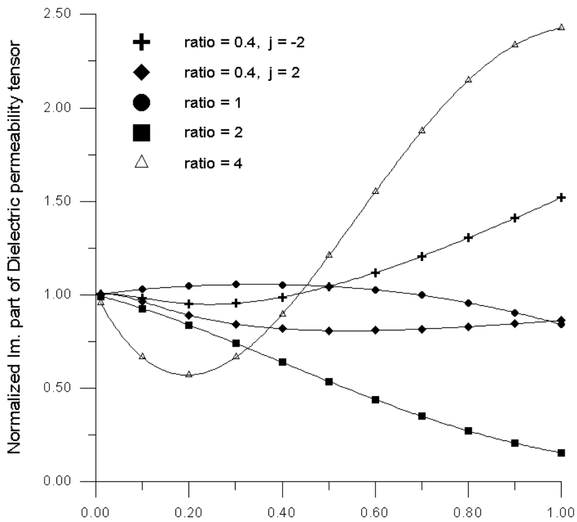

Fig.1.*Plot of the normalized of* $Im\,\varepsilon_{33}$ *against the normalized radius* $x = r/a$*, for the normalized parameters:* $v_{ph}/v_{Te} = 0.4, 1.0, 2.0, 4.0$*, parallel wave number* $|n\,q_t + j| = 0.6$*, poloidal* $j = \pm 2$ *and toroidal* $n = 2$ *wave numbers, and toroidicity* $a/R_0 = 1/3$.

Discussion and Conclusion. The parallel component of the permittivity tensor is the most important, for the analysis of the wave dissipated power and current drive,

$$P^{(e)}_m \approx Re(\tilde{j}_\| E^*_\|) \approx \frac{\omega}{8\pi} \sum_p \operatorname{Im}(\epsilon^{mp}_{33})|E_{\|,p}|^2, \;\; j^{RF}_\| = \frac{\sigma_\|\, k_\|}{e\,n_{e0}\omega} P^{(e)}_m \tag{10}$$

where $\sigma_\parallel$ is the plasma parallel conductivity. The fast wave dissipation profile[2] is usually peaked strongly because of wave field distribution. Accounting the untrapped bounce resonance effect shown in Fig.1, we find that the profile of dissipated wave power is broadened for small parallel wavenumbers. In Fig.2, using a kinetic code[6], we show the fast wave dissipation profile in TCABR for electron Landau damping in the limit $\epsilon = 0$, bounce-resonance damping, and total damping taking into account collisions. We use the TCABR parameters: $a = 18\,cm$, $b = 18.5cm$, $d = 23\,cm$, and $R_0 = 61\,cm$; parabolic density and square parabolic temperature profiles assumed where the central density, and electron and ion temperatures are $n_{e0} = 7.10^{13}\text{cm}^{-3}, T_{e0} = 500\ eV$ and $T_{i0} = 300\,eV$, respectively, while current profile is cubic parabolic with the tokamak safety parameter $q_0 = 1.1$ and the toroidal magnetic field $B_0 = 10\ kG$ in the plasma center.

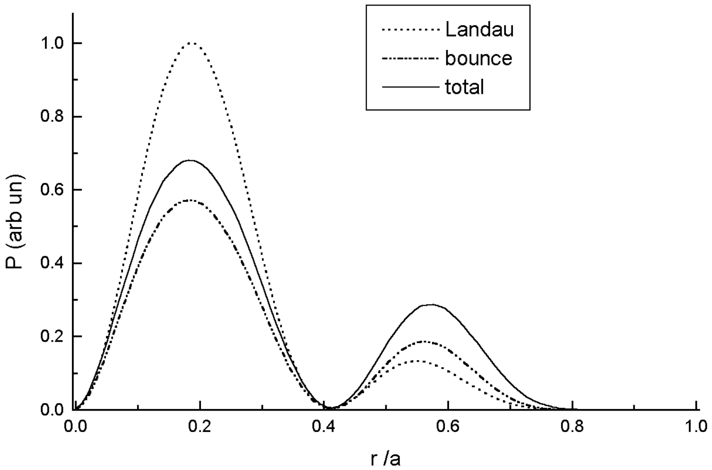

Fig.2. *Plot of the fast wave dissipation profile normalized on maximum of Landau damping against the normalized radius* $x = r/a$*, for frequency f=12.7 MHz and the normalized parameters:* $v_{ph}/v_{Te0} = 4.0$*, poloidal* $m = -1$ *and toroidal* $n = 2$ *wave numbers,* $|n\,q_t + m| = 1.2$*, and toroidicity* $a/R_0 = 0.3$*.*

In synthesis, for waves in toroidal plasmas, it may be concluded that for large phase velocities ($v_{ph}/v_{Te} \geq 3.5$) and small parallel wave numbers $|n\,q_t + l| < 1.5$, the total contribution of the untrapped electrons to the collisionless dissipation of the fast waves increases with the growing of the toroidicity parameter ($\epsilon > 0.1$).

We find also that the fast wave dissipation profile broadens because of untrapped electron bounce resonance damping and weak collision effects. These effects, which are valid for waves with $k_\parallel R_0 \leq 1$, increase both with the minor radius and are beneficial for fast wave current drive. A broadening of the profile of the fast wave current drive as compared to width predicted by Landau and TTMP damping is found experimentally in D-III-D[2] and may be explaned by the bounce resonance effect.

This work was supported by the University of São Paulo and FAPESP (Foundation of the State of São Paulo for the Support of Research)/Brazil.

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